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Superresolution Imaging Using Resonant Multiples

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Running head: **Resonant multiple migration**

ABSTRACT

A resonant multiple is defined as a multiple reflection that revisits the same subsurface location along coincident reflection raypaths. We show that resonant first-order multiples can be migrated with either Kirchhoff or wave-equation migration methods to give images with approximately twice the spatial resolution compared to post-stack primary-reflection images. A moveout-correction stacking method is proposed to enhance the signal-to-noise ratios (SNRs) of the resonant multiples before superresolution migration. The effectiveness of this procedure is validated by synthetic and field data tests.

INTRODUCTION

In a recorded seismogram, a zero-offset primary reflection is described by a pulse of temporal duration T , where T is the dominant period associated with the peak frequency of the impulse-like reflection signal. When migrated to depth with a constant velocity v , the primary reflection pulse will be distorted (Tygel et al., 1994) and the imaged reflector is an annular strip with a thickness of $\lambda/2$, where $\lambda = vT$. The true reflector's location is somewhere within the strip and cannot be known precisely with conventional primary reflection migration. This thickness $\lambda/2$ is referred to as the Abbe resolution limit, which is the classical limit of spatial resolution of a primary reflection (Tygel et al., 1994; Levin, 1998).

For a more complex subsurface velocity and source-receiver configuration, the spatial resolution of a primary-reflection image is a function of the reflector dip, reflection angle and background velocity at the trial image point (Berkhout, 1984; Tygel et al., 1994; Levin, 1998). The zero-offset post-stack migration image has a higher resolution perpendicular to the reflecting interface than the non-zero-offset pre-stack migration image because the reflection angle for zero-offset data is smaller than any of the reflection angles in non-zero-offset data. In general, the resolution of both post-stack and pre-stack images is bounded by the Abbe resolution limit. Significantly exceeding the Abbe resolution limit by super-resolution imaging will provide a finely resolved image, so that the interpreters can locate the target reflectors, such as the top salt boundary, with more accuracy.

Previous researchers have achieved superresolution imaging for a scatterer in the near field of sources or receivers using the time-reversal mirror (TRM) concept, which was first discovered in the acoustics community (Cragg and So, 2000; Blomgren et al., 2002; Lerosey

et al., 2007; Fink, 2008; Zhang and Liu, 2008). Schuster et al. (2012) and Dutta et al. (2016) harnessed the TRM technology in the context of seismic scattering and showed both theoretically and experimentally that superresolution imaging is viable by refocusing the scattered evanescent field of body waves in the near-field region of the source. Fu et al. (2017) recently showed that superresolution imaging is achievable with surface waves in the near-field region of the source. However, in exploration geophysics most of the scattering bodies of interest are in the far-field regions of both the sources and receivers, hence the near-field TRM method is not applicable.

To obtain far-field superresolution, Schuster and Huang (2014) introduced the concept of seismic resonant multiples and showed that imaging resonant multiples can give twice or more the spatial resolution than the migration of primary reflections. A resonant multiple is defined as the reverberating seismic reflection that revisits the same subsurface location along coincident ray paths. For example, Figure 1 describes the raypath of a resonant multiple in zero-offset marine data. This multiple is excited by the source at $\mathbf{s}$, reflects off the reflector at $\mathbf{x}$, bounces on the free-surface at $\mathbf{x}_o$, re-visits the same subsurface location $\mathbf{x}$ and is finally recorded by the zero-offset receiver at $\mathbf{s}$. Even though there are other types of resonant multiples as pointed by Schuster and Huang (2014), this special type of surface-related multiple is denoted as the "resonant multiple" in this paper unless noted otherwise. Resonant multiples differ from the other surface-related multiples in two aspects: (1) the upgoing and downgoing raypaths of resonant multiples overlap with themselves, and (2) resonant multiples are only recorded in the zero-offset traces. Superresolution imaging with resonant multiples can be achieved because small shifts in the positions of the reflector are amplified by the multiple reverberating raypaths (Levin, 1998; Schuster and Huang, 2014).

The reverberation property, i.e., resonance, is the key reason for superresolution. This

is why conventional surface-related multiple migration methods (Berkhout and Vershuur, 1994; Liu et al., 2011a,b; Zhang and Schuster, 2013) do not enjoy the same benefit of superresolution from zero-offset data.

For dipping reflectors, Schuster and Huang (2014) showed that the superresolution imaging is viable with interbed resonant multiples, yet they did not describe the use of surface-related resonant multiples to achieve superresolution for tilted reflectors. Guo et al. (2016) extended superresolution imaging with surface-related resonant multiples for dipping reflectors. However, there are still several remaining questions. What is the Born-modeling equation for resonant multiples in a general reflectivity model? What is the resolution improvement in the vertical and horizontal directions for dipping reflectors? How can Kirchhoff migration or wave-equation migration be practically employed for migrating resonant reflections from models with irregular layers? These questions are now answered in this paper.

Furthermore, one of the issues we address is that multiples recorded in field data usually suffer from low signal-to-noise ratios (SNRs) as they visit the subsurface twice or more. This problem jeopardizes the practical implementation of resonant multiple imaging because (1) it might be difficult to identify and separate resonant multiples from other events and (2) it can lead to images with poor SNRs. To enhance the SNRs of resonant multiples, we propose a moveout-correction stacking method, which uses a moveout-correction formula to align surface-related multiples at different offsets with the zero-offset resonant multiple based on a moveout-correction formula. The aligned multiples are then stacked together along all offsets in a common midpoint gather (CMG). This procedure is similar to the normal-moveout stacking method (Taner and Koehler, 1969; Yilmaz, 2001), except that the moveout-correction formula is now derived for surface-related multiples instead of primary

reflections. We also rigorously derive the formulas for Kirchhoff migration (Yilmaz, 2001) and reverse-time migration (RTM) (Baysal et al., 1983; Chang and McMechan, 1986) of resonant multiples.

After the introduction, the theory section derives the migration formula for resonant multiples, which is then used to explain the superresolution properties. This is followed by the moveout-correction stacking method to enhance the SNRs of resonant multiples. The numerical-examples section first shows a simple two-layer model to validate the resolution formula and moveout-correction of resonant multiples. Then the Kirchhoff and wave-equation migration methods are tested on a more complex synthetic model and a field dataset recorded by a marine experiment. In the discussion section, we list several limitations of imaging resonant multiples, and also discuss the feasibility of using interbed resonant multiples for superresolution imaging. The conclusions are drawn in the last section.

THEORY

The theory section consists of five subsections. The first subsection presents the Born-modeling equation of resonant multiples. This is followed by the migration formula for resonant multiples and we use this formula to analyse the superresolution characteristics of a resonant-multiple image. Then the third subsection shows how to use Kirchhoff migration and wave-equation migration to migrate resonant multiples. The moveout-correction stacking method is proposed in the fourth subsection. In the last subsection, the workflow is presented for implementing moveout correction to the data and then applying resonance migration to the resulting zero-offset data.

Forward Modeling of First-order Resonant Multiples

We employ the Cartesian coordinate system so that the free-surface plane is at $z = 0$, and the horizontal recording line B is along the x axis just below the free surface (see Figure 1). For a zero-offset source/receiver pair at $\mathbf{s} \in B$, the Born-modeling equation to predict resonant multiples $d(\omega, \mathbf{s})$ at the angular frequency ω is derived in Appendix A:

$$d(\omega, \mathbf{s}) = \overline{W}(\omega) \int_V g(\mathbf{x}|\mathbf{s})^2 h(\mathbf{x}) m(\mathbf{x}) d\mathbf{x}, \quad (1)$$

where $\overline{W}(\omega) = 4\omega^4 W(\omega)$, $W(\omega)$ denotes the source spectrum, V is the subsurface volume of interest, $m(\mathbf{x}) = [s_b(\mathbf{x}) \Delta s(\mathbf{x})]^2$ with $s_b(\mathbf{x})$ is the background slowness (reciprocal of velocity) model and $\Delta s(\mathbf{x})$ is the slowness perturbation. Here, $g(\mathbf{x}|\mathbf{s})$ represents the Green's function which satisfies the Helmholtz equation

$$(\nabla^2 + \omega^2 s_b(\mathbf{x})^2) g(\mathbf{x}|\mathbf{s}) = -\delta(\mathbf{x} - \mathbf{s}), \quad (2)$$

without the free-surface boundary condition. A physical interpretation of $g(\mathbf{x}|\mathbf{s})$ is the direct-wave propagator in free space for a source at $\mathbf{s}$ and a receiver at $\mathbf{x}$. In equation 1, $h(\mathbf{x})$ is the solution to the homogeneous Helmholtz equation

$$(\nabla^2 + \omega^2 s_b(\mathbf{x})^2) h(\mathbf{x}) = 0, \quad (3)$$

with the boundary condition

$$h(x, y, z = 0) + g(x, y, z = 0|\mathbf{x}) = 0, \quad \forall x, y. \quad (4)$$

Here, $h(\mathbf{x})$ represents the specular reflection from the subsurface point at $\mathbf{x}$ to the free surface, and then back to $\mathbf{x}$ (see Appendix A). The asymptotic forms of $g(\mathbf{x}|\mathbf{s})$ and $h(\mathbf{x})$ based on equations A-14 and A-16 are given by

$$\begin{aligned} g(\mathbf{x}|\mathbf{s}) &= \frac{e^{i\omega\tau_{\mathbf{s}\mathbf{x}}}}{A_{\mathbf{s}\mathbf{x}}}, \\ \text{and } h(\mathbf{x}) &= -\frac{e^{i\omega 2\tau_{\mathbf{x}\mathbf{x}_o}}}{2A_{\mathbf{x}\mathbf{x}_o}}, \end{aligned} \quad (5)$$

where i is the imaginary unit, $\tau_{\mathbf{s}\mathbf{x}}$ represents the traveltime from $\mathbf{s}$ to $\mathbf{x}$, $A_{\mathbf{s}\mathbf{x}}$ and $A_{\mathbf{x}\mathbf{x}_o}$ represent the geometrical spreading terms from $\mathbf{s}$ to $\mathbf{x}$ and from $\mathbf{x}$ to $\mathbf{x}_o$, respectively (see equations A-15 and A-16). For resonant multiples, ray $\overline{xx_o}$ must be perpendicular to the free surface at $\mathbf{x}_o$. This means that $\mathbf{x}_o$ is a function of $\mathbf{x}$ even though this relationship is not shown explicitly in the formula. Plugging equation 5 into equation 1 leads to the asymptotic modeling formula for a first-order surface-related multiple with zero-offset between the source and receiver:

$$d(\omega, \mathbf{s}) = -\overline{W}(\omega) \int_V \frac{e^{i\omega 2(\tau_{\mathbf{s}\mathbf{x}} + \tau_{\mathbf{x}\mathbf{x}_o})}}{2A_{\mathbf{s}\mathbf{x}}^2 A_{\mathbf{x}\mathbf{x}_o}} m(\mathbf{x}) d\mathbf{x}, \quad (6)$$

which is also known as a first-order surface-related resonant multiple.

Superresolution by Migrating Resonant Multiples

The migration equation for computing the resonant-multiple image $m(\mathbf{x})^{\text{mig}}$ is obtained by applying the adjoint kernel $\overline{W}(\omega)^* \frac{e^{-i\omega 2(\tau_{\mathbf{s}\mathbf{x}} + \tau_{\mathbf{x}\mathbf{x}_o})}}{2A_{\mathbf{s}\mathbf{x}}^2 A_{\mathbf{x}\mathbf{x}_o}}$ to the data in equation 6 and integrating over the source coordinates in the data space:

$$m(\mathbf{x})^{\text{mig}} = - \int_{\omega_1}^{\omega_2} \int_B \overline{W}(\omega)^* \frac{e^{-i\omega 2(\tau_{\mathbf{s}\mathbf{x}} + \tau_{\mathbf{x}\mathbf{x}_o})}}{2A_{\mathbf{s}\mathbf{x}}^2 A_{\mathbf{x}\mathbf{x}_o}} d(\omega, \mathbf{s}) d\mathbf{s} d\omega, \quad (7)$$

where ω_1 and ω_2 are the minimum and maximum frequencies of the source spectrum. The superscript $*$ represents complex conjugation.

To simplify the analysis, the true slowness perturbation is assumed to be only located at $\mathbf{x}'$, that is $m(\mathbf{x}) = \delta(\mathbf{x} - \mathbf{x}')$, and we conveniently assume that $\overline{W}(\omega) = 1$ between ω_1 and ω_2 . Under these assumptions, equation 6 becomes

$$d(\omega, \mathbf{s}) = -\frac{e^{i\omega 2(\tau_{\mathbf{s}\mathbf{x}'} + \tau_{\mathbf{x}'\mathbf{x}_o'})}}{2A_{\mathbf{s}\mathbf{x}'}^2 A_{\mathbf{x}'\mathbf{x}_o'}}, \quad (8)$$

where ray $\overline{x'x_o'}$ is perpendicular to the free surface at $\mathbf{x}_o'$. Inserting equation 8 into equation 7 gives

$$m(\mathbf{x})^{\text{mig}} = \int_{\omega_1}^{\omega_2} \int_B \frac{e^{i\omega 2(\tau_{\mathbf{s}\mathbf{x}'} + \tau_{\mathbf{x}'\mathbf{x}_o'} - \tau_{\mathbf{s}\mathbf{x}} - \tau_{\mathbf{x}\mathbf{x}_o})}}{4A_{\mathbf{s}\mathbf{x}}^2 A_{\mathbf{x}\mathbf{x}_o} A_{\mathbf{s}\mathbf{x}'}^2 A_{\mathbf{x}'\mathbf{x}_o'}} ds d\omega. \quad (9)$$

For a smooth migration velocity, $\tau_{\mathbf{s}\mathbf{x}}$ and $\tau_{\mathbf{x}\mathbf{x}_o}$ are continuous functions of $\mathbf{x}$. When the trial image point $\mathbf{x}$ is in the vicinity of the true scatterer's location $\mathbf{x}'$, $f(\mathbf{x}) = \tau_{\mathbf{s}\mathbf{x}'} + \tau_{\mathbf{x}'\mathbf{x}_o'} - \tau_{\mathbf{s}\mathbf{x}} - \tau_{\mathbf{x}\mathbf{x}_o}$ can be approximated using the first-order Taylor expansion as

$$f(\mathbf{x}) \approx f(\mathbf{x}') + (\mathbf{x} - \mathbf{x}') \cdot \nabla f(\mathbf{x})|_{\mathbf{x}=\mathbf{x}'} = -\nabla(\tau_{\mathbf{s}\mathbf{x}'} + \tau_{\mathbf{x}'\mathbf{x}_o'}) \cdot (\mathbf{x} - \mathbf{x}'). \quad (10)$$

According to the dispersion equation, the traveltime gradient is related to the wavenumber by

$$\nabla \tau_{\mathbf{s}\mathbf{x}'} = \frac{\mathbf{k}_{\mathbf{s}\mathbf{x}'}}{\omega} \quad \text{and} \quad \nabla \tau_{\mathbf{x}'\mathbf{x}_o'} = \frac{\mathbf{k}_{\mathbf{x}'\mathbf{x}_o'}}{\omega}, \quad (11)$$

where $\mathbf{k}_{\mathbf{s}\mathbf{x}'}$ and $\mathbf{k}_{\mathbf{x}'\mathbf{x}_o'}$ are the model wavenumbers at $\mathbf{x}'$ as shown in Figure 2a. Inserting

equations 10 and 11 into equation 9, we obtain

$$m(\mathbf{x})^{\text{mig}} = \int_{k_1}^{k_2} \int_B s_b(\mathbf{x})^{-1} \frac{e^{-i2(\mathbf{k}_{\text{sx}'} + \mathbf{k}_{\mathbf{x}'\mathbf{x}'_o}) \cdot (\mathbf{x} - \mathbf{x}')}}{4A_{\text{sx}}^2 A_{\mathbf{x}\mathbf{x}_o} A_{\text{sx}'}^2 A_{\mathbf{x}'\mathbf{x}'_o}} ds dk, \quad (12)$$

where k represents the length of the wavenumber vectors $\mathbf{k}_{\text{sx}'}$ and $\mathbf{k}_{\mathbf{x}'\mathbf{x}'_o}$. According to the scalar dispersion equation, $dk = s_b(\mathbf{x})d\omega$, $k_1 = s_b(\mathbf{x})\omega_1$ and $k_2 = s_b(\mathbf{x})\omega_2$. Equation 12 suggests that for a fixed source location $\mathbf{s}$ and wavenumber length k , $2(\mathbf{k}_{\text{sx}'} + \mathbf{k}_{\mathbf{x}'\mathbf{x}'_o})$ is the model wavenumber associated with the resonant-multiple image. This means that the migration image $m(\mathbf{x})^{\text{mig}}$ is a band-limited version of the true scatterer $m(\mathbf{x}) = \delta(\mathbf{x} - \mathbf{x}')$ because the direction of the wavenumber $\mathbf{k}_{\text{sx}'}$ is limited by the source/receiver location $\mathbf{s}$, and the magnitudes of $\mathbf{k}_{\text{sx}'}$ and $\mathbf{k}_{\mathbf{x}'\mathbf{x}'_o}$ are bounded between k_1 and k_2 .

If $\mathbf{x}'$ is on a reflector with a dip angle θ measured with respect to the horizontal axis, then the wavenumbers $\mathbf{k}_{\text{sx}'}$ and $\mathbf{k}_{\mathbf{x}'\mathbf{x}'_o}$ have to honor Snell's reflection law as shown in Figure 2a. Geometrically this means that the angle between $\mathbf{k}_{\mathbf{x}'\mathbf{x}'_o}$ and $\mathbf{k}_{\text{sx}'}$ is 2θ . For a laterally homogeneous velocity model, ray $\overline{x'x'_o}$ is along the vertical z direction. In this case the wavenumber $2(\mathbf{k}_{\text{sx}'} + \mathbf{k}_{\mathbf{x}'\mathbf{x}'_o})$ can be represented as

$$\begin{aligned} 2(\mathbf{k}_{\text{sx}'} + \mathbf{k}_{\mathbf{x}'\mathbf{x}'_o}) &= (k_x, k_z) \\ &= (2k \sin 2\theta, 4k \cos^2 \theta). \end{aligned} \quad (13)$$

The associated horizontal- and vertical-wavelengths of the image are

$$\lambda_x = \frac{2\pi}{2k \sin 2\theta} \quad \text{and} \quad \lambda_z = \frac{2\pi}{4k \cos^2 \theta}. \quad (14)$$

Moreover, the geometry in Figure 2a suggests that in a homogeneous velocity model, reso-

nant multiples cannot be recorded by a receiver below the free surface for a reflector with a dip angle greater than 45 degree. The model wavenumber vector $2(\mathbf{k}_{\mathbf{s}\mathbf{x}'} + \mathbf{k}_{\mathbf{x}'\mathbf{x}'_o})$ can also be decomposed into $k_{\perp}$ and $k_{\parallel}$, which are the wavenumbers perpendicular and parallel to the reflector's interface, respectively:

$$k_{\perp} = 4k \cos \theta \quad \text{and} \quad k_{\parallel} = 0. \quad (15)$$

As a comparison, the migration formula for the post-stack primary-reflection image $m(\mathbf{x})^p$ is

$$m(\mathbf{x})^p = \int_{k_1}^{k_2} \int_B s_b(\mathbf{x})^{-1} \frac{e^{-i2\mathbf{k}_{\mathbf{s}'\mathbf{x}'} \cdot (\mathbf{x} - \mathbf{x}')}}{A_{\mathbf{s}'\mathbf{x}'}^2} d\mathbf{s}' dk, \quad (16)$$

where ray $\overline{s'x'}$ is perpendicular to the reflector as shown in Figure 2b. For a fixed k , the image wavenumber $2\mathbf{k}_{\mathbf{s}'\mathbf{x}'}$ can be decomposed along the x and z directions as

$$k_x^p = 2k \sin \theta \quad \text{and} \quad k_z^p = 2k \cos \theta, \quad (17)$$

so that the associated wavelengths are

$$\lambda_x^p = \frac{2\pi}{2k \sin \theta} \quad \text{and} \quad \lambda_z^p = \frac{2\pi}{2k \cos \theta}. \quad (18)$$

Similarly, for primary reflections, the image wavenumber $2\mathbf{k}_{\mathbf{s}'\mathbf{x}'}$ can be decomposed into $k_{\perp}^p$ and $k_{\parallel}^p$, which are the wavenumbers perpendicular and parallel to the interface of the reflector:

$$k_{\perp}^p = 2k \quad \text{and} \quad k_{\parallel}^p = 0. \quad (19)$$

From equations 13 and 17, the horizontal- and vertical-wavenumber ratios r_x and r_z

between the resonant-multiple and the primary-reflection images are given by

$$r_x = k_x/k_x^p = 2 \cos \theta \quad \text{and} \quad r_z = k_z/k_z^p = 2 \cos \theta. \quad (20)$$

Similarly, from equations 15 and 19, the ratio $r_\perp$ between the wavenumbers perpendicular to the interface of the resonant-multiple and the primary-reflection images is

$$r_\perp = k_\perp/k_\perp^p = 2 \cos \theta. \quad (21)$$

Equation 20 suggests that for a mildly tilted reflector (small θ), the first-order resonant-multiple image has about twice the vertical- and horizontal-wavenumbers of the primary-reflection image. The increased vertical wavenumber is more important than the horizontal wavenumber for nearly flat subsurface structures (Tygel et al., 1994). However, r_x , r_z and $r_\perp$ decrease for larger dip angles. Equations 13 and 17 are validated in the numerical-examples section by comparing the theoretical predicted wavenumbers against the resonant-multiple images computed for reflectors with different dip angles.

Equation 13 gives the image wavenumber associated with specular resonant multiples. Next we study the image of resonant multiples diffracted by scatterers. A resonant multiple reflected from a subsurface reflector at $\mathbf{x}'$ can only be recorded by the source/receiver pair at $\mathbf{s}$ where rays $\overline{x'x'_o}$ and $\overline{s'x'}$ honour Snell's reflection law. On the contrary, resonant multiples diffracted from a scatterer at $\mathbf{x}'$ can be recorded by all source/receiver pairs, which all contribute to the image of the scatterer. Sources at different locations lead to different directions of $k_{\mathbf{s}\mathbf{x}'}$. In a homogeneous medium, the maximum horizontal wavenumber k_x is determined by the source at (x_s, z_s) with the greatest horizontal offset from the scatterer at

(x', z') . The maximum vertical wavenumber $k_z = 4k$ is determined by the source above the scatterer. Therefore, the model wavenumbers k_x and k_z can be derived based on geometrical considerations:

$$k_x = \frac{2k|x_s - x'|}{\sqrt{(x_s - x')^2 + (z_s - z')^2}} \quad \text{and} \quad k_z = 4k. \quad (22)$$

For primary diffractions, the maximum horizontal wavenumber is the same as that of the diffracted resonant multiples and the maximum vertical wavenumber is half that of the diffracted resonant multiples.

To summarize, the theory suggests that for specular reflections from an almost flat interface, the first-order resonance image has about twice the resolution perpendicular to the reflection interface compared to that of the primary image. However, this improvement of resolution decreases as the interface becomes more tilted. Furthermore, the resonance resolution parallel to the reflecting interface is no better than that for a primary. For diffractions from scatterers, on the other hand, the vertical resolution of the first-order resonant multiple image is twice that of the primary image.

Kirchhoff Migration and Wave-equation Migration of Resonant Multiples

Equation 7 is the Kirchhoff migration formula for resonant multiples. However, a part of its kernel $\frac{e^{-i\omega 2\tau_{\mathbf{x}\mathbf{x}_o}}}{A_{\mathbf{x}\mathbf{x}_o}}$ is not straightforward to calculate because $\mathbf{x}_o$, which is the reflection point of ray $\overline{xx_o}$ at the free surface, is unknown for a complex velocity model. To solve this

problem, we use the stationary phase integration (Bleistein, 1984) to approximate $\frac{e^{-i\omega 2\tau_{\mathbf{x}\mathbf{x}_o}}}{A_{\mathbf{x}\mathbf{x}_o}}$:

$$\frac{e^{-i\omega 2\tau_{\mathbf{x}\mathbf{x}_o}}}{A_{\mathbf{x}\mathbf{x}_o}} \approx \alpha \int_{fs} \frac{e^{-i\omega 2\tau_{\mathbf{x}\mathbf{x}''}}}{A_{\mathbf{x}\mathbf{x}''}} dx''^2, \quad (23)$$

$$\text{where } \alpha = e^{-i\pi/4} \sqrt{\pi / (\omega \|\frac{\partial^2 \tau_{\mathbf{x}\mathbf{x}_o}}{\partial x_o^2}\|)}^{-1},$$

in which fs denotes the integration is along the free surface. A detailed proof of equation 23 is in Appendix B.

Equation 23 suggests that the kernel $\frac{e^{-i\omega 2\tau_{\mathbf{x}\mathbf{x}_o}}}{A_{\mathbf{x}\mathbf{x}_o}}$ approximates the summation of $\frac{e^{-i\omega 2\tau_{\mathbf{x}\mathbf{x}''}}}{A_{\mathbf{x}\mathbf{x}''}}$ from every point $\mathbf{x}''$ along the free surface. This is valid irrespective of the velocity model. The right-hand-side of equation 23 can also be interpreted as the kernel of a plane wave initiated from the free surface with zero ray parameter. Omitting α and plugging equation 23 into equation 7 leads to

$$m(\mathbf{x})^{\text{mig}} = -\frac{1}{2} \int_{\omega_1}^{\omega_2} \overbrace{\left\{ \bar{W}(\omega) \int_{fs} \frac{e^{i\omega 2\tau_{\mathbf{x}\mathbf{x}''}}}{A_{\mathbf{x}\mathbf{x}''}} dx''^2 \right\}^*}_{\text{complex conjugate of the plane-wave side}} \overbrace{\int_B \frac{e^{-i\omega 2\tau_{\mathbf{s}\mathbf{x}}}}{A_{\mathbf{s}\mathbf{x}}^2} d(\omega, \mathbf{s}) ds d\omega}_{\text{data side}}. \quad (24)$$

Equation 24 suggests that the resonant-multiple image is the dot product or zero-lag cross-correlation between the plane-wave-side wavefield and the data-side wavefield. An efficient option for implementing equation 24 is that the migration velocity can be halved in order to calculate the traveltimes $2\tau_{\mathbf{x}\mathbf{x}''}$ and $2\tau_{\mathbf{s}\mathbf{x}}$.

Resonant multiples can also be migrated with wave-equation migration. For the medium with half of the migration velocity, the asymptotic Green's functions are given by $\tilde{g}(\mathbf{x}|\mathbf{x}'') =$

$\frac{e^{i\omega 2\tau_{\mathbf{x}\mathbf{x}''}}}{A_{\mathbf{x}\mathbf{x}''}}$ and $\tilde{g}(\mathbf{x}|\mathbf{s}) = \frac{e^{i\omega 2\tau_{\mathbf{s}\mathbf{x}}}}{A_{\mathbf{s}\mathbf{x}}}$. Inserting these formulas into equation 24 leads to

$$m(\mathbf{x})^{\text{mig}} = -\frac{1}{2} \int_{\omega_1}^{\omega_2} \overbrace{\left\{ \overline{W}(\omega) \int_{f_s} \tilde{g}(\mathbf{x}|\mathbf{x}'') dx''^2 \right\}^*}_{\text{complex conjugate of the plane-wave side}} \overbrace{\int_B \frac{\tilde{g}(\mathbf{x}|\mathbf{s})^*}{A_{\mathbf{s}\mathbf{x}}} d(\omega, \mathbf{s}) d\mathbf{s} d\omega}_{\text{data side}}, \quad (25)$$

where the Green's functions $\tilde{g}(\mathbf{x}|\mathbf{x}'')$ and $\tilde{g}(\mathbf{x}|\mathbf{s})$ can be computed either in the time domain or in the frequency domain using half of the migration velocity. Similar to the pre-stack reverse-time migration, where the source-side and receiver-side wavefields are extrapolated, multiplied and summed along the time axis, the plane-wave-side and data-side wavefields in equation 25 are computed for the resonant-multiple image $m(\mathbf{x})^{\text{mig}}$.

Moveout-correction Stacking of Resonant Multiples

Resonant multiples are only recorded by zero-offset hydrophones. However, in a streamer acquisition, the minimum offset is typically around 100 meters. To solve this problem, Guo et al. (2015) transformed the traveltime t of a non-zero-offset multiple in a common-midpoint gather (CMG) into the traveltime t_0 of a resonant multiple in the same CMG according to the moveout-correction formula

$$t_0 = \sqrt{t^2 - [h(1 - 2 \sin^2 \theta)/v]^2}, \quad (26)$$

where h is the offset of the pre-transformed multiple, θ is the dip angle of the reflector and v stands for the constant velocity between the free-surface and the reflector.

For a flat reflector ($\theta = 0$), equation 26 becomes the same as the normal-moveout-

correction formula of primary reflections:

$$t_0 = \sqrt{t^2 - (h/v)^2}. \tag{27}$$

For flat reflectors, equation 27 is valid not only for constant velocity media, but also for laterally homogeneous media where v stands for the root-mean-squared velocity between the free-surface and the reflector. This can be easily proved using symmetry.

Equation 26 can also be used to enhance the SNRs of resonant multiples. The first step is to change the traveltimes of all non-zero-offset multiples into zero-offset resonant multiples, and then the transformed multiples are stacked together along all offsets to enhance the SNRs. This process is similar to the normal-moveout stacking method (Taner and Koehler, 1969; Yilmaz, 2001), except that equation 26 is derived for multiples rather than primary reflections.

The moveout-correction formula is based on the following assumptions. First, equation 26 is derived for the first-order surface-related multiples in a homogeneous medium, where the two reflection points A and B are on the same reflector as shown by the straight arrows in Figure 3. Furthermore, equation 26 implicitly assumes a planar reflector, that is the dip angles at A and B are the same. This assumption is reasonable for short or medium offset data associated with a reflector with mildly varying dip angles. However, for more complex structures, such as an anticline, the assumption of a planar reflector is not always satisfied. To overcome the limitations of the first two assumptions, traveltimes interferometry (Schuster, 2009) provides another option for aligning the surface-related multiples, which predicts the traveltimes of multiples based on primary reflections. The advantage of this method is that it does not require the knowledge of the subsurface velocity model and it is

valid for any subsurface reflector and velocity models. Thirdly, equation 26 assumes that sources and receivers are at the free-surface boundary, even though in a streamer acquisition they are placed a few meters below. This error is negligible for reflectors in the far-field region of sources and receivers.

One possible disadvantage of the moveout-correction stacking method is the multiples recorded at far offset are stretched. In Figure 4c, for example, the wavelet of the moveout-corrected multiple at 2.25 km is visually wider than the multiple at zero offset. Stacking with the stretched multiples would decrease the frequency content of zero-offset resonant multiples, thus defeating the superresolution character of the image. The same problem also exists with conventional moveout-correction methods. One way to circumvent the stretching problem is to limit the offset range within which to stack. Another option is to use traveltimes interferometry to predict the traveltimes of multiples, then isolate them within local windows and stack. This procedure is further described in the discussion section.

Workflow

The workflow for migrating resonant multiples can be summarized by the following steps.

1. Separate first-order surface-related multiples from primary reflections based on their moveout differences (Yilmaz, 2001), or use a data-driven method such as surface-related multiple elimination (SRME) (Verschuur et al., 1992). For a deep-water model, the surface-related multiples can be separated from primary reflections simply based on their traveltimes differences.
2. Apply moveout-correction based on equation 26 to the data to transform non-zero-offset multiples into zero-offset resonant multiples. Then stack the moveout-corrected

traces along all offsets to enhance the SNRs. One way to implement equation 26 is to combine the dip-angle term $(1 - 2 \sin^2 \theta)$ with the velocity v by defining $\tilde{v} = v/(1 - 2 \sin^2 \theta)$, and correct the moveout of multiples based on

$$t_0 = \sqrt{t^2 - (h/\tilde{v})^2}. \tag{28}$$

The value of $\tilde{v}$ is not the goal since the main aim of the moveout correction is to align multiple data in different offsets. After moveout-correction, the long-offset multiples are sometimes not well aligned with short or medium offsets. This is because in a complex velocity model, the moveout of multiples is not a perfect hyperbola. In this case we can either apply the residual-statics methods or stack only the short and medium offset data. Another option is to apply the optimum mute selection (Yilmaz, 2001), which progressively stacks the data starting from the zero offset until the SNRs begin to decrease at certain offset. This moveout-correction stacking step can be skipped if the original zero-offset resonant multiples have sufficient SNRs.

3. Window around the post-stack resonant multiples from data. Since the traveltimes of a resonant multiple is roughly twice of the associated zero-offset primary reflection, we first double the traveltimes or halve the time-axis of the primary reflections, and then window the resonant multiples that match the similar pattern of the transformed primary reflections. After windowing resonant multiples, certain processing techniques such as median filtering can be used to remove noise inside the time window of the picked resonant multiples.
4. Migrate the resonant multiples using Kirchhoff migration based on equation 24, or using wave-equation migration according to equation 25.

NUMERICAL EXAMPLES

Resonant multiple migration is tested on synthetic datasets and a marine dataset. These tests are designed to validate the theory and show the feasibility of computing superresolution images from resonant multiples.

Synthetic Test 1

The first synthetic test is on a two-layer velocity model with a flat interface shown in Figure 4a, where 900 shot profiles with 900 receivers each shot are computed by solving the 2D acoustic wave equation with the free-surface boundary condition. The sources and receivers are located 5 meters below the free surface with a horizontal spacing of 5 meters. The recorded primary and multiple reflections share the same frequency spectrum, with the peak frequency at 5 Hz. Figures 4b and 4c show a CMG at 2.25 km before and after the moveout correction using equation 26, where the surface-related multiples (marked by the red arrow) at different offsets are aligned with zero-offset resonant multiple. The zero-offset primary reflections and resonant multiples shown in Figure 5a are migrated separately. The resonant-multiple images using the Kirchhoff and wave-equation migrations are shown in Figures 5c and 5d, respectively. Compared to the primary-reflection image in Figure 5b, the resonant-multiple images have much higher vertical resolution. Moreover, the images of the 5-Hz resonant multiples have the same vertical resolution as the 10-Hz primary-reflection image shown in Figure 5e. This experiment proves that for flat reflectors, a resonant-multiple image has twice the vertical resolution of a primary-reflection image. This conclusion is further validated by Figure 6 which compares the vertical-wavenumber spectra of resonant-multiple and primary-reflection images.

The same experiment is repeated for a 2-layer velocity model with a 15-degree tilted interface (see Figure 7a). Figures 7b and 7c show the CMG at 2.25 km before and after the moveout correction. The zero-offset primary reflections and the resonant multiples shown in Figure 8a are migrated separately to obtain the images shown in Figures 8b to 8d. Figure 9 depicts the vertical-wavenumber spectra of the four images, where the peak vertical wavenumber of the resonant-multiple image is 1.92 times that of the primary-reflection image using the same frequency data. This ratio agrees with the theoretical prediction of $r_z = 2 \cos \theta \approx 1.93$ given by equation 20.

For different dip angles from 0 to 35 degrees, the centroid wavenumbers of the resonant-multiple and primary-reflection images are plotted as stars in Figure 10. The curves in Figure 10 describe the theoretical predicted wavenumbers given by equations 13 and 17. The theoretical prediction and experimental results agree well with each other. This result validates equations 13 and 17.

Synthetic Test 2

The theory is further tested on the complex velocity model shown in Figure 11a. This model simulates a marine velocity with a salt dome, where 1600 zero-offset traces with an interval of 10 meters are generated by solving the 2D acoustic wave equation. The source is a Ricker wavelet with a 12-Hz peak frequency. The primary reflections and resonant multiples from the top and bottom of the salt dome are marked by the black and red arrows in Figure 11b, respectively. Resonant multiples are picked, median-filtered and then migrated to obtain the resonant images shown in Figures 12b and 12c. These images have much higher vertical wavenumbers than the primary-reflection image (see Figure 12a) as shown by the k_z spectra

in Figure 13.

Field Data Results

Superresolution imaging is now tested on a marine dataset. The streamer data consist of 1905 shots with a shot interval of 25 m, and each shot has 576 hydrophones with an interval of 12.5 m. The nearest source-receiver offset is approximately 147 m. Figure 14a shows the near-offset gather, where the blue dashed line separates the traveltimes of primary reflections and multiples. The moveout-correction stacking method is used to enhance the SNRs of resonant multiples (marked by the red arrows) as shown in Figure 14b. Figure 14c shows that the post-stack primary reflections and resonant multiples share about the same frequency spectra. The moveout-corrections of CMGs at different midpoints are displayed in Figure 15, and the root-mean-squared velocity for primary reflections is transformed into the migration velocity (see Figure 16) using Dix's equation (Dix, 1955). The resonant multiples are windowed from the post-stack multiple data and median filtered before migration. The resonant-multiple images have much higher vertical resolution than the primary-reflection image. Figures 17a to 17c show the post-stack primary-reflection images and the resonant-multiple images using Kirchhoff migration and wave-equation migration, respectively. We also compare the k_z spectra of these images in Figure 17d. The centroid vertical wavenumber of the resonant-multiple images is 0.260 m^{-1} , which is approximately twice that of the primary-reflection image at 0.128 m^{-1} . Figure 18 magnifies two sections of the images, where the resonant-multiple image clearly shows higher vertical resolution than the primary-reflection image.

DISCUSSION

The moveout correction of surface-related multiples is based on the assumption of a planar reflector and a laterally homogeneous velocity model. For a more complex velocity model and reflectors with varying dip angles, the non-zero-offset multiples after moveout correction are not well-aligned with the zero-offset resonant multiples. As a result, the stacking of the moveout corrected multiples might blur the resonant multiple data and reduce their frequency content as well as the horizontal resolution. To perform a more accurate moveout correction, traveltime interferometry (Schuster, 2009) predicts the traveltimes of first-order surface-related multiples $\tau_{\mathbf{sg}}^m$ recorded at $\mathbf{g}$ from the traveltimes of primary reflections $\tau_{\mathbf{sg}}^p$ based on Fermat's principle:

$$\tau_{\mathbf{sg}}^m = \min_{\mathbf{s}' \in B} (\tau_{\mathbf{ss}'}^p + \tau_{\mathbf{s}'\mathbf{g}}^p), \tag{29}$$

where B is the set of source points on the acquisition line. After obtaining the traveltimes of multiples, we can isolate multiples within local windows and and stack them across different offsets to enhance the SNRs. The advantage of this method is that it does not require the knowledge of the subsurface velocity model and it is valid for any subsurface structures and velocity models. Moreover, aligning multiples on the basis of traveltime interferometry does not suffer from the far-offset stretching problem. The disadvantage, however, is that the traveltimes of primary reflections need to be extracted, which is typically labor intensive and not always straightforward, especially if the primaries have low SNRs.

In the synthetic and field data examples, we only migrate the specular-reflection resonant multiples. One of the reasons is that in practice specular-reflection resonant multiples are easier to identify and pick compared to resonant multiples diffracted by scatterers. However, once picked from other events, diffraction resonant multiples can also be migrated to achieve

superresolution images.

One limitation of this method is that there must be no significant attenuation in the model, otherwise the multiple signals will suffer from significant loss of high frequencies compared to the primaries. Another limitation is that surface-related resonant multiples are sometimes very weak and hard to identify for land acquisition.

To alleviate the attenuation problem and extend this method to land data, we now briefly discuss the feasibility of superresolution imaging using interbed resonant multiples reverberating between two subsurface reflectors (see Figure 19). Assume the upper reflector is flat and its depth is accurately estimated, then the interbed resonant multiples are almost identical to surface-related resonant multiples, except that the free-surface boundary is replaced by the upper reflector location. For a tilted upper-reflector, as long as ray xx_o is perpendicular to reflector 1 at $\mathbf{x}_o$, the raypath will still overlap with itself so that superresolution imaging can be achieved. The detailed theory of migrating interbed resonant multiples is beyond the scope of this paper and will be studied in the future.

CONCLUSIONS

We show that superresolution imaging in the far-field regions of the sources and receivers is possible by migrating first-order resonant multiples. These multiples exist both for flat and tilted reflectors, and in theory the resolution improves with increasing order of the resonant multiple. The model wavenumber of the first-order resonant-multiple image is approximately twice that of the post-stack primary-reflection image. A moveout-correction stacking method is presented to enhance the SNRs of resonant multiples before migration.

We also derive the Kirchhoff and wave-equation migration formulas for resonant multiples.

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The moveout-correction and migration equations are tested on synthetic and field data, and the results prove that first-order resonant-multiple images can have about twice the resolution perpendicular to the reflector compared to primary-reflection images; however the spatial resolution of the resonance image parallel to the interface is no better than that for a primary reflection. One limitation of this method is that the resonant multiples must be clearly separated from other events including non-resonant multiples and primaries. Another limitation is that there must be no significant attenuation in the model, so that the multiple data have about the same frequency content as the primary reflection.

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APPENDIX A

BORN-MODELING EQUATION OF RESONANT MULTIPLES

The 3D acoustic wave equation is given by

$$(\nabla^2 + \omega^2 s(\mathbf{x})^2)P(\mathbf{x}) = -F(\mathbf{x}), \tag{A-1}$$

with the free-surface boundary condition

$$P(x, y, z = 0) = 0, \quad \forall x, y. \quad (\text{A-2})$$

Here $s(\mathbf{x})$ denotes the slowness (reciprocal of velocity) of the medium, $P(\mathbf{x})$ is the pressure wavefield and $F(\mathbf{x})$ is the source term.

The slowness model $s(\mathbf{x})$ can be split into a smooth background slowness $s_b(\mathbf{x})$ and a slowness perturbation $\Delta s(\mathbf{x})$: $s(\mathbf{x}) = s_b(\mathbf{x}) + \Delta s(\mathbf{x})$, so that wave equation A-1 becomes

$$(\nabla^2 + \omega^2[s_b(\mathbf{x})^2 + 2s_b(\mathbf{x})\Delta s(\mathbf{x})])P(\mathbf{x}) = -F(\mathbf{x}), \quad (\text{A-3})$$

where $\Delta s(\mathbf{x})^2$ is omitted because the slowness perturbation $\Delta s(\mathbf{x})$ is assumed to be small.

Rearranging equation A-3 we get

$$(\nabla^2 + \omega^2 s_b(\mathbf{x})^2)P(\mathbf{x}) = -2\omega^2 s_b(\mathbf{x})\Delta s(\mathbf{x})P(\mathbf{x}) - F(\mathbf{x}). \quad (\text{A-4})$$

From equation A-4, the wavefield $P(\mathbf{r})$ recorded by the hydrophone at $\mathbf{r}$ in a marine seismic experiment is given by the Lippmann-Schwinger equation:

$$P(\mathbf{r}) = 2\omega^2 \int_V G(\mathbf{r}|\mathbf{x}) s_b(\mathbf{x}) \Delta s(\mathbf{x}) P(\mathbf{x}) dx^3 + \int_V G(\mathbf{r}|\mathbf{x}) F(\mathbf{x}) dx^3, \quad (\text{A-5})$$

where $G(\mathbf{r}|\mathbf{x})$ is the Green's function recorded at $\mathbf{r}$ due to a monochromatic source at $\mathbf{x}$ oscillating at the angular frequency ω . Equation A-5 is integrated over the volume V , which is enclosed by the free-surface boundary and the hemisphere at infinity (see Figure A-1).

The pressure wavefield $P(\mathbf{r})$ is on both the left- and right-hand-side of equation A-5, and it

can be solved by the Neumann series solution (Morse and Feshbach, 1953; Schuster, 2017).

We only list the first three terms of the solution:

$$\begin{aligned}
 P(\mathbf{r}) = & \overbrace{\int_V G(\mathbf{r}|\mathbf{x})F(\mathbf{x})d\mathbf{x}^3}^{\text{direct arrival}} \\
 & + \overbrace{2\omega^2 \int_V G(\mathbf{r}|\mathbf{x})s_b(\mathbf{x})\Delta s(\mathbf{x}) \int_V G(\mathbf{x}|\mathbf{x}')F(\mathbf{x}')d\mathbf{x}'^3 d\mathbf{x}^3}^{\text{primary reflection}} \\
 & + \overbrace{4\omega^4 \int_V G(\mathbf{r}|\mathbf{x})s_b(\mathbf{x})\Delta s(\mathbf{x}) \int_V G(\mathbf{x}|\mathbf{x}')s_b(\mathbf{x}')\Delta s(\mathbf{x}') \int_V G(\mathbf{x}'|\mathbf{x}'')F(\mathbf{x}'')d\mathbf{x}''^3 d\mathbf{x}'^3 d\mathbf{x}^3}^{\text{first-order multiple}} \\
 & + \cdots .
 \end{aligned}
 \tag{A-6}$$

Since resonant multiples are a special type of first-order surface-related multiples, we only study the first-order multiple term $P(\mathbf{r})^m$ in equation A-6. For a point source at $\mathbf{s}$, $F(\mathbf{x}'') = W(\omega)\delta(\mathbf{x}'' - \mathbf{s})$, where $W(\omega)$ is the source frequency spectrum. For a zero-offset acquisition, the hydrophone is coincident with the source so that $\mathbf{r} = \mathbf{s}$. Based on these two conditions, the first-order multiple term $P(\mathbf{r})^m$ in equation A-6 becomes

$$P(\mathbf{s})^m = 4\omega^4 W(\omega) \int_V G(\mathbf{s}|\mathbf{x})s_b(\mathbf{x})\Delta s(\mathbf{x}) \overbrace{\int_V G(\mathbf{x}|\mathbf{x}')s_b(\mathbf{x}')\Delta s(\mathbf{x}')G(\mathbf{x}'|\mathbf{s})d\mathbf{x}'^3}^{\text{inner integral}} d\mathbf{x}^3, \tag{A-7}$$

where $G(\mathbf{x}|\mathbf{x}')$ is the solution to the Helmholtz equation

$$(\nabla^2 + \omega^2 s_b(\mathbf{x})^2)G(\mathbf{x}|\mathbf{x}') = -\delta(\mathbf{x} - \mathbf{x}'), \tag{A-8}$$

with the free-surface boundary condition

$$G(x, y, z = 0|\mathbf{x}') = 0, \quad \forall x, y. \tag{A-9}$$

We use the method of images (Jensen et al., 2000) to define $G(\mathbf{x}|\mathbf{x}')$ as

$$G(\mathbf{x}|\mathbf{x}') = g(\mathbf{x}|\mathbf{x}') + h(\mathbf{x}), \quad (\text{A-10})$$

where $g(\mathbf{x}|\mathbf{x}')$ is the Green's function that satisfies the same equation as $G(\mathbf{x}|\mathbf{x}')$, but without the free-surface boundary condition. $h(\mathbf{x})$ can be any function that satisfies the homogeneous Helmholtz equation

$$(\nabla^2 + \omega^2 s_b(\mathbf{x}))h(\mathbf{x}) = 0, \quad (\text{A-11})$$

and the boundary condition

$$g(x, y, z = 0|\mathbf{x}') + h(x, y, z = 0) = 0, \quad \forall x, y. \quad (\text{A-12})$$

If $s_b(\mathbf{x})$ is homogeneous,

$$g(\mathbf{x}|\mathbf{x}') = \frac{e^{ika}}{4\pi a} \quad \text{and} \quad h(\mathbf{x}) = -\frac{e^{ika'}}{4\pi a'}, \quad (\text{A-13})$$

with $k = \omega s_b(\mathbf{x})$, $a = |\mathbf{x} - \mathbf{x}'|$ and $a' = |\mathbf{x} - \bar{\mathbf{x}}'|$. $\bar{\mathbf{x}}' = (x', y', -z')$ is the image or mirror location of $\mathbf{x}' = (x', y', z')$. In a heterogeneous medium, the asymptotic forms of $g(\mathbf{x}|\mathbf{x}')$ and $h(\mathbf{x})$ are

$$g(\mathbf{x}|\mathbf{x}') = \frac{e^{i\omega\tau_{\mathbf{x}'\mathbf{x}}}}{A_{\mathbf{x}'\mathbf{x}}} \quad \text{and} \quad h(\mathbf{x}) = -\frac{e^{i\omega\tau_{\bar{\mathbf{x}}'\mathbf{x}}}}{A_{\bar{\mathbf{x}}'\mathbf{x}}} \quad (\text{A-14})$$

where $\tau_{\mathbf{x}'\mathbf{x}}$ and $\tau_{\bar{\mathbf{x}}'\mathbf{x}}$ are the traveltimes from $\mathbf{x}'$ to $\mathbf{x}$ and from $\bar{\mathbf{x}}'$ to $\mathbf{x}$, respectively. The geometrical spreading terms $A_{\mathbf{x}'\mathbf{x}}$ and $A_{\bar{\mathbf{x}}'\mathbf{x}}$ are given by

$$A_{\mathbf{x}'\mathbf{x}} = 4\pi R_{\mathbf{x}'\mathbf{x}} \quad \text{and} \quad A_{\bar{\mathbf{x}}'\mathbf{x}} = 4\pi R_{\bar{\mathbf{x}}'\mathbf{x}}, \quad (\text{A-15})$$

where $R_{\mathbf{x}'\mathbf{x}}$ and $R_{\bar{\mathbf{x}}'\mathbf{x}}$ are the ray lengths from $\mathbf{x}'$ to $\mathbf{x}$ and from $\bar{\mathbf{x}}'$ to $\mathbf{x}$, respectively. Because of symmetry, $\tau_{\bar{\mathbf{x}}'\mathbf{x}} = 2\tau_{\mathbf{x}\mathbf{x}_o}$ and $R_{\bar{\mathbf{x}}'\mathbf{x}} = 2R_{\mathbf{x}\mathbf{x}_o}$, where $\mathbf{x}_o$ is the intersection between $\mathbf{x}'\bar{\mathbf{x}}'$ and the free surface (see Figure A-1). $h(\mathbf{x})$ in equation A-14 can then be re-written as

$$h(\mathbf{x}) = -\frac{e^{i\omega 2\tau_{\mathbf{x}\mathbf{x}_o}}}{2A_{\mathbf{x}\mathbf{x}_o}}, \quad \text{with} \quad A_{\mathbf{x}\mathbf{x}_o} = 4\pi R_{\mathbf{x}\mathbf{x}_o}. \quad (\text{A-16})$$

Equations A-13 and A-14 suggest that the solution to the Helmholtz equation with the free-surface boundary condition consists of two Green's functions from different sources with opposite polarity: a positive source at $\mathbf{x}'$ and its negative mirror source at $\bar{\mathbf{x}}'$. Physically, the transmission arrival from a negative mirror source at $\bar{\mathbf{x}}'$ to $\mathbf{x}$ is equivalent to the specular reflection from $\mathbf{x}'$ to the free surface and back to $\mathbf{x}$ with the reflection coefficient of -1 .

Inserting equation A-10 into equation A-7 gives

$$P(\mathbf{s})^m = P(\mathbf{s})_1^m + P(\mathbf{s})_2^m, \quad (\text{A-17})$$

where

$$P(\mathbf{s})_1^m = 4\omega^4 W(\omega) \int_V G(\mathbf{s}|\mathbf{x}) s_b(\mathbf{x}) \Delta s(\mathbf{x}) \int_V g(\mathbf{x}|\mathbf{x}') s_b(\mathbf{x}') \Delta s(\mathbf{x}') G(\mathbf{x}'|\mathbf{s}) dx'^3 dx^3, \quad (\text{A-18})$$

$$P(\mathbf{s})_2^m = 4\omega^4 W(\omega) \int_V G(\mathbf{s}|\mathbf{x}) s_b(\mathbf{x}) \Delta s(\mathbf{x}) \int_V h(\mathbf{x}) s_b(\mathbf{x}') \Delta s(\mathbf{x}') G(\mathbf{x}'|\mathbf{s}) dx'^3 dx^3. \quad (\text{A-19})$$

Equations A-17 to A-19 are the Born-modeling equations for general first-order multiples, which need be adjusted according to the definition of resonant multiples. As the resonant multiple revisits the same subsurface location, $\mathbf{x}'$ must coincide with $\mathbf{x}$. This means that the inner integral in equation A-18 must be integrated over a small volume V_o enclosing $\mathbf{x}$,

instead of the whole subsurface volume V . We define V_o as a sphere with a infinitesimal radius ϵ centered at $\mathbf{x}$, so that the inner integral of equation A-18 is replaced with

$$\int_{V_o} g(\mathbf{x}|\mathbf{x}') s_b(\mathbf{x}') \Delta s(\mathbf{x}') G(\mathbf{x}'|\mathbf{s}) d\mathbf{x}'^3 = s_b(\mathbf{x}) \Delta s(\mathbf{x}) G(\mathbf{x}|\mathbf{s}) \lim_{\epsilon \rightarrow 0} \int_0^\epsilon 4\pi g(\mathbf{x}|\mathbf{x} + \mathbf{a}) a^2 da, \quad (\text{A-20})$$

where $\mathbf{x}' = \mathbf{x} + \mathbf{a}$ and a is the length of vector $\mathbf{a}$. Equation A-20 presumes that $s_b(\mathbf{x}') \Delta s(\mathbf{x}') G(\mathbf{x}'|\mathbf{s}) = s_b(\mathbf{x}) \Delta s(\mathbf{x}) G(\mathbf{x}|\mathbf{s})$ because $\mathbf{x}'$ is very close to $\mathbf{x}$ inside V_o . Inside the infinitesimal volume V_o the medium is homogeneous, so that inserting equation A-13 into equation A-20 leads to

$$\int_{V_o} g(\mathbf{x}|\mathbf{x}') s_b(\mathbf{x}') \Delta s(\mathbf{x}') G(\mathbf{x}'|\mathbf{s}) d\mathbf{x}'^3 = s_b(\mathbf{x}) \Delta s(\mathbf{x}) G(\mathbf{x}|\mathbf{s}) \lim_{\epsilon \rightarrow 0} \int_0^\epsilon e^{ika} a da = 0. \quad (\text{A-21})$$

This proves that for resonant multiples,

$$P(\mathbf{s})_1^m = 0. \quad (\text{A-22})$$

The derivation for $P(\mathbf{s})_2^m$ is less complicated. Again, for resonant multiples, $\mathbf{x}'$ must coincide with $\mathbf{x}$ because the resonant multiple revisits the same subsurface location by definition. Consequently,

$$\Delta s(\mathbf{x}') = \Delta s(\mathbf{x}) \delta(\mathbf{x}' - \mathbf{x}). \quad (\text{A-23})$$

Substituting equation A-23 into equation A-19 and using the reciprocity property of Green's function $G(\mathbf{s}|\mathbf{x}) = G(\mathbf{x}|\mathbf{s})$ yields

$$P(\mathbf{s})_2^m = 4\omega^4 W(\omega) \int_V G(\mathbf{x}|\mathbf{s})^2 h(\mathbf{x}) [s_b(\mathbf{x}) \Delta s(\mathbf{x})]^2 d\mathbf{x}^3. \quad (\text{A-24})$$

The summation of equations A-22 and A-24 gives the resonant multiples

$$P(\mathbf{s})^{\text{rm}} = 4\omega^4 W(\omega) \int_V G(\mathbf{x}|\mathbf{s})^2 h(\mathbf{x}) [s_b(\mathbf{x}) \Delta s(\mathbf{x})]^2 dx^3. \quad (\text{A-25})$$

Similar to equation A-10, $G(\mathbf{x}|\mathbf{s})$ can be decomposed into two terms using the method of images:

$$G(\mathbf{x}|\mathbf{s}) = g(\mathbf{x}|\mathbf{s}) + h'(\mathbf{x}), \quad (\text{A-26})$$

where $g(\mathbf{x}|\mathbf{s})$ is the Green's function recorded at $\mathbf{x}$ due to the source at $\mathbf{s}$ without the free-surface boundary condition. Here, $h'(\mathbf{x})$ can be physically interpreted as the propagator of the free-surface reflection from $\mathbf{s}$ to $\mathbf{x}$. Inserting equation A-26 into equation A-25 gives

$$\begin{aligned} P(\mathbf{s})^{\text{rm}} = & \overbrace{4\omega^4 W(\omega) \int_V g(\mathbf{x}|\mathbf{s})^2 h(\mathbf{x}) [s_b(\mathbf{x}) \Delta s(\mathbf{x})]^2 dx^3}^{\text{deghosted resonant multiple}} + \\ & \overbrace{8\omega^4 W(\omega) \int_V g(\mathbf{x}|\mathbf{s}) h'(\mathbf{x}) h(\mathbf{x}) [s_b(\mathbf{x}) \Delta s(\mathbf{x})]^2 dx^3}^{\text{source- or receiver-side ghost}} + \\ & \overbrace{4\omega^4 W(\omega) \int_V h'(\mathbf{x})^2 h(\mathbf{x}) [s_b(\mathbf{x}) \Delta s(\mathbf{x})]^2 dx^3}^{\text{source- and receiver-side ghost}}. \end{aligned} \quad (\text{A-27})$$

The first term of equation A-27 is the Born-modeling equation for the deghosted resonant multiple $d(\omega, \mathbf{s})$:

$$d(\omega, \mathbf{s}) = \overline{W}(\omega) \int_V g(\mathbf{x}|\mathbf{s})^2 h(\mathbf{x}) [s_b(\mathbf{x}) \Delta s(\mathbf{x})]^2 dx^3, \quad (\text{A-28})$$

with $\overline{W}(\omega) = 4\omega^4 W(\omega)$. The second and third terms of equation A-27 can be interpreted as the ghosts of multiples as shown in Figures A-2b to A-2d.

APPENDIX B

STATIONARY PHASE INTEGRATION

We show that equation 23 is valid irrespective of velocity models. According to the stationary phase integration (Bleistein, 1984),

$$\int_{f_s} \frac{e^{-i\omega\tau_{\mathbf{x}\mathbf{x}''}}}{A_{\mathbf{x}\mathbf{x}''}} dx''^2 \approx \alpha \frac{e^{-i\omega\tau_{\mathbf{x}\mathbf{x}^*}}}{A_{\mathbf{x}\mathbf{x}^*}}, \quad (\text{B-1})$$

where $\mathbf{x}^*$ is on the free surface and satisfies

$$\left. \frac{\partial \tau_{\mathbf{x}\mathbf{x}''}}{\partial x''} \right|_{\mathbf{x}''=\mathbf{x}^*} = 0, \quad (\text{B-2})$$

and x'' is the horizontal coordinate of $\mathbf{x}''$.

According to the scalar dispersion relation, the horizontal wavenumber $k_x = \omega \frac{\partial \tau_{\mathbf{x}\mathbf{x}''}}{\partial x''} = 0$ at $\mathbf{x}^*$. This means that ray $\overline{xx^*}$ is perpendicular to the free-surface at $\mathbf{x}^*$, irrespective of the velocity model. Considering that ray $\overline{xx_o}$ is also vertical to the free surface at $\mathbf{x}_o$, we can draw the conclusion that $\mathbf{x}^* = \mathbf{x}_o$, so that equation 23 is proved.

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LIST OF FIGURES

1 Raypath diagrams of zero-offset resonant multiples reflecting from (a) a flat and
 2 (b) tilted reflector in homogeneous media. The multiples are excited by the source at $\mathbf{s}$ and
 3 recorded by the hydrophone at the same location. The raypath trajectory is $\mathbf{s} \rightarrow \mathbf{x} \rightarrow \mathbf{x}_o \rightarrow$
 4 $\mathbf{x} \rightarrow \mathbf{s}$. Ray $\overline{xx_o}$ is perpendicular to the water-air interface.

5 Wavenumber analysis of migrating (a) resonant multiples and (b) zero-offset pri-
 6 mary reflections. θ is the dip angle of the tilted interface measured with respect to the
 7 horizontal axis x .

8 Raypath diagrams of different types of first-order surface-related multiples. The
 9 straight arrows represent the raypath of a multiple which bounces on the same reflector
 10 at A and B. The dashed arrows represent the raypath of a multiple which bounces on two
 11 different reflectors. h is the offset between the source and receiver.

12 (a) 2-layer velocity model with a flat interface. (b) CMG at $X = 2.25$ km before
 13 moveout-correction, and (c) after moveout-correction. The red arrows mark the resonant
 14 multiples.

15 (a) Zero-offset data, where the first arrivals are primary reflections and the second
 16 arrivals are resonant multiples. (b) Kirchhoff migration image using 5-Hz primary reflec-
 17 tions. (c) Kirchhoff migration image using 5-Hz resonant multiples. (d) Wave-equation
 18 migration image using 5-Hz resonant multiples. (e) Kirchhoff migration image using 10-Hz
 19 primary reflections.

20 Vertical wavenumber comparison of images shown in Figure 5.

21 (a) 2-layer velocity model with a tilted interface. (b) CMG at $X = 2.25$ km before
 22 moveout-correction, and (c) after moveout-correction. The red arrows mark the resonant
 23 multiples.

(a) Zero-offset data, where the first arrivals are primary reflections and the second arrivals are resonant multiples. (b) Kirchhoff migration image using 5-Hz primary reflections. (c) Kirchhoff migration image using 5-Hz resonant multiples. (d) Wave-equation migration image using 5-Hz resonant multiples. (e) Kirchhoff migration image using 10-Hz primary reflections.

Vertical wavenumber comparison of images shown in Figure 8.

(a) Vertical and (b) horizontal wavenumbers of the images with different dip-angle reflectors. The blue and red curves are respectively calculated and plotted based on equations 13 and 17. $k = 0.0242 \text{ m}^{-1}$ is half of the vertical wavenumber of the flat primary-reflection image. The stars represent the experimental results of the centroid wavenumbers of the migration images. The blue (red) curves and stars represent the wavenumbers of resonant-multiple (primary-reflection) images.

(a) Velocity model with a salt dome and (b) the simulated zero-offset data. The black arrows indicate the primary reflections from the top and the bottom of the salt dome. The red arrows point out the associated resonant multiples.

Kirchhoff migration images computed from (a) primary reflections and (b) resonant multiples. (c) Wave-equation migration image computed from resonant multiples.

Vertical-wavenumber comparison of the images shown in Figure 12.

(a) Near-offset data before moveout correction. (b) Zero-offset data after moveout correction and stacking. (c) Comparison between the frequency spectra of the post-stack primary reflections and multiples.

CMGs before and after moveout-correction at the horizontal locations of (a) 18.7 km, (b) 21.2 km and (c) 37.5 km. The red arrows mark the flattening of resonant multiples.

Migration velocity model converted from the root-mean-squared velocity using

Dix's equation.

17 a) Primary-reflection and (b) resonant-multiple images using Kirchhoff migration.
 (c) Resonant-multiple image computed by RTM. (d) Comparison between the vertical-
 wavenumber spectra of (a), (b) and (c).

18 (a) and (c) are the zoom-in views of the red and green dashed squares of the
 primary-reflection image shown in Figure 17a. (b) and (d) are the zoom-in views of the red
 and green dashed squares of the resonant-multiple image in Figure 17c.

19 Raypath diagram of an interbed resonant multiple: $\mathbf{s} \rightarrow \mathbf{x} \rightarrow \mathbf{x}_o \rightarrow \mathbf{x} \rightarrow \mathbf{s}$.

A-1 Volume V enclosed by the free-surface boundary and the hemisphere at infinity. $\mathbf{x}$
 and $\mathbf{x}'$ are inside V . $\bar{\mathbf{x}}'$ is the mirror point of $\mathbf{x}'$ with respect to the free surface.

A-2 (a) Raypath diagrams of the deghosted resonant multiples $\mathbf{s} \rightarrow \mathbf{x} \rightarrow \mathbf{x}_o \rightarrow \mathbf{x} \rightarrow \mathbf{s}$,
 (b) receiver-side multiple ghost $\mathbf{s} \rightarrow \mathbf{x} \rightarrow \mathbf{x}_o \rightarrow \mathbf{x} \rightarrow \mathbf{x}'_o \rightarrow \mathbf{s}$, (c) source-side multi-
 ple ghost $\mathbf{s} \rightarrow \mathbf{x}'_o \rightarrow \mathbf{x} \rightarrow \mathbf{x}_o \rightarrow \mathbf{x} \rightarrow \mathbf{s}$ and (d) source-and-receiver side multiple ghost
 $\mathbf{s} \rightarrow \mathbf{x}'_o \rightarrow \mathbf{x} \rightarrow \mathbf{x}_o \rightarrow \mathbf{x} \rightarrow \mathbf{x}'_o \rightarrow \mathbf{s}$.

Figure 1: Raypath diagrams of zero-offset resonant multiples reflecting from (a) a flat and (b) tilted reflector in homogeneous media. The multiples are excited by the source at **s** and recorded by the hydrophone at the same location. The raypath trajectory is **s**→**x**→**x_o**→**x**→ **s**. Ray **xx_o** is perpendicular to the water-air interface.

81x39mm (300 x 300 DPI)

Figure 2: Wavenumber analysis of migrating (a) resonant multiples and (b) zero-offset primary reflections. θ is the dip angle of the tilted interface measured with respect to the horizontal axis x .

149x102mm (300 x 300 DPI)

Figure 3: Raypath diagrams of different types of first-order surface-related multiples. The straight arrows represent the raypath of a multiple which bounces on the same reflector at A and B. The dashed arrows represent the raypath of a multiple which bounces on two different reflectors. h is the offset between the source and receiver.

120x101mm (300 x 300 DPI)

Figure 4: (a) 2-layer velocity model with a flat interface. (b) CMG at $X = 2.25$ km before moveout-correction, and (c) after moveout-correction. The red arrows mark the resonant multiples.

72x18mm (300 x 300 DPI)

Figure 5: (a) Zero-offset data, where the first arrivals are primary reflections and the second arrivals are resonant multiples. (b) Kirchhoff migration image using 5-Hz primary reflections. (c) Kirchhoff migration image using 5-Hz resonant multiples. (d) Wave-equation migration image using 5-Hz resonant multiples. (e) Kirchhoff migration image using 10-Hz primary reflections.

221x172mm (300 x 300 DPI)

Figure 6: Vertical wavenumber comparison of images shown in Figure 5.

80x38mm (300 x 300 DPI)

Figure 7: (a) 2-layer velocity model with a tilted interface. (b) CMG at $X = 2.25$ km before moveout-correction, and (c) after moveout-correction. The red arrows mark the resonant multiples.

73x19mm (300 x 300 DPI)

Figure 8: (a) Zero-offset data, where the first arrivals are primary reflections and the second arrivals are resonant multiples. (b) Kirchhoff migration image using 5-Hz primary reflections. (c) Kirchhoff migration image using 5-Hz resonant multiples. (d) Wave-equation migration image using 5-Hz resonant multiples. (e) Kirchhoff migration image using 10-Hz primary reflections.

220x185mm (300 x 300 DPI)

Figure 9: Vertical wavenumber comparison of images shown in Figure 8.

83x41mm (300 x 300 DPI)

Figure 10: (a) Vertical and (b) horizontal wavenumbers of the images with different dip-angle reflectors. The blue and red curves are respectively calculated and plotted based on equations 13 and 17. $k = 0.0242 \text{ m}^{-1}$ is half of the vertical wavenumber of the flat primary-reflection image. The stars represent the experimental results of the centroid wavenumbers of the migration images. The blue (red) curves and stars represent the wavenumbers of resonant-multiple (primary-reflection) images.

101x36mm (300 x 300 DPI)

Figure 11: (a) Velocity model with a salt dome and (b) the simulated zero-offset data. The black arrows indicate the primary reflections from the top and the bottom of the salt dome. The red arrows point out the associated resonant multiples.

120x68mm (300 x 300 DPI)

Figure 12: Kirchhoff migration images computed from (a) primary reflections and (b) resonant multiples. (c) Wave-equation migration image computed from resonant multiples.

215x364mm (300 x 300 DPI)

Figure 13: Vertical-wavenumber comparison of the images shown in Figure 12.

92x32mm (300 x 300 DPI)

Figure 14: (a) Near-offset data before moveout correction. (b) Zero-offset data after moveout correction and stacking. (c) Comparison between the frequency spectra of the post-stack primary reflections and multiples.

273x384mm (300 x 300 DPI)

Figure 15: CMGs before and after moveout-correction at the horizontal locations of (a) 18.7 km, (b) 21.2 km and (c) 37.5 km. The red arrows mark the flattening of resonant multiples.

172x87mm (300 x 300 DPI)

Figure 16: Migration velocity model converted from the root-mean-squared velocity using Dix's equation.

72x27mm (300 x 300 DPI)

Figure 17: (a) Primary-reflection and (b) resonant-multiple images using Kirchhoff migration. (c) Resonant-multiple image computed by RTM. (d) Comparison between the vertical-wavenumber spectra of (a), (b) and (c).

367x694mm (300 x 300 DPI)

Figure 18. (a) and (c) are the zoom-in views of the red and green dashed squares of the primary-reflection image shown in Figure 17a. (b) and (d) are the zoom-in views of the red and green dashed squares of the resonant-multiple image in Figure 17c.

132x84mm (300 x 300 DPI)

Figure 19: Raypath diagram of an interbed resonant multiple: **S**→**X**→**X_o**→**X**→ **S**.

93x72mm (300 x 300 DPI)

Figure A-1: Volume V enclosed by the free-surface boundary and the hemisphere at infinity. $\mathbf{x}$ and $\mathbf{x}'$ are inside V .

118x91mm (300 x 300 DPI)

Figure A-2: (a) Raypath diagrams of the deghosted resonant multiples $s \rightarrow x \rightarrow x_o \rightarrow x \rightarrow s$, (b) receiver-side multiple ghost $s \rightarrow x \rightarrow x_o \rightarrow x \rightarrow x'_o \rightarrow s$, (c) source-side multiple ghost $s \rightarrow x'_o \rightarrow x \rightarrow x_o \rightarrow x \rightarrow s$ and (d) source- and receiver side multiple ghost $s \rightarrow x'_o \rightarrow x \rightarrow x_o \rightarrow x \rightarrow x'_o \rightarrow s$.

135x139mm (300 x 300 DPI)