Accepted Manuscript

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PII: S0142-9418(18)30043-6

DOI: [10.1016/j.polymertesting.2018.02.024](https://doi.org/10.1016/j.polymertesting.2018.02.024)

Reference: POTE 5348

To appear in: *Polymer Testing*

Received Date: 10 January 2018

Accepted Date: 20 February 2018

Please cite this article as: D. Pulungan, A. Yudhanto, S. Goutham, G. Lubineau, R. Yaldiz, W. Schijve, Characterizing and modeling the pressure- and rate-dependent elastic-plastic-damage behaviors of polypropylene-based polymers, *Polymer Testing* (2018), doi: 10.1016/j.polymertesting.2018.02.024.

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Characterizing and modeling the pressure- and rate-dependent elastic-plastic-damage behaviors of polypropylene-based polymers

Ditho Pulungan¹, Arief Yudhanto¹, Shiva Goutham¹, Gilles Lubineau^{1,*}, Recep Yaldiz²,
Warden Schijve²

¹King Abdullah University of Science and Technology (KAUST), Physical Science and Engineering Division,
COHMAS Laboratory, Thuwal 23955-6900, Saudi Arabia

²SABIC, T&I Composites, P.O Box 319, 6160 AH Geleen, the Netherlands

*Corresponding author's e-mail: gilles.lubineau@kaust.edu.sa

Abstract

Polymers in general exhibit pressure- and rate-dependent behavior. Modeling such behavior requires extensive, costly and time-consuming experimental work. Common simplifications may lead to severe inaccuracy when using the model for predicting the failure of structures. Here, we propose a viscoelastic viscoplastic damage model for polypropylene-based polymers. Such a set of constitutive equations can be used to describe the response of polypropylene under various strain-rates and stress-triaxiality conditions. Our model can also be applied to a broad range of thermoplastic polymers. We detail the experimental campaign that is needed to identify every parameter of the model at best. We validated the proposed model by performing 3-point bending tests at different loading speeds, where the load-displacement response of polypropylene beam up to failure was accurately predicted.

Keywords: viscoelasticity, viscoplasticity, damage model, stress relaxation, impact polypropylene

1. Introduction

Thermoplastic polymers are widely used in applications requiring materials to have excellent impact performance, such as automotive structures and electronic components or packaging of portable devices. Improving the impact performance of thermoplastics is achieved by reinforcing them with talc particles [1] or fibers (short [2], long [3] or continuous ones [4–6]).

One of the most widely used thermoplastics is polypropylene (PP); its abundance and low cost make it the ideal candidate for industrial production. However, the behavior of PP is very complex due to its strong dependency on strain-rate, stress triaxiality (pressure), temperature and humidity [7–10]. Therefore, the predictive modeling of its behavior requires complex mathematical formulations due to those dependencies. These

dependencies are necessary to be accounted for since a macroscopically uniaxial stress-state will result in complex stress-states in the polymer matrix when reinforcement is present. In this scenario, such a complex model is required to predict the failure of PP-based composites accurately [11].

The viscoelastic viscoplastic behavior of PP has been widely studied in the literature. For example, Ariyama [12–14] performed experiments to study the viscoelastic relaxation of PP. Recently, Tscharnuter *et al.* [15] experimentally studied the uniaxial nonlinear viscoelastic and viscoplastic behavior of PP. They proposed a modeling approach using the Schapery viscoelastic and Perzyna viscoplastic models that they fit based on the stress relaxation and strain recovery data. Drozdov and Gupta [16,17] researched the nonlinear viscoelastic viscoplastic behavior of isotactic PP and its behavior concerning different temperature.

PP also experiences the stress triaxiality (pressure)-dependent yield strength [18–20]. It means that the yield strength of PP increases as the pressure increases which results in a higher yield strength in compression compared to the one in tension [21,22]. In their studies, the pressure-dependent yield strength of PP is best fitted by linear Drucker-Prager yield function. In line with this pressure-dependent behavior, the volume change during plastic strain was also found. Unlike metals, which do not experience volume change during plastic deformation, the volume change may occur during the plastic deformation (known as non-isochoric plastic deformation) of PP. Various authors have studied this phenomenon by measuring the volumetric strain of PP using digital image correlation (DIC) [1,23,24]. However, due to the different dependency between the yield strength and plastic volume change with respect to pressure, non-associative flow function is usually used [1,24].

Upon extensive plastic straining, the cavities or micro-voids phenomenon start to appear in PP. The cavities and micro-voids usually take place in the amorphous phase between the crystalline lamellae [25]. G'Sell *et al.* [23,26] have researched the influence of voids and crazing on the apparent plastic deformation. They showed that the plastic deformation of some polymers might entirely be governed by crazing and cavitation process. In terms of apparent global behavior, the presence of crazing and cavitation causes gradual degradation of the elastic stiffness and therefore, results in the stress softening. One of the most common techniques used to take into account the effect of damage on the degradation of the global behavior of polymers is continuum damage mechanics (CDM) [27,28]. It is based on the thermodynamics of the irreversible processes in which damage variable D is used to represent the “knock-down” factor that degrades the apparent macroscopic properties e.g., elastic stiffness tensor. The damage variable D can be either a scalar for isotropic damage as usually found in polymers [29] or a second/fourth order tensor for anisotropic damage as usually found in composites [30].

The material parameters for PP have been calibrated for uniaxial loading case, in which Schapery viscoelastic and Perzyna viscoplastic models were considered [15]. The modeling approach for PP that considers the cooling rate effect (degree crystallinity) was proposed by Parenteau [31]. Modeling of damage process in PP compound was studied by Temimi-Maaref *et al.* [32]. However, up to authors' knowledge, the complete experimental calibration and validation for PP that considers viscoelastic, viscoplastic and damage up to failure have not been investigated.

In this paper, we propose a phenomenological constitutive model that takes into account viscoelasticity (rate-dependent elasticity), viscoplasticity (rate-dependent plasticity) with pressure-dependent yield strength, non-isochoric plastic deformation and damage of neat polypropylene up to failure. To that end, a viscoelastic model based on generalized Maxwell model is coupled with Johnson-Cook viscoplasticity, hyperbolic Drucker-Prager yield criterion, and non-associative flow function. The model is then combined with plasticity-based scalar continuum damage model (CDM) for simulating the final failure of PP. We discussed in detail the identification procedure of the material parameters and the required experimental work. The implementation aspect of the model is explained in detail, and the model was implemented in a user-defined material subroutine (UMAT) for an implicit integration scheme of Abaqus Standard V6.12 [33]. Finally, the proposed model was applied to predict the behavior of polypropylene under 3-point-bending at two different loading speeds up to the failure point.

2. Proposed constitutive models

2.1. Decomposition of stress and strain

In this research, the PP copolymer under consideration exhibits a limited range of strain to failure (below 5%). Therefore, the proposed model adopts an infinitesimal strain framework. We additively decomposed the total strain tensor $\boldsymbol{\varepsilon}$ into a viscoelastic strain $\boldsymbol{\varepsilon}^{ve}$ and a viscoplastic strain $\boldsymbol{\varepsilon}^{vp}$ as follows

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{ve} + \boldsymbol{\varepsilon}^{vp} \quad (1)$$

[Figure 1 here]

Cauchy stress tensor $\boldsymbol{\sigma}$ is decomposed as follows

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}^{dev} + \boldsymbol{\sigma}^{vol} \quad (2)$$

where $\boldsymbol{\sigma}^{dev}$ and $\boldsymbol{\sigma}^{vol}$ are the deviatoric and volumetric stress contributions. Hydrostatic pressure p , or simply, pressure is defined as follows:

$$p = -\frac{\sigma_{kk}}{3} \quad (3)$$

and von-Mises stress q is defined as

$$q = \sqrt{\frac{3}{2} \boldsymbol{\sigma}^{dev} : \boldsymbol{\sigma}^{dev}} \quad (4)$$

We define the stress triaxiality as

$$\eta = -p/q \quad (5)$$

2.2. Viscoelasticity

We followed the generalized Maxwell viscoelastic model as formulated by Kaliske and Rothert [34]. It can be considered as a parallel arrangement between elastic spring and several Maxwell elements as shown in Figure 1. We assume that viscoelasticity only occurs due to deviatoric deformation. Therefore, the volumetric stress can be calculated as follows

$$\boldsymbol{\sigma}^{vol} = 3K_0 \boldsymbol{\epsilon}^{ve,vol} \quad (6)$$

where $\boldsymbol{\epsilon}^{ve,vol} = \frac{1}{3} \epsilon_{kk}^{ve} \mathbf{I}$ is the volumetric viscoelastic strain tensor, K_0 is rate-independent bulk modulus, and $\mathbf{I}$ is the second-order identity tensor. Meanwhile, the deviatoric stress can be calculated as follows

$$\boldsymbol{\sigma}^{dev} = \boldsymbol{\sigma}_0^{dev} + \sum_{m=1}^M \mathbf{h}_m \quad (7)$$

where $\boldsymbol{\sigma}_0^{dev} = 2G_0 \boldsymbol{\epsilon}^{ve,dev}$ is the deviatoric stress contribution from the elastic spring, $\boldsymbol{\epsilon}^{ve,dev}$ is the deviatoric viscoelastic strain tensor, $\mathbf{h}_m$ is the deviatoric stress contribution from the m -th Maxwell branch in the viscoelastic model, and M is the total number of Maxwell branch required to fit the stress relaxation response of PP.

Let us now consider the integration over the time interval $[t^{[n]}, t^{[n+1]}]$ (time step $\Delta t = t^{[n+1]} - t^{[n]}$) of the response of viscoelastic network. The deviatoric stress at $t^{[n+1]}$ at m -th Maxwell element is then as follows

$$\mathbf{h}_m^{[n+1]} = \mathbf{h}_m^{[n]} \exp\left(-\frac{\Delta t}{\tau_m}\right) + \frac{\gamma_m \left(1 - \exp\left(-\frac{\Delta t}{\tau_m}\right)\right)}{\frac{\Delta t}{\tau_m}} \left(\boldsymbol{\sigma}_0^{dev[n+1]} - \boldsymbol{\sigma}_0^{dev[n]}\right) \quad (8)$$

where τ_m and $\gamma_m = G_m/G_0$ are the relaxation time and shear stiffness ratio of the m -th Maxwell element (see Figure 1).

In this case, the tangential viscoelastic tensor can be separated as a volumetric and deviatoric contributions expressed as

$$\mathbf{C}^{ve} = K_0 \mathbf{i} \otimes \mathbf{i} + 2G_0 \left\{ 1 + \sum_{m=1}^M \frac{\gamma_m \left(1 - \exp\left(-\frac{\Delta t}{\tau_m}\right) \right)}{\frac{\Delta t}{\tau_m}} \right\} \mathbf{P}^{dev} \quad (9)$$

where $\mathbf{P}^{dev} = \mathbf{I} - \frac{1}{3}(\mathbf{i} \otimes \mathbf{i})$ is the fourth order deviatoric projection tensor and $\mathbf{I}$ is the fourth order identity tensor. For interested readers, we refer to Ref. [34] for all related viscoelasticity derivations.

2.3. Viscoplasticity

A hyperbolic Drucker-Prager (hDP) yield function and Johnson-Cook rate-dependent hardening model [33] are coupled to represent the non-recoverable time-dependent (viscoplastic) and pressure-dependent deformation of PP. The hDP yield function can be expressed as follows

$$f = \sqrt{q^2 + (d_0 - p_{t0} \tan \beta)^2} - (p \tan \beta + d) \quad (10)$$

where p and q are the already introduced hydrostatic pressure and von Mises equivalent stress, respectively.

d is the yield strength hardening at pure shear (also known as cohesion strength) and it can be identified based on the uniaxial tension yield stress σ_t as follows

$$d = \sqrt{(d_0 - p_{t0} \tan \beta)^2 + \sigma_t^2} + \frac{1}{3} \sigma_t \tan \beta \quad (11)$$

d_0 is the initial value of d , p_{t0} is the strength of PP at ideal triaxial hydrostatic loading (i.e. $\sigma_{11} = \sigma_{22} = \sigma_{33} = \sigma_t$ and $\sigma_{12} = \sigma_{13} = \sigma_{23} = 0$) as shown as the location of the apex of the yield surface in the $q - p$ plane, and β is a material parameter that controls the dependency of the yield strength with respect to pressure.

[Figure 2 here]

Figure 2 shows the illustration of hDP yield surface. The value of d_0 can be calculated as follow

$$d_0 = \frac{\left(\frac{1}{9} \tan^2 \beta - 1\right) \sigma_{t0}^2 - p_{t0}^2 \tan^2 \beta}{\left(\frac{2}{3} \sigma_{t0} - 2p_{t0}\right) \tan \beta} \quad (12)$$

To take into account the isotropic hardening and viscoplasticity of PP, following Johnson-Cook rate-dependent hardening model, the tensile yield strength σ_t is now a function of equivalent viscoplastic strain $\bar{\epsilon}^{vp}$ and equivalent viscoplastic strain-rate $\dot{\bar{\epsilon}}^{vp}$ that is

$$\sigma_t = \sigma_{t0} \left(1 + \mu \ln \left(\frac{\dot{\bar{\epsilon}}^{vp}}{c} \right) \right) \quad (13)$$

where σ_{t0} is static yield stress as function of equivalent viscoplastic strain $\bar{\epsilon}^{vp}$ which can be input as tabular form while c and μ are two material parameters fitting the rate-dependent hardening.

A non-associative flow rule was used to capture the plastic volume change accurately. In this study, the following flow rule g is used.

$$g = \sqrt{q^2 + (\epsilon \tan \psi)^2} - p \tan \psi \quad (14)$$

where $\epsilon = (d_0 - p_{t0} \tan \beta) / \tan \beta$ is a parameter, referred to as the eccentricity and ψ is the dilation angle at high pressure p . Then, the viscoplastic strain increment can be calculated from a flow rule as follows

$$\dot{\bar{\epsilon}}^{vp} = \dot{\lambda} \frac{\frac{\partial g}{\partial \sigma}}{\left| \frac{\partial g}{\partial \sigma} \right|} = \dot{\lambda} \mathbf{n} \quad (15)$$

$\dot{\lambda}$ is the scalar viscoplastic multiplier obtained from viscoplastic consistency conditions, and $\mathbf{n}$ is the direction of the viscoplastic flow. In this study, we calibrated the value of ψ by performing FEM simulation on a single cubic element so that the plastic Poisson's ratio between FEM results and DIC results on the tested sample are reasonable. Similar procedure has also been explained by Epee *et al.* [1].

The direction of viscoplastic flow based on hyperbolic Drucker-Prager can be expressed as follows

$$\mathbf{n} = \mathbf{n}^{dev} + \mathbf{n}^{vol} = \frac{3}{2} \frac{\sigma^{dev}}{AB} + \frac{1}{3} \frac{\tan \psi}{B} \mathbf{i} \quad (16)$$

where $\mathbf{n}^{dev}$ and $\mathbf{n}^{vol}$ are deviatoric and volumetric part of the viscoplastic flow direction, respectively, and

$$A = \sqrt{(\epsilon \tan \psi)^2 + q^2} \quad (17)$$

$$B = \sqrt{\frac{3}{2} \frac{q^2}{(\epsilon \tan \psi)^2 + q^2} + \frac{\tan^2 \psi}{3}} \quad (18)$$

2.4. Plasticity-based scalar damage model

Isotropic damage for the PP is modelled following the standard formulations in Abaqus [33]. In this model, the damage starts to propagate when the scalar quantity ω reaches 1, where ω is defined as:

$$\omega = \int_0^t \frac{\dot{\bar{\epsilon}}^{vp}}{\bar{\epsilon}_{init}^{vp}(\eta, \bar{\epsilon}^{vp})} dt \quad (19)$$

where $\bar{\epsilon}^{vp}$ is defined later in the next section. The damage initiation criterion $\bar{\epsilon}_{init}^{vp}$ can be function of triaxiality and strain-rate to take into account the triaxiality- and rate-dependent damage often found in PP. In our paper, only the triaxiality-dependent damage of PP was taken into account by setting different damage initiation values for each loading condition. This value was identified in tension ($\eta = 1/3$). Based on our previous expertise, we multiplied this value by 3 for pure shear configurations ($\eta = 0$) and by 7 for uniaxial compression configurations ($\eta = -1/3$) based on the proprietary data and these values were in a good approximation of the results of the experimental data reported by Jerabek *et al.* [22]. Upon damage initiation, a scalar damage variable D is used to degrade the elastic stiffness of the material. We assumed that damage starts at the maximum stress in the stress-strain curve. To include the damage model into the viscoelastic viscoplastic model in the Eqs. (6), (7), and (10), we are using the classical effective stress tensor $\tilde{\sigma}$ and effective strain tensor $\tilde{\epsilon}$ defined as

$$\tilde{\sigma} = \frac{\sigma}{1 - D} \quad \text{and} \quad \tilde{\epsilon} = \epsilon \quad (20)$$

It is widely known that when damage starts, the classical continuum damage mechanics framework suffers from localization problems. This means that the energy required to create additional damage is proportional to the size of the element and as the size approaches zero, the dissipated energy due to damage approaches zero. It also results in spurious mesh dependency. These phenomena are unphysical and must be avoided. To

ensure mesh-objective simulations, the characteristic length of the element L_c was introduced to the fracture energy expression:

$$G_f = \int_{\bar{\varepsilon}^{vp}|D=0}^{\bar{\varepsilon}^{vp}|D=1} L_c \sigma_t d\bar{\varepsilon}^{vp} = \int_0^{\bar{u}^{vp}|D=1} \sigma_t d\bar{u}^{vp} \quad (21)$$

For this, instead of working with the stress-strain relationship, the stress vs. relative plastic displacement concept is used such that

$$D = D(\bar{u}^{vp}) \quad \text{where} \quad \bar{u}^{vp} = L_c(\bar{\varepsilon}^{vp} - \bar{\varepsilon}_{|D=0}^{vp}) \quad (22)$$

Therefore, one needs to supply the evolution of D with respect to effective plastic displacement $\bar{u}^{vp}$. In this particular case, a simple linear evolution of D w.r.t. $\bar{u}^{vp}$ is assumed due to the brittleness of the material. Hence, the evolution of damage is fully defined by $\bar{u}_{crit}^{vp}$ i.e., the value of $\bar{u}^{vp}$ at $D = 1$.

2.5. Numerical integration

The objective of the integration algorithm is to calculate the updated stress tensor at time $t^{[n+1]}$, given a set of state variables from the previous converged increment $t^{[n]}$ and a total strain increment $\Delta \varepsilon$.

First, let us integrate Eq. (15) over the increment Δt by using a Backward Euler scheme. Assuming ε^{vp} varies linearly over Δt , the viscoplastic strain increment can be expressed as follows

$$\Delta \varepsilon^{vp} = \dot{\lambda}^{[n+1]} \Delta t \mathbf{n}^{[n+1]} \quad (23)$$

$\mathbf{n}^{[n+1]}$ can be calculated by Eq. (16) and $\dot{\lambda}^{[n+1]}$ can be obtained by solving consistency condition i.e., both f and Δf are zero during the viscoplastic yielding. During the viscoplastic yielding, the change in yield function can be expressed in terms of three factors: the change of deviatoric stress $\Delta \sigma^{dev}$, the change in pressure Δp , and the change in the yield strength $\Delta \sigma_t$ itself as expressed below

$$\Delta f = \frac{\partial f}{\partial \sigma^{dev}} : \Delta \sigma^{dev} + \frac{\partial f}{\partial p} \Delta p + \frac{\partial f}{\partial \sigma_t} \Delta \sigma_t \quad (24)$$

$$\begin{aligned} \Delta f = & \frac{3}{2} \frac{\sigma^{dev}}{\sqrt{(\epsilon \tan \beta)^2 + q^2}} : \Delta \sigma^{dev} - \tan \beta \Delta p \\ & - \left(\frac{\sigma_t}{\sqrt{(\epsilon \tan \beta)^2 + \sigma_t^2}} + \frac{\tan \beta}{3} \right) \Delta \sigma_t \end{aligned} \quad (25)$$

Next, we need to evaluate $\Delta\sigma^{dev}$, Δp , and $\Delta\sigma_t$ as follows

$$\begin{aligned}\Delta\sigma^{dev} &= 2G_0 \left\{ 1 + \sum_{m=1}^M \frac{\gamma_m \left(1 - \exp\left(-\frac{\Delta t}{\tau_m}\right) \right)}{\frac{\Delta t}{\tau_m}} \right\} \Delta\epsilon^{ve,dev} = 2G_t \Delta\epsilon^{ve,dev} \\ &= 2G_t (\Delta\epsilon^{dev} - \Delta\epsilon^{vp,dev}) = 2G_t \Delta\epsilon^{dev} - \frac{3G_t \Delta t \sigma^{dev}}{AB} \dot{\lambda}^{[n+1]}\end{aligned}\quad (26)$$

$$\Delta p = -K_0 \Delta\epsilon_{kk}^{vp} = -K_0 (\Delta\epsilon_{kk} - \Delta\epsilon_{kk}^{vp}) = -K_0 \Delta\epsilon_{kk} + \frac{K_0 \tan \psi \Delta t}{B} \dot{\lambda}^{[n+1]}\quad (27)$$

The change in tensile yield strength can be calculated as follows

$$\Delta\sigma_t = \frac{\partial\sigma_t}{\partial\bar{\epsilon}^{vp}} \Delta\bar{\epsilon}^{vp} + \frac{\partial\sigma_t}{\partial\dot{\bar{\epsilon}}^{vp}} \Delta\dot{\bar{\epsilon}}^{vp} = H \Delta\bar{\epsilon}^{vp} + H' \Delta\dot{\bar{\epsilon}}^{vp}\quad (28)$$

where H and H' are strain- and strain-rate-hardening, respectively. Since the change of yield strength $\Delta\sigma_t$ depends on the change of equivalent viscoplastic strain and equivalent viscoplastic strain-rate, we need to define the equivalent viscoplastic strain rate $\dot{\bar{\epsilon}}^{vp}$ first.

First, we need a variable for equivalent viscoplastic strain-rate that matches the viscoplastic strain-rate in uniaxial testing, i.e., $\dot{\bar{\epsilon}}^{vp} = \dot{\epsilon}_{11}^{vp}$. The definition of equivalent viscoplastic strain-rate can be taken by involving a constant α to be identified such that,

$$\dot{\bar{\epsilon}}^{vp} = \alpha \sqrt{\dot{\epsilon}^{vp} : \dot{\epsilon}^{vp}} = \alpha \dot{\lambda} \sqrt{\mathbf{n} : \mathbf{n}} = \alpha \dot{\lambda} \sqrt{\mathbf{n}^{dev} : \mathbf{n}^{dev} + \mathbf{n}^{vol} : \mathbf{n}^{vol}}\quad (29)$$

Note that $\mathbf{n}^{dev} : \mathbf{n}^{vol} = \mathbf{n}^{vol} : \mathbf{n}^{dev} = 0$. With straightforward algebraic calculation, we find that

$$\mathbf{n}^{dev} : \mathbf{n}^{dev} = \frac{3}{2} \frac{q^2}{A^2 B^2}\quad (30)$$

and

$$\mathbf{n}^{vol} : \mathbf{n}^{vol} = \frac{1}{3} \frac{\tan^2 \psi}{B^2}\quad (31)$$

Therefore, Eq. (29) can be expressed as follows

$$\dot{\bar{\epsilon}}^{vp} = \alpha \dot{\lambda} \sqrt{\frac{3}{2} \frac{q^2}{A^2 B^2} + \frac{1}{3} \frac{\tan^2 \psi}{B^2}} \quad (32)$$

The viscoplastic strain rate in the first direction during uniaxial tensile test is

$$\dot{\epsilon}_{11}^{vp} = \dot{\lambda} n_{11} = \dot{\lambda} \left(\frac{3}{2} \frac{\sigma_{11}^{dev}}{AB} + \frac{1}{3} \frac{\tan \psi}{B} \right) \quad (33)$$

where $\sigma_{11}^{dev} = \frac{2}{3} q$ during uniaxial tensile load. Therefore,

$$\dot{\epsilon}_{11}^{vp} = \dot{\lambda} \left(\frac{q}{AB} + \frac{1}{3} \frac{\tan \psi}{B} \right) \quad (34)$$

and finally, it leads to

$$\alpha = \frac{\left(\frac{q}{AB} + \frac{1}{3} \frac{\tan \psi}{B} \right)}{\sqrt{\frac{3}{2} \frac{q^2}{A^2 B^2} + \frac{1}{3} \frac{\tan^2 \psi}{B^2}}} \quad (35)$$

Note that for $\psi = 0$, the value of $\alpha = \sqrt{\frac{2}{3}}$ that returns to the value for von Mises flow function. In this model, the relation between equivalent viscoplastic strain rate and scalar viscoplastic multiplier is expressed as follows

$$\dot{\bar{\epsilon}}^{vp} = \dot{\lambda} \left(\frac{q}{AB} + \frac{1}{3} \frac{\tan \psi}{B} \right) \quad (36)$$

By integrating the equation above, the equivalent viscoplastic strain increment is

$$\Delta \bar{\epsilon}^{vp} = \left(\frac{q}{AB} + \frac{1}{3} \frac{\tan \psi}{B} \right) \dot{\lambda}^{[n+1]} \Delta t \quad (37)$$

and the change in equivalent viscoplastic strain rate is

$$\Delta \dot{\bar{\epsilon}}^{vp} = \left(\frac{q}{AB} + \frac{1}{3} \frac{\tan \psi}{B} \right) \dot{\lambda}^{[n+1]} \quad (38)$$

Now we can evaluate the increment in yield strength given by

$$\Delta\sigma_t = \frac{\partial\sigma_t}{\partial\bar{\epsilon}^{vp}} \Delta\bar{\epsilon}^{vp} + \frac{\partial\sigma_t}{\partial\dot{\bar{\epsilon}}^{vp}} \Delta\dot{\bar{\epsilon}}^{vp} = \left(\frac{q}{AB} + \frac{1}{3} \frac{\tan\psi}{B} \right) (H \Delta t + H') \dot{\lambda}^{[n+1]} \quad (39)$$

Substituting Eqs. (26), (27) and (39) into Eq. (25), we then get the following expression

$$\begin{aligned} \Delta f = & \frac{3}{2} \frac{\sigma^{dev}}{\sqrt{(\epsilon \tan \beta)^2 + q^2}} \\ & \left(2G_t \Delta \epsilon^{dev} - \frac{3G_t \Delta t \sigma^{dev}}{AB} \dot{\lambda}^{[n+1]} \right) \\ & - \tan \beta \left(-K_0 \Delta \epsilon_{kk} + \frac{K_0 \tan \psi \Delta t}{B} \dot{\lambda}^{[n+1]} \right) \\ & - \left(\frac{\sigma_t}{\sqrt{(\epsilon \tan \beta)^2 + \sigma_t^2}} + \frac{\tan \beta}{3} \right) \\ & \left(\frac{q}{AB} + \frac{\tan \psi}{3B} \right) \left(H + \frac{H'}{\Delta t} \right) \dot{\lambda}^{[n+1]} \Delta t \end{aligned} \quad (40)$$

Finally, to perform Newton iteration to obtain $\dot{\lambda}^{[n+1]}$ that satisfy consistency condition, the $\frac{\partial \Delta f}{\partial \dot{\lambda}^{[n+1]}}$ is required. It is obtained by taking derivative Eq. (40) w.r.t. $\dot{\lambda}^{[n+1]}$ as follows

$$\begin{aligned} \frac{\partial \Delta f}{\partial \dot{\lambda}^{[n+1]}} = & -3G_t \frac{q^2}{AB \sqrt{(\epsilon \tan \beta)^2 + q^2}} \Delta t \\ & - \frac{K_0 \tan \beta \tan \psi}{B} \Delta t \\ & - \left(\frac{\sigma_t}{\sqrt{(\epsilon \tan \beta)^2 + \sigma_t^2}} + \frac{\tan \beta}{3} \right) \left(\frac{q}{AB} + \frac{\tan \psi}{3B} \right) \left(H + \frac{H'}{\Delta t} \right) \Delta t \end{aligned} \quad (41)$$

2.6. Consistent tangent stiffness tensor

In this section, we derived consistent tangent stiffness tensor $\frac{\partial \Delta \sigma}{\partial \Delta \epsilon}$. The tangent stiffness matrix will be decomposed into the deviatoric and hydrostatic parts as below

$$\frac{\partial \Delta \boldsymbol{\sigma}}{\partial \Delta \boldsymbol{\varepsilon}} = \left[\frac{\partial \Delta \boldsymbol{\sigma}^{dev}}{\partial \Delta \boldsymbol{\varepsilon}^{dev}} + \frac{\partial \Delta \boldsymbol{\sigma}^{vol}}{\partial \Delta \boldsymbol{\varepsilon}^{dev}} \right] : \frac{\partial \Delta \boldsymbol{\varepsilon}^{dev}}{\partial \Delta \boldsymbol{\varepsilon}} + \left[\frac{\partial \Delta \boldsymbol{\sigma}^{dev}}{\partial \Delta \boldsymbol{\varepsilon}^{vol}} + \frac{\partial \Delta \boldsymbol{\sigma}^{vol}}{\partial \Delta \boldsymbol{\varepsilon}^{vol}} \right] : \frac{\partial \Delta \boldsymbol{\varepsilon}^{vol}}{\partial \Delta \boldsymbol{\varepsilon}} \quad (42)$$

where

$$\frac{\partial \Delta \boldsymbol{\varepsilon}^{dev}}{\partial \Delta \boldsymbol{\varepsilon}} = \mathbf{P}^{dev} \quad (43)$$

and

$$\frac{\partial \Delta \boldsymbol{\varepsilon}^{vol}}{\partial \Delta \boldsymbol{\varepsilon}} = \frac{1}{3} (\mathbf{i} \otimes \mathbf{i}) \quad (44)$$

where $\mathbf{P}^{dev}$ and $\mathbf{i}$ were previously defined. Substituting Eqs. (43) and (44) into Eq. (42) results in

$$\begin{aligned} \frac{\partial \Delta \boldsymbol{\sigma}}{\partial \Delta \boldsymbol{\varepsilon}} &= \frac{\partial \Delta \boldsymbol{\sigma}^{dev}}{\partial \Delta \boldsymbol{\varepsilon}^{dev}} : \mathbf{P}^{dev} + \frac{\partial \Delta \boldsymbol{\sigma}^{vol}}{\partial \Delta \boldsymbol{\varepsilon}^{dev}} : \mathbf{P}^{dev} + \frac{1}{3} \left(\frac{\partial \Delta \boldsymbol{\sigma}^{dev}}{\partial \Delta \boldsymbol{\varepsilon}^{vol}} \right) : (\mathbf{i} \otimes \mathbf{i}) \\ &\quad + \frac{1}{3} \left(\frac{\partial \Delta \boldsymbol{\sigma}^{vol}}{\partial \Delta \boldsymbol{\varepsilon}^{vol}} \right) : (\mathbf{i} \otimes \mathbf{i}) \end{aligned} \quad (45)$$

By algebraic calculation, we can finally express the tangent stiffness matrix for the viscoelastic viscoplastic problem as follows

$$\begin{aligned} \frac{\partial \Delta \boldsymbol{\sigma}}{\partial \Delta \boldsymbol{\varepsilon}} &= \left[2G_t \mathbf{I} + \left(K_0 - \frac{2G_t}{3} \right) \mathbf{i} \otimes \mathbf{i} \right] - \frac{9G_t^2 \Delta t}{AB\lambda_{den} \sqrt{(\epsilon \tan \beta)^2 + q^2}} \boldsymbol{\sigma}^{dev} \otimes \boldsymbol{\sigma}^{dev} \\ &\quad - \frac{3G_t K_0 \tan \psi \Delta t}{B\lambda_{den} \sqrt{(\epsilon \tan \beta)^2 + q^2}} \mathbf{i} \otimes \boldsymbol{\sigma}^{dev} - \frac{3G_t K_0 \tan \beta \Delta t}{AB\lambda_{den}} \boldsymbol{\sigma}^{dev} \otimes \mathbf{i} \\ &\quad - \frac{K_0^2 \tan \beta \tan \psi \Delta t}{B\lambda_{den}} \mathbf{i} \otimes \mathbf{i} \end{aligned} \quad (46)$$

where $\lambda_{den} = \frac{-\partial \Delta f}{\partial \lambda^{[n+1]}}$ and $G_t = G_0 \left\{ 1 + \sum_{m=1}^M \frac{\gamma_m (1 - \exp(-\frac{\Delta t}{\tau_m}))}{\frac{\Delta t}{\tau_m}} \right\}$ due to the viscoelasticity.

In the presence of damage, we approximate the damaged stiffness tensor by simply multiply the viscoelastic parameters G_t and K_0 in Eq. (46) by $(1 - D)$. Note also that the above stiffness matrix is symmetric when associative flow rule is considered, i.e., $\beta = \psi$.

2.7. *Implementation algorithm*

In this subsection, the numerical algorithmic treatment of the proposed constitutive model is outlined. The numerical algorithm consists of an initial viscoelastic predictor that is used to evaluate the plastic yielding and the subsequent computation of the internal variables at the current state in case of actively plastic yielding and/or damage growth. The implementation of the model follows a staggered scheme approach. The modularity of the approach makes it easy to modify the code to glue to another sophisticated model in the future. In this method, the viscoelastic trial stress is predicted first by assuming that the total strain increments are fully viscoelastic strain increment. Then, based on the trial viscoelastic-stress, the yield function will be evaluated whether the current viscoelastic stress exceeds the yield stress or not. If the trial viscoelastic-stress exceeds the yield stress, then the viscoplastic subroutine and damage subroutine will be called to calculate the increment of viscoplastic strain and new damage state variables. It is important to note that since the plasticity and damage are coupled through the effective plastic displacement, this implies that there will be no damage growth if the plasticity is not actively yielding. Algorithm 1, Algorithm 2 and Algorithm 3 highlight the computational procedure for the implementation of the presented constitutive model into Abaqus UMAT [33].

[Algorithm 1 here]

[Algorithm 2 here]

[Algorithm 3 here]

3. **Experimental setup and results**

3.1. *Materials and processing*

We studied impact copolymer polypropylene (PP) provided by SABIC in the form of injection-molded plates. Impact copolymer polypropylene (PP) is homopolymer polypropylene that has been mixed with rubber particles. The plates were reprocessed by placing it into an aluminum mold and then hot-pressed by a static press machine (Pinette Emidecau Industries 15T) to ensure a controlled thermal history and crystallinity level. The reprocessing was carried out at a pressure of 6.8 bar with increasing temperature from 25 °C to 230 °C (heating rate of 10 °C/min). The temperature was then kept constant for 20 minutes and cooled down to the temperature of 25 °C at the cooling rate of 40 °C/min. The size of the reprocessed plate was 250 mm long, 110 mm wide with three different thicknesses: 2 mm (for tensile specimens), 4 mm (for shear and three-point bending specimens without notch) and 8 mm (for three-point bending specimens with notch). All samples were carefully cut from the processed plates to the intended specimen sizes using a Hurco milling machine.

3.2. Mechanical tests for calibrating material parameters

Table 1 lists all the material parameters and the required experiments to identify the model entirely. In total, the proposed model requires 12 material parameters and five different types of experimental tests.

First, we performed shear-relaxation tests to calibrate the viscoelastic parameters G_0 , G_m and τ_m . The loading regime of shear-relaxation tests were carried out at three different strain rates i.e., 0.0001/s, 0.001/s, and 0.01/s, to enable extrapolation of the relaxation response at higher rates. The crosshead displacement was then kept constant for 1 hour to impose constant strain at the specimen. During this phase, the stress relaxation was measured by the load cell. Note that we chose a constant engineering shear strain around 0.8% so that the imposed strain is small enough and thus, the relaxation response is solely due to viscoelasticity. Secondly, tensile-relaxation tests were carried out to determine K_0 by fitting the long-term response of a pre-loaded sample. Tensile-relaxation specimens were loaded at the strain rate of 0.0001/s up to crosshead displacement of 0.1 mm, which corresponds to 0.2% strain. This displacement was also held for 1 hour and the stress-relaxation was measured. Although the crosshead displacement was kept constant for both shear- and tensile-relaxation tests, the strain experienced by the specimen slightly changed during the test. This could affect the parameter identification results if the strain change was not considered during the identification. Therefore, in our approach, the strain change was measured by DIC and to be used as input for the identification.

[Figure 3 here]

Monotonic tensile tests at three different strain rates of 0.001/s, 0.01/s and 0.1/s were performed to calibrate viscoplastic parameters σ_{t0} , μ , and c . In addition, a non-associative plastic flow parameter ψ and damage initiation parameter $\bar{\epsilon}_{init}^{vp}$ were also calibrated based on the stress-strain curves from monotonic tensile tests at a strain rate of 0.001/s. We tested dumb-bell shaped specimens that comply with the ISO 527-2 1BA standard as shown in Figure 3 (i.e., 79 mm length, 5 mm width, 30 mm gage length, 2 mm thickness). The tensile tests were performed using an Instron 5944 universal tensile test machine (2 kN load cell).

We also performed monotonic shear tests at a strain rate of 0.001/s to obtain the hDP viscoplasticity parameter β . It should be noted that to calibrate β , both monotonic tensile test and monotonic shear test results are required. The shear test was performed using a V-notch rail (VNR) shear test method (ASTM Standard D7078). Although the standard generally recommends this method for composite materials, we found it very suitable for our experimental work with polypropylene. Other researchers have also employed VNR test method for obtaining shear properties of polypropylene [35]. However, one of the

potential disadvantages of the V-notch rail test method is the risk of specimen slippage during the test. We thus developed our own VNR fixture by modifying the gripping area to be precisely fitted with the specimen. A specimen with a thickness of 4 mm and VNR shear test setup are shown in Figure 4. We attached the VNR fixture on the Instron 5882 (100 kN load cell). We used DIC to obtain the strain fields on a circular region of interest (ROI) between two V-notches. Monotonic shear test results are shown in Figure 5a, and an example of shear strain distribution during the monotonic test is shown in Figure 5b.

[Figure 4 here]

[Figure 5 here]

Another parameter for the hDP viscoplasticity model is p_{t0} . Physically, p_{t0} determines the strength of the PP under triaxial tensile stress-state. However, due to the difficulty of performing triaxial tensile test, p_{t0} was assumed to be 95% of its maximum theoretical value (i.e., the value when the linear Drucker-Prager yield function is used) as follows:

$$p_{t0} = \frac{0.95 \left(1 + \frac{1}{3} \tan \beta\right) \sigma_{t0}}{\tan \beta} \quad (47)$$

Damage evolution parameter $\bar{u}_{crit}^{vp}$ was obtained by performing three-point bending (3PB) test on a V-notch specimen with a tip radius of 1.5 mm. Figure 6 shows the specimen specification and experimental setup. We attached the 3PB fixture on Instron 5882 (500 N load cell), and performed the test at 1.2 mm/min to obtain corresponding load-displacement curves. The V-notch specimen was simply supported by two pins (60 mm apart) and subjected to a force exerted by a middle pin that was connected to the load cell. DIC technique was also adopted to measure the strain field during the test.

[Figure 6 here]

3.3. Mechanical tests for validating the proposed models

We validated the proposed model by testing PP beams without notch under three-point bending (3PB) tests at two different loading rates, i.e., 0.5 mm/min and 50 mm/min. We opted 3PB setup for validation since this setup introduces both tension and compression stress states at the specimen. The presence of these stress states will examine the pressure-dependency plasticity aspect of the model. The tests at different loading rates are targeted to examine the rate-dependent aspect of the model, i.e., the viscoelasticity and viscoplasticity. The specimen and the 3PB bending setup are shown in Figure 6.

4. Calibration of material parameters

The identification of material parameters was carried out for each physical effect considered (i.e., viscoelastic, viscoplastic, pressure-dependent plasticity and damage). Each effect requires specific experimental testing. The viscoelastic parameters are based on tensile and shear-relaxation tests while the viscoplastic parameters are based on the monotonic tensile test at various strain rates. The calibration of the pressure-dependent yield strength is based on monotonic tensile and shear tests while the plastic Poisson ratio (non-isochoric plastic volume change) was calibrated based on DIC strain measurements on monotonic tensile specimens. Finally, damage parameters are calibrated using monotonic tensile test and a three-point bending test of the beam with a notch. For each physical effect, the parameters are usually obtained through numerical optimization. In this case, we used a built-in optimization solver on a standard spreadsheet software based on the generalized reduced gradient (GRG) method.

4.1. Viscoelasticity parameters

The calibration of the viscoelasticity model is a two-step process. First, we fit shear-relaxation responses to obtain the shear moduli. In our work, the shear-relaxation responses are fitted with 1 elastic spring and 7 Maxwell elements. Due to experimental limitation, although the strain was prescribed constant, the strain still decreased during the stress relaxation. Therefore, the apparent stress relaxation is not solely due to the relaxation itself but partially due to the decrease of the applied strain itself (see Figures 7a, 8a, and 9a). Therefore, the strain evolution has to be taken into account when performing identification of the shear moduli, i.e., the strain evolution was used as the input for the identification process. Figures 7b, 8b, and 9b show the fitted shear stress relaxation responses using the generalized Maxwell viscoelastic model. Once all the shear-relaxation moduli (G_0 to G_7 and τ_1 to τ_7) are calibrated properly, we simulate the tensile-relaxation tests and fit the results by adjusting the value of K_0 (see Figure 10). Note that in this phase, we fixed the shear relaxation moduli as obtained previously.

[Figure 7 here]

[Figure 8 here]

[Figure 9 here]

[Figure 10 here]

4.2. Viscoplasticity parameters

The tensile stress-strain curves at various strain-rates are required to fit the viscoplasticity. We used Johnson-Cook viscoplastic model (see Eq. (13)) to fit the viscoplastic behavior of the PP at higher rates. First, the yield stress w.r.t. equivalent viscoplastic strain had to be extracted from the stress-strain curves. For the sake of simplicity, we assumed that the viscoelastic strain-rate was constant during the tensile process, and therefore we could treat it as a linear elastic case when extracting the equivalent viscoplastic strain. Figure 11a shows the yield strength of PP with respect to the equivalent viscoplastic strain at a strain rate of 0.001/s. Using this reference data, we then used a spreadsheet software to calibrate μ and c so that the extrapolated stress-strain curves based on Eq. (13) matched the experimentally measured stress-strain curves at strain rates of 0.01 and 0.1/s. Figure 11b shows the experimental stress-strain curves at various strain rates fitted by the model.

[Figure 11 here]

4.3. Pressure-dependent plasticity parameters

A unit cube FE model was developed to calibrate the material parameters related to the pressure-dependent behavior of the PP. First, we calibrated the material parameter ψ which relates to the non-isochoric plastic deformation of PP. To calibrate the value of ψ , the FE model was simulated with various value of ψ so that the plastic Poisson ratio matches the DIC results. Figure 12a shows the comparison of the numerical values of the plastic Poisson ratio with those obtained experimentally. Note that in extracting this data, we treat it as in linear elastic case as before and hence we can extract the elastic strain from the total strain in both axial and transverse directions. By comparing the numerical results with the experimentally measured plastic Poisson ratio, the value of ψ should be in the range of $10^\circ < \psi < 12.5^\circ$ i.e., ψ is fixed at 11.5° . Figure 12a showed oscillated plastic Poisson ratio in the beginning of the curves since the plastic strain in this region is almost zero or small values. Therefore, oscillating plastic Poisson ratio is observed and it is merely an experimental artifact due to division by zero or small value of plastic strain. Once the ψ value is well calibrated, then we can proceed to calibrate the parameter β . β is calibrated by simulating the unit cube FE model under shear loading. By changing the value of β from 31.5 to 15, we found that the value of β should be around 15° as shown in Figure 12b.

[Figure 12 here]

4.4. Damage parameters

The damage initiation is defined in terms of equivalent viscoplastic strain. For the sake of simplicity, we assumed that damage initiated when the peak stress occurred in the stress-

strain curve of PP under a uniaxial tensile test. Therefore, the value of damage initiation $\bar{\varepsilon}_{init}^{vp}$ was the equivalent viscoplastic strain at the peak stress. For our material, $\bar{\varepsilon}_{init}^{vp} = 0.0163$. Meanwhile, it was difficult to calibrate correctly the damage evolution from a simple tensile test because the sample was brittle. To overcome this problem, we used a three point bending test of PP beam with a notch. The presence of the notch on the distal face of the beam enabled the progressive and stable crack propagation. Therefore, we could easily develop the associated finite element model to calibrate the damage evolution parameters. Our goal was to find the parameter $\bar{u}_{crit}^{vp}$ so that the softening upon the peak load would match experimental results. Figure 13 shows the load-displacement curves of PP beams with notch obtained experimentally. In this particular case, $\bar{u}_{crit}^{vp} = 0.016$ mm was found to be a good approximation.

[Figure 13 here]

5. Numerical examples and experimental validations

We performed 3PB tests on the PP beams without a notch. The objective of the test was to validate the pressure-dependent aspect of the model since both tension and compression stress states appear during the test. We performed the simulations at two different strain rates to validate the rate-dependent aspect of the model. Table 2, Table 3, Table 4 and

Table 5 list all the material parameters used in the simulation for the validation. Figure 14 and Figure 15 show the load-displacement curves predicted by the proposed model for loading rates of 0.5 mm/min and 50 mm/min, respectively. The results obtained by our model were in excellent agreement with experimental data.

[Figure 14 here]

[Figure 15 here]

6. Conclusions

A coupled generalized Maxwell viscoelastic, Johnson-Cook viscoplastic with hyperbolic Drucker-Prager yield and flow functions and plasticity-based damage model for PP-based thermoplastic polymers has been proposed and discussed in detail. The calibration of our model is based on the experimental results of uniaxial tensile tests, V-notch rail (VNR) shear tests, and three-point bending tests on PP copolymer at various strain-rates. In all tests, the strain field is measured by DIC. The use of DIC allows for more accurate measurement as it eliminates the effect of the machine compliance; it also enables a calculation of the Poisson ratio. The material parameters are sequentially calibrated for each physical effect considered (i.e., viscoelasticity, viscoplasticity, pressure-dependent plasticity, and damage).

The model is then used to predict the response of PP beams under three-point bending at two different loading rates. We found the model prediction to be in excellent agreement with experimental results. Such a detailed model is needed for accurately predicting the failure behavior of PP, especially in micromechanics simulations of PP-based thermoplastic composites.

Acknowledgments

The research reported in this publication was supported by SABIC and King Abdullah University of Science and Technology (KAUST). First author would like to thank Mr. Yongwei Wang for deep math discussions.

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Figure 15. Model prediction vs. experimental three-point bending tests of PP beam without a notch at a loading rate of 50 mm/min.

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Parameters	M. T.			M. S.	T. R.	S. R.			3PB-N	Remark
Strain rate (1/s)	10^{-3}	10^{-2}	10^{-1}	10^{-3}	10^{-4}	10^{-4}	10^{-3}	10^{-2}	1.2 mm/min	
Viscoelastic										
K_0					V					
G_0						Average				
G_m						V	V	V		
τ_m						V	V	V		
Viscoplastic										
σ_{t0}	V									
μ		V	V							
c		V	V							
Pressure-dependent plasticity										
β	V			V						
ψ	V									
p_{t0}										Numerical
Damage										
$\bar{u}_{crit}^{vp}$ or $D(\bar{u}^{vp})$									V	
$\bar{\varepsilon}_{init}^{vp}$ (tensile)	V									
Monotonic tensile test (M. T.) Monotonic shear test (M. S.) Tensile-relaxation test (T. R.) Shear-relaxation test (S. R.) Three-point Bending of beam with notch (3PB-N)										

Table 2. Viscoelastic-related parameters for PP

No	Parameters	Value	Unit
1	K_0	1850 ± 150	MPa
2	G_0	336.05 ± 18.90	MPa
3	G_1	154.53 ± 89.84	MPa
4	G_2	141.43 ± 89.73	MPa
5	G_3	135.87 ± 87.88	MPa
6	G_4	100.48 ± 9.99	MPa
7	G_5	94.93 ± 9.55	MPa
8	G_6	88.70 ± 10.54	MPa
9	G_7	80.68 ± 12.45	MPa
10	τ_1	0.01	s
11	τ_2	0.1	s
12	τ_3	1	s
13	τ_4	10	s
14	τ_5	100	s
15	τ_6	1000	s
16	τ_7	10000	s

Table 3. Viscoplastic-related parameters for PP

No	Parameters	Value	Unit
1	σ_{t0}	See Fig. 11(a)	MPa
2	μ	0.034	-
3	c	0.001	/s

Table 4. Hyperbolic Drucker-Prager plasticity-related parameters for PP

No	Parameters	Value	Unit
1	β	15.0	Degree
2	ψ	11.25	Degree
3	p_{t0}	95	%

Table 5. Damage-related parameters for PP

No	Parameters	Value	Unit
1	$\bar{u}_{crit}^{vp}$	0.016	mm
2	$\bar{\varepsilon}_{init}^{vp}$	0.0163	-

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Algorithm 2. Viscoplastic subroutine

Algorithm 3. Plasticity-based damage model

Tensile test specimen

Tensile test setup

ACCEPTED MANUSCRIPT

V-notch specimen

Dimension in mm

V-notch rail shear test setup

(a)

(b)

3PB test specimens

3PB test setup

Algorithm 1: FE implementation of the proposed viscoelastic viscoplastic damage model

- 1 Input: $\Delta\epsilon$, Δt , $\sigma^{[n]}$, $\mathbf{h}_m^{[n]}$, $\bar{\epsilon}^{vp[n]}$, $\epsilon^{vp[n]}$, $D^{[n]}$
 - 2 Initialize: $\kappa = 1$, $\Delta\epsilon^{ve} = \Delta\epsilon$, $\bar{\epsilon}^{vp[n+1](\kappa)} = \bar{\epsilon}^{vp[n]}$, $\epsilon^{vp[n+1](\kappa)} = \epsilon^{vp[n]}$, $D^{[n+1](\kappa)} = D^{[n]}$
 - 3 Calculate the trial stress $\sigma^{[n+1](\kappa)}$ based on viscoelasticity Eq. 2, Eq. 6 and Eq. 7
 - 4 Calculate pressure $p^{[n+1](\kappa)}$ Eq. 3 and von Mises stress $q^{[n+1](\kappa)}$ Eq. 4
 - 5 Calculate cohesion strength $d^{[n+1](\kappa)}$ Eq. 11
 - 6 Evaluate the yield function $f^{[n+1](\kappa)}$ Eq. 10
 - 7 **if** $f^{[n+1](\kappa)} > TOL$ **then**
 - 8 **repeat**
 - 9 Call viscoplastic subroutine
 - 10 Call damage subroutine
 - 11 $\kappa = \kappa + 1$
 - 12 **until** $|f^{[n+1](\kappa)} - f^{[n+1](\kappa-1)}| < TOL$ *and* $|D^{[n+1](\kappa)} - D^{[n+1](\kappa-1)}| < TOL$;
 - 13 **else**
 - 14 EXIT
 - 15 **end**
 - 16 Calculate tangent stiffness tensor Eq. 46
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Algorithm 2: Viscoplastic subroutine

- 1 Input: $\Delta\epsilon$, Δt , $\sigma^{[n+1](\kappa)}$, $\bar{\epsilon}^{vp[n+1](\kappa)}$, $\epsilon^{vp[n+1](\kappa)}$, $D^{[n+1](\kappa)}$
- 2 Initialization: $\xi = 1, \dot{\lambda}^{[n+1](\xi)} = 0$, $\bar{\epsilon}^{vp[n+1](\xi)} = \bar{\epsilon}^{vp[n+1](\kappa)}$, $\epsilon^{vp[n+1](\xi)} = \epsilon^{vp[n+1](\kappa)}$,
 $D^{[n+1](\xi)} = D^{[n+1](\kappa)}$
- 3 repeat
 - 4 Calculate $n^{[n+1](\xi)}$ Eq. 16
 - 5 Calculate viscoplastic strain increment $\Delta\epsilon^{vp}$ Eq. 23
 - 6 Calculate viscoelastic strain increment $\Delta\epsilon^{ve} = \Delta\epsilon - \Delta\epsilon^{vp}$ Eq. 1
 - 7 Calculate the trial stress $\sigma^{[n+1](\xi)}$ based on viscoelasticity Eq. 2, Eq. 6 and Eq. 7
 - 8 Calculate pressure stress $p^{[n+1](\xi)}$ Eq. 3 and von Mises stress $q^{[n+1](\xi)}$ Eq. 4
 - 9 Calculate the equivalent plastic strain increment Eq. 37
 - 10 Calculate cohesion strength $d^{[n+1](\xi)}$ Eq. 11
 - 11 Evaluate the yield function $f^{[n+1](\xi)}$ Eq. 10
 - 12 Calculate $\dot{\lambda}^{[n+1](\xi+1)} = \dot{\lambda}^{[n+1](\xi)} - \frac{f^{[n+1](\xi)}}{\left[\frac{df}{d\lambda}\right]^{[n+1](\xi)}}$
 - 13 $\xi = \xi + 1$
- 14 until $|f^{[n+1](\xi)}| < TOL \times d^{[n+1](\xi)}$ and $|\dot{\lambda}^{[n+1](\xi)} - \dot{\lambda}^{[n+1](\xi-1)}| < TOL$;

Algorithm 3: Damage subroutine

- 1 Input: $\bar{\varepsilon}^{vp}[n+1](\kappa)$, $\sigma^{[n+1]}(\kappa)$, and $D^{[n]}$
 - 2 Initialization: $\xi = 1$, $\bar{\varepsilon}^{vp}[n+1](\xi) = \bar{\varepsilon}^{vp}[n+1](\kappa)$, $D^{[n+1]}(\xi) = D^{[n]}$
 - 3 Calculate $\bar{u}^{vp[n+1]}(\xi)$ Eq. 22
 - 4 Solve Eq. 22 for variable $D^{[n+1]}(\xi)$
 - 5 **if** $D^{[n+1]}(\xi) < D^{[n]}$ **then**
 - 6 $D^{[n+1]}(\xi) = D^{[n]}$
 - 7 **end**
 - 8 **if** $D^{[n+1]}(\xi) > 1$ **then**
 - 9 $D^{[n+1]}(\xi) = 1$
 - 10 **end**
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Highlights

- A coupled viscoelastic viscoplastic damage model for polypropylene-based polymers is successfully developed and well validated against experimental results
- A systematic strategy and the required experimental works to calibrate parameters of the proposed model is presented in a sequential manner of each physical effect.
- The pressure-dependent plasticity and non-associative plastic flow are taken into account by calibrating uniaxial tensile and shear tests.
- Concise model derivations and numerical implementation are provided so engineers or researchers can implement our model in any finite element softwares.
- The proposed model can be applied to any polymer that exhibit the same behaviors such as viscoelasticity, viscoplasticity, pressure-dependent plasticity and damage.