A Distance Weighted-based Approach for Self-Organized Aggregation in Robot Swarms

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Abstract—In this paper, a Distance-Weighted K Nearest Neighboring (DW-KNN) topology is proposed to study self-organized aggregation as an emergent swarming behavior within robot swarms. A virtual physics approach is applied among the proposed neighborhood topology to keep the robots together. A distance-weighted function based on a Smoothed Particle Hydrodynamic (SPH) interpolation approach is used as a key factor to identify the K-Nearest neighbors taken into account when aggregating the robots. The intra virtual physical connectivity among these neighbors is achieved using a virtual viscoelastic-based proximity model. With the ARGoS based-simulator, we model and evaluate the proposed approach showing various self-organized aggregations performed by a swarm of N foot-bot robots.

Keyword—Swarm Robotics; Self-organized Aggregation; Smoothed Particles Hydrodynamics (SPH); Virtual Viscoelastic Model; DW-KNN.

I. INTRODUCTION

In swarm robotics, aggregation is a fundamental behavior in performing other collective tasks such as collective movement, self-assembly and pattern formation, or to exchange information. Aggregation with robot swarms can be achieved using cue-based mechanism driven by external gradients created around the surrounding area to attract agents to the direction of the optimal intensity [1]. It can be also achieved in self-organized manner where the aggregation process is enabled without any external cue [2]. In such case individuals mostly aggregate in some random areas as a result of interactions between parts of the system. In this paper, we are specifically interested in studying self-organized aggregation within robots swarm. We mainly take inspiration from the topological distance approach revealed in studies in birds flocking and fish schooling. Ballerini et al [3], [4] showed empirically that the unpredictable amazing complex patterns formed by a large scale of flock of birds emerge from a topological distance approach rather than a metric distance one. This means that the interaction between birds is shown to be only with the nearest six to seven neighbors rather than the entire number of neighbors in the field vision of a given bird. Computer simulations also outline that a significant higher cohesion of the aggregation is achieved under the topological interaction compared to the standard metric one. Similar

empirical results in fishes, and elephants, for example, can count up to three or four neighbors [5].

In swarm robotics, few models have applied such a topological approach to study collective behaviors within robotics swarm [6], [7]. Some researchers also proposed selecting strategies before interacting with nearby robots so that only a fixed number of neighbors are used [8], [9]. However, most of the topologies used in these studies rely on simple distances constraints. In this work, we propose a new neighborhood topology to study swarm robotics self-organized aggregation that emerge from an approach based on intra virtual physical connectivity among neighbors. In our previous work [7], the interactions between the robots (or nodes) were governed by a K-Nearest Neighboring (KNN) topology. However, relying on such topology is not always the perfect solution in many real-world scenarios. Specifically when the network nodes are holding supplementary properties that differentiate them from others. In such case the interactions between the nodes are best described by weighted connections rather than simple ones.

Here we use the density of the robot in the swarm as an additional property and we define a new neighboring relationship based on the Distance-Weighted K-Nearest Neighbors (DW-KNN)(Section. III-B). In certain applications such as driving a large number of robots from one area to another, the density of the robots could play a key role while the inter-robot distance plays only a secondary role as a metric useful for proximity control or collision avoidance. With the DW-KNN topology, virtual physical connections are dynamically created and destroyed among the K-Nearest weighted neighbors rather than the KNN one. To achieve this topology, we introduce a Smoothed Particle Hydrodynamic (SPH) density based approach (Section. III-A) to evaluate the density of the robot in the swarm and to weight the neighbors. SPH is a mesh-free Lagrangian method that is based on an interpolation technique to approximate physical quantities that are moving with particles [10]; here we approximate the density as a physical quantity moving with the robots. We use our virtual viscoelastic based proximity model (Section. III-C) [7], [11] as intra virtual physics connectivity among neighbors; the virtual viscoelastic connections are *Voigt* links, with each one consists of a *Hookean* elastic spring and a *Newtonian* damper connected in parallel. We also incorporate a repulsive primitive to avoid

This work is based upon a collaboration work supported by the King Abdullah University of Science and Technology (KAUST) Office of Sponsored Research (OSR) under Award No: OSR-2015-CRG4-2582, and the LESIA Laboratory, Department of Computer Science, University of Mohamed Khider, Biskra, Algeria.

obstacles and collision among the robots, which are not virtually, linked together (Section. III-E). The overall control model (Section. IV) with a swarm of foot-bot robots is designed and evaluated using the ARGoS simulator [12]. The results obtained in (Section. V) by simulating up to 150 foot-bot robots show the variation in self-organized aggregation emerged from the execution of the overall control model. The performance of the model is studied using three different metrics (Section. V-B): the Distance-Weighted distribution quality (F_w), the aggregation quality (F_{ag}), and the Averaged Mean Distance Error (AMDE) metric.

II. RELATED WORKS

In swarm robotics, different approaches have been evaluated to address the problem of self-organized aggregation. Garnier et al. [13] adopted a probabilistic approach to achieve aggregation within a swarm of 20 physical Alice robots in homogeneous environments. Soysal and Sahin [14] suggested a probabilistic aggregation method in which state finite machine is used to combine a set of simple behaviors (obstacle avoidance, approach, repel, and wait). Ando et al. [15] proposed a deterministic approach using graph theory to study aggregation in a group of mobile robots with a limited sensing range. Trianni et al. [16] studied aggregation with simple robots called s-bots using evolutionary robotics mechanism. The method was able to emerge clustering behaviors in both static and dynamic behavioral strategies. With that model, Dorigo et al. [17] revealed that effective evolved controllers could be achieved for both aggregation and coordinated motion behaviors in a swarm of s-bots. In a similar setup, Soysal et al. [18] investigated the effects of a number of parameters, such as the robots number, the size of arena, and run time. In another study, Melvin et al. [19] proposed two algorithms - a reactive controller with no memory and a recurrent controller with memory- to study aggregation in a swarm of e-puck robots. The algorithms are based on classical evolutionary programming technique, and use simple binary sensor that is proven to be enough to achieve error-free aggregation since a sufficient sensing range is provided. Other aggregation studies approached another vision based on virtual physics methods. Gasparri et al. [20] adopted an attractive/repulsive virtual force model to study aggregation in a swarm of multi-robot systems basing on local interaction. Later, the authors extended this model to cope with actuator saturation [20], and to integrate obstacle avoidance [21].

Most of the virtual physics based works cited above use the inter-robot distance as the only factor in aggregating the robots; however in certain applications, supplementary factors such as the density of the robots could emerge as important. In such situations, the robot's density plays a crucial role compared to inter-robot distance. In this context, the related methodologies mainly use the SPH model for fluid simulation to control robots swarm behaving as a fluid. This technique has successfully been used to demonstrate how multi-robotic agents modeled as a stream of incompressible/compressible fluid are capable of being applied in tasks such as: group motion and shape control, and group segregation [22], pattern generation [23], [24] and unknown area coverage.

Differently from using the SPH model as a virtual fluid force law to design a swarm robots proximity control. Here we use the SPH model to evaluate the density of the robot in the swarm, and then based on this density we define a DW-KNN neighboring topology to aggregate the robots. With the DW-KNN topology, self-organized aggregations are achieved using two key factors: the inter-robots distance and the robot's density.

III. METHOD

We consider a set of N simple mobile robots that are randomly distributed in an indoor arena. Given $\mathcal{N}$ as the set of neighbours of the i^{th} robot, we assume that each individual robot R_i can, within a range D_r , measures the range d_{ij} and the bearing θ_{ij} of its j^{th} neighbor. Within the same range, we assume also the robot can communicate with its neighbours. The robot model is based on the one proposed in [7], where the robot is subject to the following virtual force:

$$a_i = p_i + r_i. \quad (1)$$

p_i is defined as the proximity control vector and r_i is defined as the repulsive control vector. We compute p_i using virtual viscoelastic links among the Distance-weighted k-Nearest Neighbors. The weighted distances are determined via scaling the distances by the corresponding densities received from the neighbors; each robot density is calculated using a SPH density estimator control. The subsections below give more detail about the basic preliminaries behind the overall control model:

A. SPH Density Estimator Control

We apply a SPH based interpolation method to estimate the density of the robot in the swarm. The robot density ρ_i is evaluated as in [22] as the weighted sum of distances over its proximate robots within a particular range $D_r = 2h$,

$$\rho_i = \sum_{j \in \mathcal{N}} \mathcal{W}(d_{ij}, h), \quad (2)$$

where $\mathcal{W}$ is a weight function (or kernel function), and h is the smoothing length. In SPH, the most well used weight functions are the B-spline functions. Here we use the $M4$ cubic spline functions truncated at $2h$ [25]:

$$f(q) = \sigma \begin{cases} \frac{1}{4}(2-q)^3 - (1-q)^3, & 0 \leq q < 1 \\ \frac{1}{4}(2-q)^3, & 1 \leq q < 2 \\ 0, & q \geq 2 \end{cases}, \quad (3)$$

where $q = d_{ij}/h$ and $\sigma = 10/(7\pi h^2)$ is a normalized constant. Note that h should be chosen in a way that controls the compact support of the SPH; here the function support of $f(q)$ is determined by $2h$ [22, 25].

B. Distance-weighted k-Nearest Neighbours Control

We take inspiration from the theory of Distance-Weighted KNN classification approach, in which a weight based distance function, is used for heavily weighting the closest neighbors more than the farthest ones. Here we use SPH densities to

weight the distances. For that, each robot R_i of the swarm should communicate its density ρ_i to its neighbours. Once the focal robot receives the communicated densities, a distance-weighted function w_{ij} is computed using the following equation:

$$w_{ij} = \rho_j d_{ij}, \quad (4)$$

where ρ_j denotes the density received from the j^{th} neighbor. Further, the robot build a Distance-Weighted K-nearest neighbors connectivity, $\mathcal{N}^{DWKnn}(t)$, via sorting the neighbours by the nearest K weighted distances, where $K \in \{1, 2, \dots, N-1\}$ refers to how many neighbours are taken into account. The cardinality of the neighbourhood, $|\mathcal{N}^{DWKnn}(t)|$, is K and $j \in \mathcal{N}^{DWKnn}(t)$: $w_{ij} \leq w_{im}, \forall i \in \mathcal{N}^{DWKnn}(t)$. In the case where $K = N-1$ the mesh is then the all-to-all connected network.

Note that by using the function in eq.4, both the distance and the density are applied as equal key factors. Therefore a neighbor robot R_j with a heavy density and located far away from the robot R_i could have more impact than another one located near R_i with a weak density.

C. Proximity Control

The proximity control is applied to keep and arrange the robots together with a certain distance. To achieve this model, we use virtual viscoelastic Links (VVL) between the focal robot (R_i) and the set of its Distance-Weighted KNN, $\mathcal{N}^{DWKnn}(t)$. Each VVL is a Voigt model consisted of a *Hookean* elastic spring connected in parallel with a *Newtonian* damper. The proximity model vector p_i is computed basing on our proposed model [7]:

$$p_i = \sum_{j \in \mathcal{N}^{DWKnn}(t)} [k_{ij}^s (d_{ij} - d_0) + k_{ij}^d v_{ij}] \quad (5)$$

$$k_{ij}^s = k_s / \sqrt{d_{ij}}, \quad k_{ij}^d = k_d \sqrt{k_{ij}^s}$$

where k_s is the spring constant, d_0 is the equilibrium length of the spring, k_d is the damping coefficient and v_{ij} is the velocity of the robot R_i with regards to R_j .

D. Repulsive Control

To avoid obstacles, for example the four walls that surround the arena, we incorporate a repulsive control vector modeled as a potential field that is generated around each obstacle. The field has a strong influence when a robot is close and a decreasing effect as the robot moves further away. We compute the repulsive control vector as follows [7], [11]:

$$r_i = \sum_{j \in O_i} \begin{cases} k_r \left(\frac{1}{L_j} - \frac{1}{L_0} \right) \left(\frac{1}{L_j^2} \right), & L_j \leq L_0 \\ 0, & \text{elsewhere} \end{cases} \quad (6)$$

where O_i is the set of the obstacles. k_r is a scaling constant, L_j is the distance between the robot R_i and the nearest edge of the j^{th} obstacle, and L_0 is the obstacle influence threshold.

E. Motion Control

At each control step, each robot R_i transforms the total vector, a_i , into signals that update the forward speeds (v_{li} and v_{ri}) of its left and right wheels respectively as follows [7], [11]:

$$\begin{bmatrix} v_{li} \\ v_{ri} \end{bmatrix} = \begin{bmatrix} 1 & b/2 \\ 1 & -b/2 \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix}, \quad (7)$$

$$\omega_i = k_\omega \left(\frac{180\alpha}{\pi} \right), \quad v_i = \frac{v_{max}}{\sqrt{|\omega_i|+1}}, \quad (8)$$

where v_i and ω_i are respectively the forward and the angular speed of the robot, b is the robot inter-wheels distance, α is the angle formed by the x and y components of the vector a_i , k_ω is a gain constant, and v_{max} is the maximum forward speed that a robot can reach. The angular speed should be also within the range $[-\omega_{max}, \omega_{max}]$.

IV. SELF-ORGANIZED OVERALL CONTROL MODEL WITH ROBOTS SWARM

We investigate self-organized aggregation with a swarm of simulated versions of foot-bot robots. The foot-bot is a 2-wheeled differential mobile robot that comes with a set of actuators and sensors. The overall control model implemented in the foot-bot robot is shown in Fig. 1.

Figure 1. The overall control model for self-organized robots swarm.

To achieve the SPH Density Estimator Control (SPHDEC), the robot uses the range and bearing sensing and communication device called (RAB) to measure the distances d_j to its neighbors within the perception range D_r . Using the equations pointed in the SPHDEC model, the foot-bot is able to compute its density which is immediately sent to its neighbours via the RAB. Since the foot-bot can exchange only 10 bytes of data, the density is scaled by 10^3 before it's sent to the neighbors.

To achieve the Distance-weighted k-Nearest Neighbors Control (DWKNNC), the robot receives the densities of its neighbors and computes its corresponding weighted distances; then based on the value of K , the robot can get the K-Nearest Weighted distance neighbors.

To realize the Proximity Control (PC), the robot uses again the RAB to communicate the linear speed to the neighboring robots and to measure the relative range and bearing (d_j and θ_j). For attaining the Repulsive Control (RC), we use proximity sensors to get the distance L_j of the closet detected obstacle and its corresponding angle ϕ_j . Finally, based on the

resulted vector a_i , the Motion Control (MC) actuates the linear speed of the robot wheels.

V. SIMULATION RESULTS

The simulations performed in this study were based on the ARGoS platform [12]. ARGoS is capable to simulate large-scale swarms of robots of any kind efficiently. In the following subsections, we illustrate the configuration setup used in the simulation and some of the simulation results, we explain the metrics used to evaluate the proposed method, and finally we present some of the results within the metrics.

A. Experimental setup

In this study, we use a swarm of N foot-bots that are arbitrary placed in an arena of $(10 * 6)m^2$, and are heading arbitrary orientations. In the experiments and in order to emulate the real devices in the foot-bot robot, noise is added to the range and bearing measurements of the RAB device. Noise is modelled as a Gaussian distribution with zero mean and 0.01 deviation. The loss of packets during communication is also taken into consideration by setting the loss probability to 3%. We conduct three separate experiments for different number of foot-bot robots ($N = \{50, 100, 150\}$) for duration of 2000 time steps each (note that in ARGoS $1 \text{ ts} = 0.1 \text{ sec}$). Starting from an initial distribution of the robots, a set of constants and parameters (see Table I) are initiated at each experiment.

TABLE I. PARAMETERS AND CONSTANTS USED IN THE SIMULATION.

Parameter	Description	Value
b	Inter-wheels distance	0.14 m
v_{max}	Maximum forward speed	0.10 m/s
ω_{max}	Maximum angular speed	$180^\circ/\text{s}$
d_0	Equilibrium length of the spring	0.3 m
L_0	Obstacle influence threshold	0.1 m
k_r	Obstacle scaling constant	1.75 force unit
k_d	Damper gain constant	1.25 force unit
k_ω	Angular speed gain	$1.5^\circ/\text{s}$
k_s	Spring gain constant	1.9 force unit
D_r	Maximum perception range	1.5 m
h	The smoothing length	0.75 m

We demonstrate in Fig. 2 the evolution of self-organized aggregations that emerge from the execution of the overall control model at diverse time steps ($t = 400$, $t = 1200$ and $t = 2000$) while starting from the same initial positions ($t = 0$). The figure shows the differentiation of the obtained self-organized aggregations through only varying the neighborhood topology. Fig. 3c highlights results obtained from a DW-2NN topology; Fig. 3b presents results achieved from a DW-3NN topology, whereas results with DW-4NN and DW-5NN topologies are mentioned in Fig. 2c and Fig. 2d.

B. Performance Metrics

In this section we are interested in studying the performance of our proposed model. Specifically we are

focusing in evaluating how the robots achieve a self-organized aggregation while keeping a certain distance among them by only relying on the DW-KNN topology. For that, we use the following metrics to measure the degree of attainment of this objective:

1) *Distance-Weighted Distribution Quality*: We define a new metric to measure the quality of how the overall weighted-distances of the entire swarm is evolved. First the weighted-distances averaged over the different robots and neighbors $AWD(t)$ is calculated as follows:

$$AWD(t) = \frac{1}{NK} \sum_{i=1}^N (\sum_{j=1}^K w_{ij}(t)), \quad (9)$$

Then the distance-weighted quality metric, $F_w(t)$, is gotten by the following equation:

$$F_w(t) = 1 - \frac{1}{\sqrt{AWD(t)+1}}. \quad (10)$$

2) *Aggregation Quality*: The aggregation quality [17], $F_{ag}(t)$ (t), is related to the average distance of the foot-bots from their center of mass. To measure this metric, first the distance $c_i(t)$ of each robot R_i from the center of mass of the group at simulation cycle t is computed:

$$c_i(t) = \left\| X_i(t) - \frac{1}{N} \sum_{j=1}^N X_j(t) \right\|, \quad (11)$$

where $X_i(t)$ is the position vector of the i^{th} foot-bot at time step t . This value is used to compute the aggregation quality $Ag_i(t)$ of the i^{th} foot-bot as follows:

$$Ag_i(t) = \begin{cases} 0, & c_i(t) < \tilde{r}(n) \\ \frac{R(n)-c_i(t)}{R(n)-\tilde{r}(n)}, & \tilde{r}(n) \leq c_i(t) \leq R(n) \\ 1, & c_i(t) > R(n) \end{cases}, \quad (12)$$

where $R(n) = \tilde{r}(n) + 100$, and $\tilde{r}(n) = r_s(\sqrt{n} - 1)$ is defined as an approximation of the radius of the smallest circle that has n robots positioned on the perimeter, and with r_s is the radius of a foot-bot [17].

Then the average aggregation quality, $F_{ag}(t)$, is defined as follows:

$$F_{ag}(t) = \frac{1}{NK} \sum_{i=1}^N (\sum_{j=1}^K Ag_i(t)). \quad (13)$$

3) *Averaged Mean Distance Error*: The Averaged Mean Distance Error metric $AMDE$ is the inter-robots distance error averaged over the different robots and neighbors. It is used mainly to measure the degree of how the robots swarm is keeping a certain desired distance among themselves while they are moving together. The metric is defined as follows:

$$AMDE(t) = \frac{1}{NK} \sum_{i=1}^N (\sum_{j=1}^K (d_{ij}(t) - d_0)). \quad (14)$$

Figure 2. Self-organized aggregation of a swarm of $N = \{50, 100, 150\}$ foot-bots running different DW-KNN topologies.

4) *Results within the metrics:* To evaluate more accurately our proposed approach, we performed 25 runs per each experiment; the experiment duration of each run is set again to 2000 time steps. We plot the results obtained from the simulation of $N = \{50, 100, 150\}$ foot-bot robots within the DW-2NN, DW-3NN, DW-4NN and the DW-5NN topology.

Fig.3 shows the evolution in time of $F_w(t)$, $F_{ag}(t)$ and $AMDE(t)$ for the four DW-KNN case studies respectively. In all plots, the lines show the median values of the 25 runs. As stated in the figures, for all topologies the swarm robot system converges into a stable values for $F_w(t)$ and $F_{ag}(t)$ and a minimum steady value of $AMDE(t)$ for all N robots. This means that the final self-organized aggregations are achieved where all the robots are having nearly the same density when maintaining the same desired inter-robot distance among the DW-KNN robots.

VI. CONCLUSIONS AND FURTHER WORK

When studying collective behaviors of a large number of robots, the inter-robot distance can be of a key importance, but additional properties such as the density of robots could have

more impact on the collective behavior of the whole swarm. Here we rely on both the distance and the density as equal key factors, and we propose a DW-KNN topology to study self-organized aggregation within robots swarm. With this topology, we use the virtual viscoelastic based proximal model proposed in [7] to keep and arrange the neighbors together. The DW-KNN is achieved via defining a distance-weighted function that is based on the SPH interpolation approach to estimate the robot's density in the swarm. Various self-organized patterns are achieved using the ARGoS simulator, and performance analysis shows the efficacy of the proposed approach. Future research will focus on the performance analysis of the approach in presence of more noise in communication sensors, and on the impact of the robot's density in self-organized aggregation when incorporating different SPH density estimation methods; more attention should be also given to study collective decision making on how a robot adapt the value of K to dynamically switch its neighboring relationship.

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Figure 3. Evolution of (a) $F_w(t)$, (b) $F_{ag}(t)$ and (c) $AMDE(t)$ with the DWKNN Topology.

(a)

(b)

(c)

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