Dynamically adaptive data-driven simulation of extreme hydrological flows

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Abstract

Hydrological hazards such as storm surges, tsunamis, and rainfall-induced flooding are physically complex events that are costly in loss of human life and economic productivity. Many such disasters could be mitigated through improved emergency evacuation in real-time and through the development of resilient infrastructure based on knowledge of how systems respond to extreme events. Data-driven computational modeling is a critical technology underpinning these efforts. This investigation focuses on the novel combination of methodologies in forward simulation and data assimilation. The forward geophysical model utilizes adaptive mesh refinement (AMR), a process by which a computational mesh can adapt in time and space based on the current state of a simulation. The forward solution is combined with ensemble based data assimilation methods, whereby observations from an event are assimilated into the forward simulation to improve the veracity of the solution, or used to invert for uncertain physical parameters. The novelty in our approach is the tight two-way coupling of AMR and ensemble filtering techniques. The technology is tested using actual data from the Chile

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tsunami event of February 27, 2010. These advances offer the promise of significantly transforming data-driven, real-time modeling of hydrological hazards, with potentially broader applications in other science domains.

Keywords: Data assimilation, Ensemble Kalman filter, Adaptive mesh refinement, Tsunami, Okada model, Shallow water equations, Uncertainty quantification

1. Introduction

Coastal areas are often densely populated [1] and are most vulnerable to extreme hazards such as storm surges, hurricanes and tsunamis. As of the 2010 census, 39% of the US population or 123.3 million people are concentrated in areas directly on the shoreline [2]. Furthermore, the world coastal population is projected to rise to around 949 million (then projected global population is 8.7 billion) by 2030, and to 1.4 billion people (then projected global population is 11.3 billion) by 2060 [3]. This means that a large portion of the global population is vulnerable to the destruction that may be caused by these extreme events. The effects of hazardous flows can be mitigated by using reliable computational fluid dynamics models that can accurately forecast the events and disseminate information quickly, to aid in preparedness and timely preemptive measures when necessary.

In this paper, we investigate computational algorithms to advance the science of real-time prediction of coastal and overland hydrological hazards. The numerical models that simulate large scale geophysical flows involve solving simultaneous partial differential equations that govern the flow of fluids. Under certain assumptions, the governing equations can be used to simulate flows on a synoptic scale, replicating the path and movement of extreme hydrological flows such as hurricanes and tsunamis [4–7]. The input to these models are in the form of initial condition of the flow, boundary conditions, forcing parameters such as wind stresses, topography of sea bed, salinity etc. Simulations of hydrological hazards are computationally challenging in at least two aspects: they are inherently multi-scale (both spatially and temporally) [8, 9], and there are significant uncertainties in the underlying physical models [10–12]. These uncertainties are inevitable. This means that uncertainty in any of the inputs can lead

25 to solutions that may not replicate reality perfectly. For instance, in tsunami modeling,
26 the sea floor motion that generates the tsunami is not well determined because the fault
27 displacement is dependent on the subsurface geologic structure. A good measure is to
28 assess the results through validation with field data. This can be in the form of satellite
29 imagery or data from buoys in oceans. However, measurements are not only scarce,
30 but also carry some uncertainties due to various factors such as sensor precision or
31 observation location.

32 Data assimilation is a technique whereby the observations along with the uncertain-
33 ties are combined with the forward models in a statistical way to compute the best pos-
34 sible estimate of the state system. There are two main approaches to assimilating data
35 into numerical models - 1) variational methods such as 3D-Var [13] and 4D-Var [14]
36 and 2) sequential methods [15, 16]. This paper uses the two step sequential approach
37 based on the Kalman filter as proposed by [15, 16] in which data are incorporated as
38 they are available. The first step is forecasting using the forward model, followed by an
39 analysis step, statistically averaging the observation with the model forecast according
40 to their estimated uncertainties. For linear systems, under the assumption of unbiased
41 model and incoming uncorrelated Gaussian distributed observation and model errors,
42 the method yields the best linear unbiased estimate. For non-linear dynamics, the Ex-
43 tended Kalman filter was proposed in [17]. These methods, namely Kalman and Ex-
44 tended Kalman filters, propagate the state of the system as well as the associated error
45 covariance, a measure of the uncertainty in the state estimate. Propagation of the error
46 covariance can be computationally prohibitive when the dimension of the state is large.
47 This is typical for large scale hydrodynamic flows, where the dimension of the state
48 can be $\mathcal{O}(10^8)$ [18]. This issue is addressed by ensemble based methods such as the
49 Ensemble Kalman filter [19, 20]. These schemes use Monte Carlo methods to estimate
50 the background error covariance. The method performs an ensemble run of the forward
51 model in the forecast stage from which the sample covariance is calculated and then
52 used at the analysis step. The ensemble based technique asymptotically approaches the
53 Kalman filter in the limit of large ensembles and Gaussian error distribution [21]. The
54 computational cost has been well described in [20].

55 Over the years, a variety of ensemble filters have been developed and implemented

56 in various fields [22–27]. Several review articles [20, 28–32] enumerate the advance-
57 ments in the field, leading to development of various filters, such as optimal interpo-
58 lation [33], EnSRF [34], EAKF [35, 36], SEIK [37, 38], ETKF [39], SEEK [40, 41],
59 RRSQRT [42], ESTKF [43]. Many of these filters have been developed with the un-
60 derlying idea of a low-rank approximation of the error covariance matrix.

61 One of the challenges of data assimilation is that factors such as neglected uncer-
62 tainties, poorly known model errors and a low rank representation of the estimation
63 error via a finite ensemble size lead to underestimation of the error covariances of the
64 filter. The underestimation of the variance is mitigated by the use of covariance infla-
65 tion [44–47]. In addition, localization can be applied to address rank deficiency and
66 spuriously large cross-covariances between different state variables. Two methods of
67 localization often used are 1) Covariance localization [48], where the error covariance
68 matrix is transformed based on physical distance of grid points in the numerical model,
69 and 2) Domain localization [49], where the physical domain is divided into individual
70 local domains for updates. Also, [50] demonstrates a regulated localization scheme
71 that weighs elements of observation error covariance matrix by a localization function
72 of varying width.

73 This paper aims to develop a data assimilation framework that employs the above
74 described aspects of ensemble based filters with large scale hydrodynamic models, and
75 effectively improve forecasting. Ensemble based filters have been implemented in var-
76 ious large scale oceanic models [26, 51–54]. [31] reviews the developments in regional
77 data assimilation over the last 15 years. With respect to employing data assimilation
78 with tsunami simulations, significant literature exists. [55] discusses the need and chal-
79 lenges of real time tsunami forecasting with respect to accuracy, speed and robustness
80 of tsunami warning systems. [56] assumes a linear system and assimilates data using
81 optimal interpolation for the 2012 Haida Gwaii earthquake. In this study, the Kalman
82 gain (called the smoothing matrix) does not change with time and is a function of
83 the station distribution. [57, 58] employed the particle filter method to assimilate tide
84 gauge data into a tsunami simulation model. As part of the NOAA tsunami forecasting
85 system [59], the best estimates of tsunami amplitudes and arrival times for potential
86 at-risk communities are obtained by assimilating real-time event data with the Method

87 of Splitting Tsunami (MOST) model [60, 61].

88 Some of these large scale ocean models employ adaptive mesh refinement (AMR)
89 technique. Adaptive meshing uses algorithms that dynamically refine the grids spa-
90 tially, temporally or both, to obtain fine scale solutions in the areas of interest that can
91 be identified based on user criteria [62–65]. There are two main approaches of imple-
92 menting AMR. Both the methods identify the regions where fine scale solutions are
93 of interest. One class of AMR technique splits the elements of the grid into finer el-
94 ements on the same grid [66]. Another class of AMR techniques involve multi-level
95 mesh structure that is comprised of various refinement levels [64, 67]. AMR technique
96 proves to be advantageous towards reducing computational cost. In large scale hydro-
97 dynamic models, AMR can be used to dynamically obtain fine scale solutions near the
98 coast, where we are most interested in the solution, while retaining a coarse mesh in
99 deep oceans. For instance, in the case of tsunami simulations, we can dynamically
100 refine the mesh along the moving wave front [68, 69].

101 In the area of data assimilation with AMR, several methods involving 3D-Var and
102 4D-Var have been proposed by [70, 71]. In a very recent study, [72] implemented
103 the ensemble based filter for data assimilation. The ensembles were interpolated to
104 a reference mesh, while accounting for the interpolation errors using conservative in-
105 terpolation operators. However, no prior work has analyzed the implementation of
106 ensemble based filter for data assimilation with multi-level AMR, and this paper is
107 the first attempt to provide primary insights in this direction. Our goal is to build an
108 advanced data-driven AMR model for efficient simulation and forecasting of coastal
109 marine dynamics.

110 In this paper, we describe the traditional ensemble Kalman filter and an AMR capa-
111 ble numerical model - GEOCLAW. Coupling ensemble based assimilation with AMR
112 can be challenging as the AMR algorithm may dictate the ensemble members to adapt
113 in their own way. This makes it difficult to compute quantities such as the mean and
114 the error covariance of the state vector, essential for the assimilation step. We discuss
115 the hurdles faced in detail and devise a methodology for a suitable solution. In the
116 process, we define the assimilation field and the observation operator. We propose a
117 methodology wherein data are assimilated at all the multi-scale levels. Further, we ap-

ply the data assimilation framework on tsunami models and demonstrate an algorithm for the initial ensemble generation, which is essential for any ensemble based filtering technique. The final sections put forth validation results from a real event - the Chile tsunami of February 27, 2010. We perform twin experiments by testing the assimilation process with synthetic measurements. The experiments will look into the aspects of localization radius, observation configurations, minimum ensemble size and the initial ensemble affecting state estimates. This is followed by an AMR-coupled assimilation run of the observed real event, taking into account the available gauge data.

2. Ensemble based Kalman filters

The traditional ensemble Kalman filter EnKF described in [19, 20] is formulated as a sequential process of the forecast step followed by the analysis step. Unlike EnKF, filters such as SEIK [38] and ETKF [39] are implemented as ensemble square-root-filters that split the analysis step into a state update step and a resampling step. The state update step computes the analysis state and its error covariance matrix and the resampling step generates the ensemble members that exactly represent the analysis state and its error covariance matrix. In the current study, we have implemented the Error Subspace Transform Kalman filter (ESTKF) [43], a variant of the SEIK filter that performs ensemble transformation in the error subspace. The ESTKF combines the advantages of SEIK filter and ETKF [43]. ESTKF computes ensemble transformations that are identical to ETKF, but at a lower computational cost. Further, unlike the SEIK filter, the ensemble transformation in ESTKF is independent of the order of ensemble members in the ensemble matrix. This was demonstrated in the studies ¹ where the applications are close enough to the current one. This section outlines the ESTKF algorithm that is described in detail in ([43]).

Consider a dynamical system M_k that forecasts the true state from time t_k to t_{k+1} as,

$$x_k^t = M_k(x_{k-1}^t) + \eta_k,$$

¹<http://pdaf.awi.de/trac/wiki/WhichFiltertouse>

where x_k^t is the true state and η_k is the system noise at time t_k assumed to be unbiased. Further, consider the observation y_k of the state at time t_k given by,

$$y_k = H_k x_k^t + \epsilon_k,$$

where ϵ_k is the measurement error at time t_k assumed to be unbiased. H_k is the operator that maps the field of interest from the state space to the observation space. With an ensemble of size N , we define the forecasted state matrix X_k^f as,

$$X_k^f = \begin{bmatrix} \vdots & \vdots & \vdots \\ x_{k,1}^f & \dots & x_{k,p}^f & \dots & x_{k,N}^f \\ \vdots & \vdots & \vdots \end{bmatrix},$$

where $x_{k,p}^f$ is the p^{th} forecasted ensemble member at time t_k . We also define $\bar{X}_k^f = \bar{x}_k^f \mathbf{1}_d$, where the mean of the forecasted ensemble states is $\bar{x}_k^f = \frac{1}{N} \sum_{i=1}^N x_{k,i}^f$ and $\mathbf{1}_d$ is the matrix such that all the entries are 1s. At time t_k , P_k^f is the forecast error covariance and $R_k = \epsilon_k \epsilon_k^T$ is the **observation error covariance**. **The observation error comprises of two components, the measurement error and the representation error[73]. For the various test cases discussed later in the paper, we assume that both the components have been accounted by a constant root mean square error value.** Similar to the forecast variables defined above, we define X_k^a as the analysis state matrix at time t_k comprising of the analysis state ensemble members $x_{k,p}^a$. In ESTKF, the forecast error covariance matrix at time t_k is represented as,

$$P_k^f = L_k G L_k^T,$$

where,

$$L_k = X_k^f \Omega,$$

$$G = \frac{1}{N-1} \mathbb{I}_{(N-1) \times (N-1)},$$

$$\Omega_{i,j} = \begin{cases} 1 - \frac{1}{N} \frac{1}{\frac{1}{\sqrt{N}} + 1} & , i = j, i < N \\ -\frac{1}{N} \frac{1}{\frac{1}{\sqrt{N}} + 1} & , i \neq j, i < N \\ -\frac{1}{\sqrt{N}} & , i = N \end{cases}.$$

142 $\Omega_{i,j}$ is the projection matrix that has a full rank, zero column sums with the columns
 143 being orthonormal. $\mathbb{I}$ is the Identity matrix. The state vector $X_k^f \in \mathbb{R}^{n \times N}$, $P_k^f \in \mathbb{R}^{n \times n}$,
 144 $L_k \in \mathbb{R}^{n \times (N-1)}$, $G \in \mathbb{R}^{(N-1) \times (N-1)}$ and $\Omega \in \mathbb{R}^{N \times (N-1)}$. The ESTKF algorithm
 145 can be outlined as follows:

146 1. Forecasting from t_{k-1} to t_k :

Starting from a given analysis state matrix X_{k-1}^a , integrate each ensemble member $x_{k-1,p}^a$, with the dynamical system M_k as,

$$x_{k,p}^f = M_k(x_{k-1,p}^a).$$

147 2. Updating at time t_k :

Compute the analysis state estimate $\bar{x}_k^a$ as,

$$\bar{x}_k^a = \bar{x}_k^f + L_k A_k (H_k L_k)^T R_k^{-1} (y_k - H_k \bar{x}_k^f),$$

where A_k is the transform matrix given by,

$$A_k^{-1} = \rho(N-1)\mathbb{I} + (H_k L_k)^T R_k^{-1} (H_k L_k).$$

148 ρ is the inflation factor used to enhance the filter robustness and to partially
 149 account for model errors [38, 40]. More sophisticated methods to treat model
 150 errors have been discussed in [74].

151 3. Resampling:

This step is the second order exact sampling that computes the ensemble states.

$$X_k^a = \bar{x}_k^a \mathbf{1}_d + \sqrt{N-1} X_k^f \Omega C_k \Omega^T.$$

152 where C_k is the square root of A_k .

153 3. Model description: GEOCLAW

The mathematical model used is the 2D depth averaged shallow water equations in x and y directions, governed by,

$$h_t + (hu)_x + (hv)_y = 0,$$

$$(hu)_t + (hu^2 + 1/2gh^2)_x + (huv)_y = -ghB_x - Du,$$

$$(hv)_t + (huv)_x + (hv^2 + 1/2gh^2)_y = -ghB_y - Dv,$$

154 where h is the total depth of water column, u and v are depth averaged velocities in the
 155 x and y directions respectively, B is the bathymetry, D is the drag coefficient and g is
 156 the acceleration due to gravity.

157 The equations are solved using GEOCLAW, an open-source package written in For-
 158 tran [69], with a user interface in Python. GEOCLAW is a part of the general Clawpack
 159 (Conservation Laws) package [68]. It uses high-resolution shock-capturing finite vol-
 160 ume methods on logically Cartesian grids to solve non-linear hyperbolic systems of
 161 partial differential equations [75].

162 GEOCLAW uses AMR algorithms as discussed in [69, 75–77], to simulate multi-
 163 scale geophysical flows. AMR is implemented using a patch-based mesh refinement
 164 approach, shown in Figure ,1, where multiple levels of structured meshes are used to
 165 reach a fine resolution [64]. For instance, mesh levels can be created that can range
 166 from a few meters at the coastline, to a few hundred kilometers in the deep ocean.

167 For notation, a *Level* k refinement is called a k^{th} level patch. Thus, the coarsest
 168 mesh is called a *Level* 1 patch. For tsunami modeling, GEOCLAW has the capability
 169 to use anisotropic refinement in time as well.

170 4. Coupling GEOCLAW and data assimilation

171 This section describes the challenges faced in the development of the online cou-
 172 pled system of GEOCLAW with the ensemble assimilation techniques and presents pro-
 173 posed solutions. Section 4.1 addresses the "supermesh" methodology used for handling
 174 the ensemble members with different AMR structure, which is essential for the assim-
 175 ilation step. Section 4.2 describes the quantity of interest for assimilation and the con-
 176 struction of the state vector. Section 4.3 describes the construction of the observation
 177 operator.

178 4.1. Mesh union strategy

179 The primary difficulty in using AMR in conjunction with ensemble filtering schemes
 180 is that the regridding operation during the forecast allows every ensemble member to

181 adapt its grid in its own way; hence, at the end of the forecast stage, one ensemble
 182 member may obtain a grid structure that is different from the other ensemble members.
 183 Figure 1 illustrates a hypothetical scenario of two ensemble members at an analysis
 184 stage, with different refined patches at level 2 and level 3. Thus, at finer levels, the en-
 185 semble members have varying number of cells and thereby end up with a varying state
 186 vector length. This means that statistical quantities such as covariance of the state,
 187 essential for the update step, are no longer straightforward to compute.

188 This motivates us to develop a methodology that can enforce all the ensemble mem-
 189 bers to produce the same mesh structure at all levels at the analysis stage. At the as-
 190 simulation stage, (1) construct a union of meshes of all the ensemble members, also
 191 called a supermesh, (2) communicate the information of every ensemble member to
 192 the supermesh, (3) perform analysis, (4) communicate the updated information back
 193 from the supermesh to the respective ensemble member, and (5) continue the forecast.

Remark i - Construction of a supermesh is done by modifying the default re-
 gridding operation to a constrained regridding operation. The mesh structure of each
 ensemble member is communicated to every other ensemble member. Thus, at any
 given point in the domain, we know the maximum refinement level that exists for at
 least one ensemble member. Formally, let $\phi_{(x,y)}^{(i)}$ and $\delta_{(x,y)}^{(i)}$ be the maximum refinement
 level at a point (x, y) for the i^{th} ensemble member, before and after the analysis step
 respectively. That is,

$$\delta_{(x,y)}^{(i)} \triangleq \max_i (\phi_{(x,y)}^{(i)}).$$

194 After the analysis step, the mesh structure for all the ensemble members is the
 195 same. Hence, $\delta_{(x,y)}^{(i)} \equiv \delta_{(x,y)}$. This way we ensure that during the assimilation, all the
 196 members have the same mesh structure at the start of the assimilation step.

197 **Remark ii** - AMR algorithm in GEOCLAW constructs a finer mesh to Level $k + 1$
 198 by first flagging cells in Level k . At Level k , the flagged cells are clustered based on a
 199 clustering parameter that defines the cut-off percentage of flags required to bound the
 200 flags within a rectangular region and construct a patch [78]. This implies that the union
 201 of two ensemble members may not lead to an "exact union". Nevertheless, since all
 202 the ensemble members have the same flags at all levels, the deterministic nature of the

203 clustering algorithm guarantees the same finer patches for the finer levels. When this
204 procedure is applied recursively from coarsest mesh to finer mesh, all the ensemble
205 members end up with the same mesh structure.

206 **Remark iii** - A regrid operation in the forecast step (after assimilation) removes
207 the unnecessary refinement and the ensemble members undergo forecasting in the usual
208 manner thereafter.

209 **Remark iv** - In practice, it is observed that the mesh structures of the ensemble
210 members do not vary in high degree from each other.

211 Implementation of the above strategy of the union of meshes leads to the final mesh
212 shown in Figure 1, on which assimilation will be performed.

213 Here we illustrate the generation of the supermesh grid via a GEOCLAW run of a
214 model problem that simulates the water flow in a parabolic bowl. The initial condition
215 of the water surface elevation is a Gaussian hump shown in Figure 2. We run the simu-
216 lation for 3.2 seconds. The maximum AMR level that is possible during the simulation
217 is set to 2. We generate two ensemble members with different initial amplitudes. Fig-
218 ure 3 shows the snapshots of water surface elevation at instances when the supermesh
219 is formed. The left and the middle column represent the two ensemble members, and
220 the right column is the case if the supermesh was formed. In the figure, the rectangular
221 patches without any cells (clean patches) are the level 2 patches. It is seen that from
222 2.56 seconds, as the water front is moving radially outward, different initial amplitudes
223 causes the ensemble members to adapt with different meshes, forming a supermesh
224 accordingly. As mentioned in Remark ii above, the supermesh grid structure is not a
225 perfect union of grids of the ensemble members.

226 4.2. Construction of state vector

For the current study, the state vector is constructed in a way that all the cells at all
the levels of the supermesh are combined into one single array. We define a patch as
the collection of grids at a given level. If P is the maximum refinement level in δ , then

we define the state to be assimilated (ψ), water surface elevation as

$$\psi \triangleq \begin{bmatrix} \text{level 1 patch} \\ \vdots \\ \text{level } k \text{ patch} \\ \vdots \\ \text{level } P \text{ patch,} \end{bmatrix},$$

where we have augmented the state vector at various levels into a new state vector. For instance, the state vector constructed from the hypothetical union of mesh in Figure 1 would be of size $64 + 112 + 160 = 336$. This means that if our state vector has a cell value at a finer level, then it also contains the underlying coarser cell value.

4.3. Construction of observation operator

In a multi-level AMR structure, a measurement may come from a location where more than one level of refinement exists. A measurement recorded at a physical location that belongs to the finest cell will also belong to the underlying coarse cells. Given that the state vector combines the variables from all the levels at a given physical location, the observation operator is constructed in such a way that the same observation realization is used in all the levels at that physical location.

For instance, consider the mesh structure in Figure 4 that represents a single coarse cell denoted by v_1 at Level 1. The coarse cell v_1 is refined to finer cells denoted by v_2, v_3, v_4, v_5 at Level 2. The cell v_2 is further refined to cells v_6, v_7, v_8, v_9 at Level 3. The state vector for this simple structure is $\psi = [v_1, v_2, \dots, v_8, v_9]^T$. Depending on the location of the incoming measurement, multiple scenarios occur. **Scenario 1** - If there exists just a single incoming observation M_1 at a point belonging in cell v_6 , then clearly, M_1 belongs to v_2 and v_1 as well. Hence the measurement recorded for v_1, v_2 and v_6 is M_1 . **Scenario 2** - If there exists two incoming observations M_1 and M_2 at the locations belonging to cells v_6 and v_9 , respectively, then both the measurement values belong to v_2 and v_1 as well. Hence, the measurement values recorded at v_6 is M_1 , at v_9 is M_2 and at v_1 and v_2 is taken as the average of M_1 and M_2 . This idea can be extended to other scenarios as well.

250 4.4. Data assimilation framework

251 We have used the data assimilation software - Parallel Data Assimilation Frame-
252 work (PDAF) [79], a Fortran library for various ensemble filters such as EnKF, ETKF,
253 ESTKF, SEIK, SEEK and the localized filters such as LETKF, LSEIK, LESTKF. The
254 library supports parallel ensemble forecasts and assimilation in MPI and OpenMP. As
255 seen in Figure 5, each GEOCLAW run is a serial run representing a single ensemble
256 member. The forecast of the ensemble members are executed in parallel under MPI
257 parallelism. For the assimilation stage, the forecasted state vectors are gathered to the
258 root for the update. The updated states are scattered back to the cluster for next forecast
259 stage. For the current study, a single ensemble member is assigned to a single core dur-
260 ing the forecasting. Hence, a total of N ensemble members is performed by N cores
261 in a cluster, wherein, each core is assigned to one ensemble member. The assimilation
262 is performed by a single core as well. It should be noted that the PDAF library has
263 the capability to adjust the number of cores per ensemble member during the forecast
264 stage and the number of cores in the update step. The simulations were carried on a
265 local desktop with 8 cores and not in a distributed environment. A virtual MPI environ-
266 ment was created such that the 8 cores would mimic the user-defined number of cores.
267 Hence, the wall clock time will not be a good estimate for analyzing execution time. In
268 the present study, the coupling is done in such a way that the forecasts are distributed
269 amongst the processors and the analysis is done by a single processor.

270 5. Initial ensemble generation

271 5.1. Motivation

272 Generating a representative initial ensemble to start the assimilation systems is im-
273 portant to obtain good performance with EnKF. Several approaches have been proposed
274 [37, 80, 81]. For instance, [37] discusses the idea of generating the initial ensemble
275 for large scale ocean and atmosphere models via second-order exact sampling from a
276 model trajectory that is obtained from years of spin-up. However, unlike the ocean and
277 atmospheric models, where the initial ensemble of states can be produced via model
278 spin-ups, tsunamis occur relatively instantly due to the rapid bathymetry changes. We

279 address this inability to generate ensembles via model spin-ups by an algorithm that
 280 can effectively map the uncertainty in the bathymetry changes at the event of tsunami
 281 to the initial ensemble.

The tsunami simulation models use bathymetry changes due to the associated earthquake. For instance, the Okada model [82] relates the topography change Δd , to various fault parameters,

$$\Delta d \sim f(\textit{strike}, \textit{length}, \textit{width}, \textit{depth}, \textit{slip}, \textit{rake}, \textit{dip}, \textit{longitude}, \textit{latitude}).$$

282 This means that any uncertainty in the fault parameters can produce the possible bathymetry
 283 changes that will in turn produce the subspace of initial states of the tsunami.

284 5.2. Procedure

285 We assume that the fault parameters have a multivariate Gaussian distribution with
 286 a specified mean and a covariance. We also assume that the fault parameters are not
 287 correlated to each other and hence the covariance matrix is diagonal with each entry
 288 representing the variance of a fault parameter. Under these assumptions, first perturb
 289 the fault parameters using a relatively large sample N_s . The subspace of the perturbations
 290 generated, represents the possible bathymetry deformations. Thus, a forward
 291 integration of the model up to a small time (called checkpoint time) with each of the
 292 samples as the initial condition, generates the subspace of initial tsunamis. Similar
 293 to the idea proposed by [37, 81], we adopt second-order exact sampling from various
 294 model states. But unlike the temporal snapshots of the states, we use the snapshots of
 295 the various possible tsunamis that we have generated, to develop a perturbation matrix
 296 of a desired size. These perturbations can then be later added to the mean water surface
 297 elevation at the checkpoint time and in turn generate the initial ensemble.

298 5.3. Practical implementation

299 Large scale ocean model softwares use checkpoint and restart capabilities, wherein,
 300 a forecast can be resumed from a checkpoint time. We rely on this capability to implement
 301 the initial ensemble generation algorithm. Let us denote t_o , t_{cp} and t_{end} as the
 302 initial time, the checkpoint time and the simulation final time respectively.

- 303 1. Perturbation generation (Checkpoint run) - At time $t_o = 0$, draw a relatively
304 large random sample (say $N_s = 100$) of the fault parameters of the Okada model,
305 under the assumption that the parameters are uncorrelated to each other and are
306 normally distributed with a user specified mean and a diagonal covariance. Inte-
307 grate using each of the fault up to a reasonably small time, called the checkpoint
308 time t_{cp} to generate the subspace of possible tsunamis.
- 309 2. Use the 100 model states (N_s) and a user desired ensemble size (n_s) to gen-
310 erate the perturbation matrix. Add the perturbations to the mean water surface
311 elevation at time t_{cp} .
- 312 3. Assimilation run - With the assimilation facility activated, resume the code at the
313 checkpoint time t_{cp} till the final time t_{end} .

314 **Remark i** - For a given problem, the eigen decomposition has to be performed only
315 once at the initial time $t_o = 0$. The eigenvectors and the values are small in size and
316 can be stored on disk. This can be a part of the preprocessing stage.

317 **Remark ii** - A small checkpoint time t_{cp} implies that forecasting N_s (here 100)
318 ensemble members till the checkpoint time is not so computationally expensive.

319 6. Assimilation experiments on Chile 2010 tsunami event

320 On February 27, 2010 at 03:34 local time (06:34 UTC), an 8.8 magnitude earth-
321 quake off the coast of Chile triggered a massive tsunami, causing catastrophic damage
322 to life and property.

323 In this section, we will apply the discussed methodologies of initial ensemble gen-
324 eration and AMR coupled data assimilation framework on a test setup that simulates the
325 event. We first describe the model parameters for simulating the actual tsunami event.
326 To assess the assimilation system, we perform twin experiments, wherein we treat the
327 above run as the "truth" and use the water surface elevation data as measurement values
328 for the assimilation runs. We will demonstrate that assimilation is effective on model
329 runs that have the initial conditions that are different from the true initial conditions.
330 Note that the initial condition here refers to the fault parameters causing the associated

331 earthquake. We will also perform experiments to assess the effect of localized assim-
332 ilation and the placement of measurement sensors on the assimilated solution. This
333 is followed by an additional twin experiment that analyzes the effect of initial ensem-
334 ble generation and the ensemble size on assimilation runs. Further, we also validate
335 our results using real gauge data of the actual event. In the process we will demon-
336 strate the efficacy of assimilation with AMR to obtain improved state estimates and the
337 corresponding uncertainty.

338 6.1. Model parameters

339 For the tsunami simulation, the computational domain is the South Pacific ocean
340 off the Chilean coast shown in Figure 6. The domain contains islands/features such as
341 Easter Island and the Galapagos Islands. In the coarsest mesh, these features are not
342 visible. During the simulation, as the mesh refines, the features become more appar-
343 ent. The model parameters, including the mesh parameters, time stepping parameters,
344 and topography are given in Table 1. The marked points 15, 12 and 32412 represent
345 locations where the quantities of interest, such as water surface elevation, are recorded.
346 In Figure 6, the two black lines given by the horizontal line at -30 and the vertical line
347 at -90 , represent the partition of the domain at the coarsest grid level into 4 patches.
348 This is related to the specifics of the GEOCLAW software.

Table 1: Model parameters for simulating Chile tsunami event of 2010

Computational domain	$x \in [-120^\circ, -60^\circ]$ and $y \in [-60^\circ, 0^\circ]$
Coarsest discretization	$100 \text{ cells} \times 100 \text{ cells}$
Simulation time	13 hours
Time stepping	Initial time step = 0.016 seconds and desired $cfl = 0.75$
Topography	10 minute ETOPO2 from NGDC ²

²National geophysical data center (NGDC) GEODAS grid translator 2010

Table 2: Topography parameters for ensemble generation and twin experiment

Statistic	Strike	Length	Width	Depth	Slip	Rake	Dip	Longitude	Latitude
<i>Truth</i> *	16.0	450.0	100.0	35.0	15.0	104.0	14.0	-72.668	-35.826
<i>Biased</i> *	—”—	—”—	—”—	—”—	35.0	—”—	—”—	—”—	—”—

* Mean topography. Units [*meter, kilometer, kilometer, kilometer, meter, meter, meter, meter, °, °*]

349 6.2. Reference solution

350 The reference solution is simulated using the “true” fault parameters of the Okada
351 model, given in Table 2 that produces the associated slip in the fault region and the sea
352 floor deformation shown in Figure 7a. The bathymetry change causes a tsunami with
353 the water surface elevation snapshots given in Figure 8a. The figures show a primary
354 wave traveling in the north-west direction. At approximately 4.0 hours, a section of the
355 wave traveling along the north, reflects off the coast of Peru to form a secondary wave
356 traveling in the south-west direction.

357 6.3. Twin experiment

358 In this experiment, we set up a model run with biased fault parameters in compar-
359 ison to the truth. The twin experiment is aimed to demonstrate that the run with the
360 biased fault parameters can be effectively assimilated using the measurements from the
361 “true” solution.

362 6.3.1. Biased run

363 The bias in the fault parameters given in Table 2 is a slip of 35.0 meters compared
364 to 15.0 meters for the truth. The additional slip in the fault regions relates to a larger
365 bathymetry change shown in Figure 7b, leading to a stronger tsunami. This can be
366 seen in the snapshots in Figure 8b for a free run under this biased initial condition.
367 Note that free run means a model run that is not assimilated. Comparing the “true”
368 state in Figure 8a with the free run state in Figure 8b, the tsunami generated from the
369 biased fault parameters has a larger amplitude than the true state, till the waves exit the
370 computational domain.

371 6.3.2. Initial ensemble generation

372 To generate the initial ensemble of states, we implement the algorithm discussed
 373 in Section 5. We treat the biased slip fault parameter as the mean slip for the free run.
 374 We take a sample of 100 (N_s) slips assuming that the slip follows a normal distribution
 375 of mean 35.0 meters and a standard deviation of 4 meters. Figure 9 shows the sample
 376 distribution of the slips generated with a mean of 20.0 meters added to the true slip of
 377 15.0 meters. A forward integration of the model until the checkpoint time (t_{cp}) of 2.0
 378 hours, with each unique fault parameter as an initial condition, results in the subspace
 379 of tsunamis. Proceeding with a singular value decomposition of the covariance matrix,
 380 followed by the construction of the perturbation matrix, we generate the initial ensemble
 381 of desired size. For illustration, Figure 10 is the mean water surface elevation at
 382 the checkpoint time (t_{cp}) of 2.0 hours for the model run simulated using the biased
 383 fault parameter as the initial condition. Figure 11 shows the first 9 members of the 16
 384 ensembles (n_s) that are generated after using the initial ensemble generation algorithm.
 385 At the checkpoint time (t_{cp}) of 2.0 hours, we resume the simulation, while assimilating
 386 the ensemble members with the measurement data from the “true” solution.

For all our experiments, we have applied ESTKF to assimilate the synthetic data every 15 minutes, setting the inflation factor to 1.1. We assess the assimilated water surface elevation, in comparison to the model free run (non-assimilative run). The error norm used is,

$$\|e(t)\|_2 = \|\psi_{true}(t) - \hat{\psi}(t)\|_2,$$

387 where $\hat{\psi}$ can be the assimilated state or the free run state.

388 6.3.3. Experiment 1: Effect of observation configuration and localization radius

389 In this experiment, we analyze the effect of the number of measurements and localization
 390 radius on the assimilated state estimates. We have chosen various observation
 391 grids ranging from the region containing 4 (a sparse network), 12, 35 and 42 (a dense
 392 network) sensors, distributed in the Pacific ocean as shown in Figure 12. For each of
 393 the observation grids, we use 16 (n_s) ensemble members with localization radius of 3° ,
 394 5° , 10° , 15° , 20° , 25° and 35° , along with a global assimilation. The observation error

395 is assumed to be normally distributed with the standard deviation of observation error
396 set to 0.005 meters. At every assimilation step, the state estimate after the assimilation
397 is compared to the truth and the errors are calculated.

398 At the fault region, varying slip parameters for the truth and the biased model runs
399 leads to a differing bathymetry profile. For error calculation for this experiment, we
400 omit the region around the fault, that is $(-76.0^\circ, -68.0^\circ) \times (-40.0^\circ, -32.0^\circ)$. The
401 error plots for all the mentioned observation configurations are given in Figure 13.
402 It was found that the error plots produced without ignoring the fault region give a
403 similar trend as the plots in Figure 13. However, ignoring the fault region makes the
404 trend more pronounced. The figures plot the difference between the free run and the
405 truth, the difference between the globally assimilated state estimate and the truth, and
406 the difference between the locally assimilated state estimate and the truth for various
407 localization radii.

408 Figure 13 indicates that irrespective of the observation configuration, as the local-
409 ization radius increases, the estimated error between the “true” solution and the as-
410 similated state decreases till a *critical localization radius*. Assimilation with further
411 increase in the localization radius (continuing towards a global assimilation) results in
412 errors between assimilation run and the truth being greater than the error between free
413 run and the truth, implying that assimilation starts to degrade the solution and performs
414 worse than the case when the state was not being assimilated. Also, even with a small
415 ensemble of 16 members (n_s), for the assimilation runs with localization radius less
416 than the critical radius, the assimilation run estimates have smaller error compared to
417 the error between non-assimilative free run and the truth, until at least approximately
418 30 assimilation steps, equal to 450 minutes (or 7.5 hours) of simulation. This corre-
419 sponds to the time period of 2 hours to 9.5 hours, which may be critical for evacuation
420 measures. For instance, the preparation time required for incoming tsunami at Hawaii
421 is around 10 hours when the tsunami is generated from Chile and 4 hours when the
422 tsunamis is generated from Alaska³. Hence, assimilation proves to be effective during
423 this critical time by providing improved estimate of the state.

³<http://ptwc.weather.gov/hawaii.php>

424 It is found that after 10 hours of simulation, the error norms are practically the
 425 same, whether data are assimilated or not. That is the assimilated run converges to the
 426 free run. This can be clearly observed in the assimilation of 42 observations in Fig-
 427 ure 13d. This is because, after around 10 hours, the tsunami wave front has traveled
 428 across the computational domain, leaving behind no significant wave fronts in the re-
 429 gion. However, in assimilation of other observation grids, the errors seem to increase
 430 after nearly 30 assimilation steps. This is attributed to multiple factors. One of the rea-
 431 sons is that a smaller localization radius does not cover the regions near the coast and
 432 some small errors persist. The assimilation with 42 observation grids witnessed local
 433 assimilation of the coastal regions as well and this reduced the error further. Thus, for
 434 this setup, it is found that "small" localization radius with measurements closer to the
 435 coastal areas can prove to be effective for assimilation. Increased errors near the end of
 436 the simulation can also be attributed to errors due to small ensemble size and ensemble
 437 variance inflation at every assimilation step. In Section 6.3.4, we will demonstrate that
 438 by taking larger ensemble size, this error can further be reduced.

439 For the observation configuration with 4, 12, 35 and 42 observations, the critical
 440 localization radius is approximately 25° (approximately 2500 kilometers of land), 15°
 441 (approximately 1500 kilometers of land), 15° and 15° , respectively. The state estimate
 442 for each of the assimilation runs with the critical localization radius is given in Fig-
 443 ures 14a, 14b, 14c and 14d respectively. From the state estimates with assimilation
 444 using observation configuration of 4 sensors and 12 sensors (Figure 14a and 14b), at
 445 5.0 and 6.0 hours, the tsunami wavefront has smaller estimated water surface elevation
 446 (amplitude) when assimilation is performed with 12 observations, when compared to
 447 assimilation with 4 observations. The smaller amplitude conforms to the truth at the
 448 time of 5.0 and 6.0 hours in Figure 8a. This suggests that in this particular experi-
 449 ment, assimilating with 12 observations is more effective than using 4 observations.
 450 We further compare the filter state estimate (Figure 14b and 14c) obtained by assimi-
 451 lating observations with configuration of 12 sensors to the filter state estimate obtained
 452 by assimilating observations with the configuration of 35 sensors. The observation
 453 configuration with 35 sensors has more sensors within the same area compared to the
 454 observation configuration of 12 sensors. Further, the edge that connects the rightmost

sensors is equally away from the coast for both the configurations. It is observed that there is not much improvement in the estimated state, even though more observations are assimilated. The observation configuration with 42 sensors has 7 additional sensors to the right, compared to the 35 observation configuration. That is, the edge joining the rightmost sensors for observation configuration with 42 sensors are closer to the coast compared to the observation configuration of 35 sensors. Comparing water surface elevation snapshots at around time of 4.0, 5.0 hours in Figure 14b and 14c, assimilation using 42 sensors results in waves with higher estimated water surface elevation traveling along the coast when compared to assimilation with 35 observations. This reaffirms the fact that assimilation of sensor data closer to the coast may be helpful.

In summary, this experiment illustrates that the assimilation runs do not always guarantee that the analysis state will have lesser errors than the free run solution. In fact, in this experiment, the assimilation runs with high localization radius perform worse than the free run. One of the possible reasons might be that with high localization radius, the analysis step updates the state vector unreasonably in the regions that are physically far from the observation coordinates. Other reasons include existence of various confounding variables such as ensemble size, observation configuration, standard deviation of observation standard error and most importantly the difference of the initial condition and the truth. To get insights into this phenomenon, we have analyzed assimilation run with various localization radii, while keeping the above-mentioned variables fixed. It is found there exists an optimum localization radius that guarantees that the assimilation is effective. Thus, this experiment of assessing the optimum localization radius serves as a method for model calibration.

6.3.4. Experiment 2: Effect of initial ensemble generation

In the previous experiment, the initial ensemble was generated via perturbations in the slip parameter only. Further, though the prior slip information of 35.0 meters was significantly away from the true slip of 15.0 meters, the above experiment demonstrated the effectiveness of assimilation. This experiment differs in a few aspects - 1) the initial ensemble is generated with perturbations in all the fault parameters, 2) the observation grid chosen is 142 sensors in Figure 15, covering regions that are closer to the shore,

3) the synthetic observations are generated by a model run that takes a random sample of fault parameters from the multivariate Gaussian distribution assuming the mean of the fault parameters is given by μ_{topo} in Table 3 and covariance matrix is diagonal with the diagonal entries given by σ_1^2 , 4) the sample of 100 (N_s) fault configurations for the initial ensemble generation algorithm assumes a distribution of fault parameters with mean parameters given by μ_{topo} in Table 3 and variance terms of the parameters given by σ_2^2 , 5) the observation error is assumed to be normally distributed with the standard deviation of observation error set to 0.001 m.

Table 3: Topography parameters for ensemble generation for twin experiment 2

Statistic	Strike	Length	Width	Depth	Slip	Rake	Dip	Longitude	Latitude
μ_{topo}^a	16.0	450.0	100.0	35.0	15.0	104.0	14.0	-72.668	-35.826
$\sigma_1^2{}^b$	4.0	150.0	50.0	10.0	5.0	20.0	4.0	3.0	3.0
$\sigma_2^2{}^c$	1.6	45.0	10.0	3.5	1.5	10.4	1.4	2.0	2.0

^a Mean topography. Units [*meter, kilometer, kilometer, kilometer, meter, meter, meter, meter, °, °*]

^b Variance for generating initial ensemble for twin experiment (*Units*²)

^c Variance for generating synthetic observations (*Units*²)

We can analyze the effect of perturbing all the fault parameters in the process of generating the initial ensemble mentioned in Section 5. Figure 16 plots the eigen decomposition of the covariance matrix of 100 (N_s) model states, with the red line indicating the cumulative percentage variance explained by the eigen modes. From the figure, it can be seen that the first 10 eigenvalues explain more than 85% of the total variance. We can also analyze the effect of the sample size on the percentage variance explained by the eigen modes in Table 4. The table suggests that as the sample size increases, a higher number of eigen modes account for the unique variance among the states. It is found that the subspace of the initial ensemble that can be generated is richer when compared to just perturbing the slip as in twin experiment 1.

Table 4: Percentage captured variance with varying ensemble size in initial ensemble generation in twin experiment 2 of the Chile tsunami simulation

Ensemble	% explained variance by eigen modes												Total
	1	2	3	4	5	6	7	8	9	10	11	12	
16	32	21	12	10	7	5	3	3	1	0	0	0	100
32	27	18	11	9	7	6	4	3	2	2	1	1	91
40	27	19	10	8	7	6	4	3	2	2	1	1	90
64	24	18	10	8	6	6	4	3	2	2	1	1	85
80	24	17	10	7	6	5	4	4	3	2	2	1	85
100	24	16	10	7	6	5	4	4	3	2	2	1	84

503 Since we have taken a reasonably dense observation network, with observations
504 covering near the coastal regions, the error calculations need not ignore the fault re-
505 gions. Moreover, it is found that lower observation standard deviation of 0.001 meters
506 gives reasonably good performance for the global assimilation as well. However, this
507 is subjective to various factors defining the test setup such as the initial conditions,
508 observation network etc. Figure 17a plots the difference between the “true” state and
509 free run (non-assimilative run) state, and the difference between the “true” state and the
510 globally assimilated state estimate for various ensemble sizes at the assimilation times.
511 By analyzing the effect of ensemble size, it is found that an assimilation run with 64
512 members consistently produces more accurate state estimates than the non-assimilative
513 free run. Further, as the ensemble size increases, the error decreases. It is found that the
514 number of members required depends on how far the initial ensemble is from the truth.
515 That is, by repeating the experiment with different sets of synthetic observations, it is
516 found that in some cases assimilation run with just 32 ensemble members are sufficient
517 to produce consistently less error than the free run. This is tested by another set of syn-
518 thetic measurements obtained from a model run that uses the topography parameters
519 sampled from a multivariate normal distribution with mean and variance as μ_{topo} and
520 σ_2^2 respectively in Table 3. Assimilation is performed using the same initial ensemble
521 that was generated earlier. From Figure 17b, assimilating the new measurement values

522 results with 32 ensemble members providing consistently smaller errors than the free
523 run. This suggests that the minimum ensemble size required for effective assimilation
524 depends upon how far the first guess of the initial condition is from the truth.

525 For the case just described (Figure 17b), Figure 18a shows the estimated state at
526 various assimilation steps and Figure 18b shows that over the simulation time, suc-
527 cessive assimilations reduce the root mean square in the water surface elevation. This
528 translates to the fact that with every assimilation step, we obtain state estimates with
529 better certainty. Also, some uncertainty persists near the fault regions that decreases
530 with further assimilation steps.

531 6.4. Real gauge experiment

532 The DART buoys measure the pressure at the sea floor in deep water to provide
533 an early indication of the magnitude of a tsunami. Gauge data of the real Chile 2010
534 event is available via DART (Deep-ocean Assessment and Reporting of Tsunamis)
535 buoy 32412⁴. For this experiment, we will illustrate data assimilation using 2 levels.
536 Also, we will use observation localization with support radius $20^\circ \approx 2000km$.

537 The standard deviation of observation error is set to 0.005 meters. Assimilation is
538 done every 15 minutes with 16 (n_s) ensemble members and the inflation factor is set
539 to 1.1. Figure 19 compares the assimilation mean state (right side) to the free run state
540 (left side). The rectangular boxes in the snapshot represent refined regions by a factor
541 of 2 in both directions. The figures show that the amplitude of the tsunami wavefront
542 propagating in the west direction is reduced when the gauge data is taken into account.
543 Also, since the assimilation is done on a supermesh, there are more level 2 regions in
544 the assimilated state than the corresponding free run state. This is expected because
545 at the assimilation step, refinement is performed in regions where at least one of the
546 ensemble members has a level 2 region at the end of its forecast step. At time of 5.0
547 hours, unlike the free run, the best estimate of the water surface elevation snapshot for
548 assimilation run reveals a series of waves traveling in south-east direction towards the
549 coast. This may have implications that data assimilation is effective at capturing waves

⁴<http://www.ndbc.noaa.gov/dart.shtml>

that might otherwise have not been resolved in non-assimilative simulations. Further, the AMR capability captures these waves and refines along the regions until it reaches the coast. We observe that during the entire simulation, more refined regions are automatically obtained along the coast when assimilation is performed, in comparison to the non-assimilative free run. Thus, the AMR is effective in capturing the oscillations created by the state update. Most importantly, Figure 20 outlines the effect of assimilation through the gauge output for the free run and simulation with assimilation. With assimilation, the gauge output (bottom) closely follows the real observation data. This was not possible otherwise.

7. Summary and conclusions

We have used ensemble Kalman filter based data assimilation to demonstrate global assimilation and local assimilation on a numerical model with multi-level adaptive mesh refinement. Specifically, we have developed a data assimilation framework by coupling the GEOCLAW model with the data assimilation library PDAF. A strength of the research presented, is the fact that GEOCLAW and PDAF are open software. Further, we demonstrated the use of a supermesh technique to tackle the situation of ensemble members having different mesh structure. For data assimilation of tsunami models, we have outlined an algorithm to generate the initial ensemble state and performed various twin experiments assessing the effect of observation configuration with respect to number of observations, localization radius and ensemble size on errors between assimilated state and free run. This indicates that assimilation is effective. Additionally, we showed that more accurate state estimates can be obtained while reducing uncertainty with assimilation steps. We have further provided the assimilation of actual gauge data for the Chile earthquake of Feb 27, 2010. We concurrently showed the execution of assimilation with adaptive mesh refinement up to 2 levels. We demonstrated that the AMR capability captured the oscillations in the assimilated state and tracked the waves as they traveled towards the coast. This was reaffirmed with the estimated water surface elevation closely following the true values at the DART gauge. The AMR-coupled data assimilation framework has been tested with

579 higher refinement levels (of up to 5). The results presented in this paper are generated
580 from the code for GEOCLAW at [https://zenodo.org/badge/latestdoi/](https://zenodo.org/badge/latestdoi/30735579)
581 [30735579](https://zenodo.org/badge/latestdoi/30735579) and the code for AMRCLAW at [https://zenodo.org/badge/](https://zenodo.org/badge/latestdoi/92109632)
582 [latestdoi/92109632](https://zenodo.org/badge/latestdoi/92109632). Research is underway to develop test cases that can bench-
583 mark the efficacy of the AMR with ensemble data assimilation. Even though the pre-
584 liminary results are applied on a tsunami model, other applications being considered
585 are assimilation of hurricane storm surges as well.

586 The study is aimed to provide some primary insights in the development of data
587 driven computational framework using AMR capable numerical models. In particu-
588 lar, ensemble data assimilation requires integration of several model runs, This can be
589 computationally expensive for large scale hydrodynamic models. On the contrary, the
590 numerical codes with AMR technique offer advantages with respect to computational
591 speed as areas that are not of interest such as the deep ocean can retain coarse mesh.
592 Hence, ensemble data assimilation with AMR is ideal for use in real-time forecasting
593 scenarios, where time of simulations is of essence and may serve as a bottleneck for
594 initiating evacuation measures. The coupling methodology demonstrated in this study,
595 can be extended to various models currently in use. The possible areas to further ex-
596 plore are storm surges and wave runup. Research is underway to test the developed
597 capability with other historical events. This will aid in the development of a potential
598 real-time forecasting tool, that offers computational speed and uncertainty quantifica-
599 tion simultaneously.

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