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MUDCAKE GROWTH: MODEL AND IMPLICATIONS

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Abstract: Oil and gas account for 60% of the world's energy consumption. Drilling muds that are used to advance oil and gas wells must be engineered to avoid wellbore integrity problems associated with mud cake formation, to favor cake erosion during cementing, and to prevent partial differential sticking. We developed a robust mud cake growth model for water-based mud based on wide stress-range constitutive equations within a Lagrangian reference system to avoid non-natural moving boundary solutions. The comprehensive mud cake growth model readily accommodates environmental factors (e.g., temperature, pH, and ionic concentration) and defines the yield stress distribution for displacement-erosion analyses. Results show that the mud cake thickness is more sensitive to time than to filtration pressure, therefore, time controls the non-uniform distribution of mudcake thickness during drilling. Long filtration time, high permeability, high salinity, high in-situ temperature and low viscosity exacerbate fluid loss and give rise to thick filter cakes. The analysis of residual cake thickness during cement displacement must take into account the effective stress dependent mudcake formation and the time-dependent mud thixotropy. Thixotropy dominates the mud yield stress at high void ratios, e.g. $e > 20$. The offsetting force that causes differential pressure sticking increases sub-linearly as a power function of the still-time.

30 **Keywords:** mudcake; drilling mud; cement displacement; thixotropy; differential pressure
31 sticking

1. Introduction

The drilling mud removes cuttings from the wellbore, reduces friction between the drill pipe or casing and the wellbore, cools and cleans the bit, coats the formation with a thin, low-permeability filter cake, maintains wellbore stability, and minimizes formation damage. Liquids filtrate into the formation, and solid particles remain behind and form the filter cake against the well wall.

Early research demonstrated that the cumulative flow-through volume is proportional to the square root of time when drilling mud is pressed against a filter paper under conditions of constant pressure and temperature (Larsen 1938). The filter cake evolves over time but remains inherently inhomogeneous due to the large effective stress gradient that develops across the cake and the ensuing large gradient in porosity and permeability.

The study of mudcake formation and implications has involved experimental as well as analytical and numerical methods. Experimental studies use various techniques that include: dissection (Meeten 1993, Sherwood and Meeten 1997), X-ray tomography (Elkatatny et al. 2012) and scanning electron microscopy (Hartmann et al. 1988). Cake formation models typically start with solid and liquid mass conservation equations and incorporate constitutive equations that relate void ratio, effective stress and permeability (Wakeman 1981, Sherwood et al. 1991, Stamatakis and Tien 1991, Chenevert and Dewan 2001, Johansson and Theliander 2007). Constitutive equations adopted in previous analyses distinguish the “cake phase” from the “slurry phase” and result in “moving boundary” type solutions (Outmans 1963, Collins 1976, Stamatakis and Tien 1991).

This study advanced a comprehensive mudcake growth model for water-based mud based on robust constitutive equations, and used this model to evaluate the influences of time, pressure and environmental factors such as temperature and salinity on the filtration behavior of drilling muds. Subsequent analyses explored critical drilling and completion issues that include mud shearing and differential pressure sticking.

2. Model Development

Consider a mud-filled cylinder and a piston that pushes the mud in the cylinder against a pervious formation (Figure 1a). While liquids filtrate into the formation, solid particles remain behind against the interface. Let's adopt a Lagrangian reference systems fixed in the solid to describe the physical process (see also Sherwood et al. 1991, Sherwood and Meeten 1997). The boundaries of the Lagrangian element move with the same velocity as the solid velocity during compression. Figure 1b shows an infinitesimal element between z and $z+dz$. The fluid mass conservation relates the change in flux through the element to the change in the element void ratio e [Note: void ratio is the volume of voids normalized by the volume of solids and it is related to porosity n as $n=e/(1+e)$]:

$$[v_f(z + dz) - v_f(z)]dt = -dz[e(t + dt) - e(t)] \quad (1)$$

so that

$$\frac{\partial e}{\partial t} = -\frac{\partial v_f}{\partial z} \quad (2)$$

Darcy's law relates the fluid velocity v_f [m/s] to the permeability of the porous medium k [m²], the viscosity of the liquid phase μ [Pa·s] and the pressure gradient $\partial u/\partial z$ [Pa/m].

$$v_f = -\frac{k}{\mu(1+e)} \frac{\partial u}{\partial z} \quad (3)$$

73 Let's substitute Darcy's law into the mass conservation equation to obtain the governing
74 equation for the mud compressed against the pervious wall:

$$\frac{\partial e}{\partial t} = \frac{\partial}{\partial z} \left[\frac{k}{\mu} \frac{1}{1+e} \frac{\partial u}{\partial z} \right] \quad (4)$$

75 Initially, the mud has a uniform void ratio e_0 . The piston imposes a pressure u_0 , therefore the
76 filtration pressure is $u_0 - u_\infty$, where u_∞ is the constant pressure in the far field inside the formation.

77 As the liquid invades the formation, the effective stress σ' in the mud increases next to the wall
78 in response to changes in fluid pressure u ,

$$\frac{\partial \sigma'}{\partial z} = -\frac{\partial u}{\partial z} \quad (5)$$

79 Therefore, Equation 4 becomes:

$$\frac{\partial e}{\partial t} = -\frac{\partial}{\partial z} \left[\frac{k}{\mu} \frac{1}{1+e} \frac{\partial \sigma'}{\partial z} \right] \quad (6)$$

80 The mud permeability decreases as the void ratio decreases during cake formation, and the
81 permeability $k=f(e)$ remains inside the derivative. We note that the 1D-formulation developed
82 here presumes a cake thickness much smaller than the well radius. The supplemental material
83 associated with this manuscript includes the non-dimensional form of the partial differential
84 equation (Eq. 6) and identifies the governing dimensionless ratios.

85

3. Clay Behavior – Constitutive equations

The effective stress varies from $\sigma'=0$ MPa in the slurry, to values that often exceed $\sigma'>1$ MPa in the mudcake against the formation. Consequently, the void ratios fall from $e>40$ to $e<0.5$, and the permeability decreases over 5 orders of magnitude. Surprisingly, these dramatic changes take place within a few millimeters.

Furthermore, in situ conditions are characterized by high temperature and the risk of high salinity contamination. Therefore, we need adequate constitutive models for mud compaction and permeability as a function of effective stress, temperature, and salinity. Table 1 summarizes constitutive equations adopted in previous drilling mud studies. We seek robust, physics-inspired models with a limited number of physically meaningful parameters.

Clay Structure

Electrical inter-particle forces determine particle aggregation in mineral suspensions. These forces depend on the mineral and fluid chemistry. The drilling industry prefers mono-ionic sodium montmorillonite, where Na^+ cations populate the counter-ion cloud around particles. The fluid pH determines clay edge and surface charges, while the fluid ionic concentration controls the double layer thickness and the balance between attraction and repulsion forces. Consequently, the pore fluid pH and ionic concentration will determine the tendency to disperse or to aggregate into edge-to-face, edge-to-edge, or face-to-face aggregations (Van Olphen 1977, Santamarina et al. 2002). Other environmental factors such as temperature affect particle association and filtration properties.

The highest void ratio that a packing of platy particles can attain depends on the particle slenderness α , defined as the length L_p divided by the thickness t_p . For example, the void ratio for a card-castle structure can be as high as $e_{max}=(\alpha-1)/2$ (Sharma and Zongming 1991, Santamarina et al. 2001). The slenderness of montmorillonite particles may reach $\alpha=100$, which corresponds to a void ratio $e=50$. This agrees with experimental studies using shear waves (Klein and Santamarina 2005) and rheological properties (Abend and Lagaly 2000, Santagata et al. 2008).

Compressibility

We adopted a wide stress-range compressibility model to avoid the artificially defined cake-slurry interface in the moving boundary problem (Peck and Terzaghi 1948, Chong and Santamarina 2016):

$$e = e_c - C_c \log \left(\frac{1kPa}{\sigma' + \sigma'_L} + \frac{1kPa}{\sigma'_H} \right)^{-1} \quad (7)$$

where asymptotic void ratios $e \rightarrow e_L$ and $e \rightarrow e_H$ define the threshold effective stresses σ'_L and σ'_H ,

$$\sigma'_H = 10^{(e_c - e_H)/C_c} kPa \text{ when } \sigma' \rightarrow \infty \quad (8)$$

$$\sigma'_L = \frac{\sigma'_H}{10^{(e_L - e_H)/C_c - 1}} \text{ when } \sigma' \rightarrow 0 \quad (9)$$

In mud analyses, e_L is the initial void ratio e_0 of the slurry as mixed, $e_L = e_0 \leq e_{max}$. Figure 2.a shows experimental data and model prediction for void ratio versus effective stress $e-\sigma'$. In the absence of data at very low effective stress, we note that the void ratio at the liquid limit e_{LL} is a good estimate of the void ratio at $\sigma' = 1kPa$, $e_{1kPa} \approx e_{LL} = G_s \bullet LL / 100$ (Chong and Santamarina 2016).

Fluid flow: permeability vs. void ratio

Permeability decreases as the pore size decreases when the slurry compacts to form the cake. This study adopts a power equation to relate permeability k to the void ratio e (Kozeny 1927, Carman 1937, Taylor 1948, Chapuis 2003, Ren and Santamarina 2017):

$$k = k_0 \left(\frac{e}{e_0} \right)^\beta \quad (10)$$

where k_0 is the permeability at the reference void ratio e_0 and the β -exponent captures the sensitivity of permeability to changes in the void ratio. Figure 2.b shows that this constitutive equation adequately matches experimental data for bentonite. While data are missing at very low effective stress, we highlight that the void ratio at high effective stress dominates the fluid flux across the filter cake.

The proposed constitutive models for compressibility and permeability capture the gradual mud response with effective stress, avoid the non-natural assumptions in moving boundary solutions, and correctly anticipate observed experimental trends (see comparison in supplemental material).

Temperature: clay rheology and fluid viscosity

Temperature alters the rheological characteristics and the compaction behavior of clay pastes through a combination of competing effects: increased platelet Brownian motion and hindered bond formation, increased Debye-Huckel length and inter-particle repulsion, faster aggregation towards minimum potential energy configurations, and decreased fluid viscosity.

Experimental evidence shows that the yield stress of bentonite suspensions is weakly dependent on temperature below 60°C and rapidly rises at higher temperature due to the effect of dispersion and increased effective mineral surface (Annis 1967, Alderman et al. 1988). Conversely, a well-dispersed bentonite suspension displays a decrease in the high-shear viscosity as temperature increases (Annis 1967, Alderman et al. 1988).

Figure 3 shows the shear plane around dispersed and face-to-face aggregated platelets. Consider n_0 platelets of individual thickness t_p across a representative length scale L_0 . When platelets group into n aggregates, the free space L_{free} for fluid shearing depends on the degree of dispersion $\beta=n/n_0$ and the thickness to the shear plane t_{shear} :

$$L_{free} = L_0 - n_0 t_p - 2n t_{shear} = L_0 - n_0 (t_p + 2\beta t_{shear}) \quad (11)$$

Clearly, the fluid free space decreases and the shear resistance $\tau=\mu v/L_{free}$ increases with increased dispersion β .

Fluid chemistry: pH and ionic concentration

Repulsion is shielded, van der Waals attraction prevails and platelets aggregate face-to-face at high ionic concentration, typically above seawater concentration ~0.5mol/L (see fabric maps in Santamarina et al. 2001; Palomino et al. 2005). The aggregation of platelets at high ionic concentrations leads to more compacted fabrics, a pronounced decrease in the liquid limit (Figure 4.a) and lower compressibility C_C (Figure 4.b).

Calcium ions replace sodium ions in counter ion clouds, shrink double layers and prompt aggregation. Therefore, mud contamination with calcium ions from gypsum, anhydrite, lime or

cement decreases the mud viscosity, increases fluid loss and results in more compact and less compressible cakes (Figure 5).

4. Cake Formation – Model Predictions

The governing differential equation for cake formation (Eq. 6), constitutive equations (Eq. 7, 10), and mud behavioral trends addressed above (Figures 2 to 5) come together in this section, as we explore the mud response during drilling.

Filtration pressure and filtration time

Let's consider a mud filled borehole. Figure 6 presents the void ratio profiles under a constant filtration pressure of 2MPa after 50s, 200s, 800s and 3200s (constitutive parameters listed in Table 2). The cake thickness increases with time. The non-linear e - x trends reflect the coupling between the non-linear compressibility and permeability models.

Figure 7 provides void ratio contours along a wellbore after 2.8hr (constant filtration time at all depths). The minimum void ratio attained when the cake rests against the formation ($x=0$) depends on the filtration pressure. Contour lines for large void ratios $e>10$ away from the wall are not sensitive to the filtration pressure due to the high compaction next to the formation and the associated low permeability.

Depth

The filtration time varies with depth during the drilling process. Let's consider a drilling stage from 400m to 600m with a rate of penetration $ROP=20\text{m/hr}$ (Figure 8.a). The filtration pressure is 0.8MPa at 400m and 1.2MPa at 600m with a mud weight $=1200\text{kg/m}^3$. Drilling alone takes 10 hours so that the filtration time is 10 hours at depth 400m and 0 hour at depth 600m immediately-post drilling (still-time=0hr). Let's assume a void ratio $e=10$ as a mud cake thickness criterion. With zero still-time, the corresponding mud cake thickness is 5.8mm at depth 400m and 0mm at depth 600m. After 24hr of still-time, the filtration time is 34hr at 400m, 24hr at 600m, and the corresponding cake thickness has increased to 10.8mm and 9.1mm respectively (Figure 8.b). The mud cake thickness becomes more uniform as the still time increases.

Viscosity of the continuous phase

The viscosity of water decreases with temperature, but it can be augmented by more than two orders of magnitude with the addition of viscosifiers. Let's assume that the change in the viscosity of water does not alter the absolute permeability and compressibility of the bentonite skeleton.

Results in Figure 9.a&b demonstrate that an increase in the liquid viscosity effectively reduces the fluid loss and leads to a thinner and tighter filter cake. In particular, simulation results show that both fluid loss and cake thickness double when the water viscosity decreases from 1cP ($\sim 20^\circ\text{C}$) to 0.3cP ($\sim 90^\circ\text{C}$), in good agreement with previous experimental results

(Arthur and Peden 1988). Clearly, filtration properties must be determined at in-situ temperature conditions.

Permeability

The influence of the permeability on filtration is explored in Figure 10 for three different values of the reference permeability k_0 that could result from ionic contamination ($k_0=1\times10^{-5}$ mD, 5×10^{-5} mD and 1×10^{-4} mD. Note: all other constitutive parameters in Table 2 remain the same). Results indicate that more permeable muds form thicker filter cakes and cause more severe fluid loss during filtration. This analysis supports previous experimental results and field observations related to severe fluid losses in the presence of Ca^{2+} or high ionic concentration contamination.

5. Implications

Cementation: Mud replacement and cake removal

Well integrity relies on successful cementing. Inadequate filter cake removal leaves behind a layer of residual cake which may become a potential pathway for gas flow along the annulus (Bonett and Pafitis 1996). The mudcake is sheared off until the shear stress imposed by the invading cement exceeds the shear strength of the mudcake. The imposed shear stress depends on the displacement velocity in the annulus between the casing and the well wall, and the viscosity of the cement.

On the other hand, the shear strength of the bentonite depends on effective stress in the cake and thixotropic hardening in the slurry. The yield stress of bentonite suspensions is a function of the time t and the void ratio e of bentonite:

$$\tau_y = A \left(\frac{t}{hr} \right)^B \text{Pa} \quad (12)$$

where A is the yield stress τ_y when $t=1hr$, and B is the sensitivity of τ_y to time. Experimental results in Figure 11 show that both A and B parameters are a function of the void ratio e :

$$A = 0.42 \exp \left(\frac{156}{2.6 + e} \right)$$

$$B = 0.37 - \frac{6.2}{2.6 + e}$$

The effective stress dependent shear strength τ_y in the cake is estimated from the effective stress σ' following a standard Cam Clay formulation (Schofield and Wroth 1968):

$$\tau_y = 0.2\sigma' \quad (13)$$

We combined these two models to anticipate the shear strength profile of the slurry-mudcake system. Figure 12 shows the yield stress and shear strength profiles of a filter cake formed under the filtration pressure=2MPa after 0.1hr and 2.8hr. Results show the combined effect of effective stress and thixotropy: thixotropy dominates the yield stress of muds with high void ratios $e>20$.

Let's assume a $OD=200mm$ casing inside a $ID=280mm$ well so that the annulus is 40mm thick. The black lines in Figure 13 are the shear stress profiles imposed by the cement slurry driven by an effective pressure gradient of 3kPa/m and 12kPa/m along the well. The intersection point between the yield stress and the shear stress provides a conservative estimate of the

residual cake thickness; in this case 11mm for a pressure gradient of 3kPa/m and 8mm for a pressure gradient of 12kPa/m. The high pressure gradient required to remove the compacted mudcake is not attainable under normal flow conditions. Therefore, the shear stress imposed by the cement slurry is not sufficient for effective mudcake removal.

Differential Pressure Sticking

“Stuck pipe” events have been one of the drilling industry’s major challenges due to lost time and associated costs (Outmans 1958). In fact, stuck pipes account for about 25% of the non-productive time. A third of the stuck pipes are due to differential pressure sticking (Muqeem et al. 2012).

Mudcake formation has a strong influence on differential sticking incidents. Let’s use the cake growth model developed above to determine the force exerted on the tube surface due to unbalanced fluid pressure against the tube. In a polar coordinate system centered with the well (r, θ), the contour of a centered circle is $r=R$ and the contour of an eccentric circle with an offset c satisfies $r^2+c^2-2r\cdot c\cdot\cos\theta=R^2$ (Figure 14.a-inset). The total force T generated by the fluid pressure is the integral of the fluid pressure u against the casing perimeter:

$$T = \int_0^{2\pi} u(r, \beta) R \cos(\beta) d\beta \quad (14)$$

Consider an $ID=300\text{mm}$ borehole filled with mud under a 1MPa filtration pressure. The $OD=200\text{mm}$ pipe is offset $c=49\text{mm}$ towards a side. Let’s assume that the presence of the pipe does not influence the void ratio and pore pressures profiles. Figure 14.a shows the pore pressure

against the well as a function of filtration time. There is a significant fluid pressure drop at $\beta = 180^\circ$, where the pipe is closest to the well wall. The total force T acting on the pipe increases sub-linearly with time as the mudcake grows $T = 1.05 \cdot (t/\text{hr})^{3/4}$ kN/m (Figure 14.b). This trend resembles earlier experimental studies (Annis and Monaghan 1962, Reid et al. 1996, Sherwood 1998). Indeed, minimal time is recommended to reduce stuck pipe complications (Dupriest et al. 2011).

6. Conclusions

This study advanced a comprehensive mudcake growth model for water-based mud implemented with robust, physically-informed constitutive equations. In particular, we adopted wide stress-range constitutive models to avoid numerical discontinuities, and selected compatible model parameters for constitutive equations.

The mud cake thickness is more sensitive to time than to filtration pressure. Therefore, time controls the non-uniform distribution of mudcake thickness during drilling. Environmental factors such as temperature, pH, ionic concentration and cation contamination have a significant influence on cake formation. Long filtration time, high permeability, and low viscosity exacerbate fluid loss and increase the cake thickness.

Mud replacement is critical to the quality of the cementing job and the integrity of wellbores. The analysis of residual cake thickness takes into account the effective stress dependent mudcake

formation and the time-dependent mud thixotropy. Thixotropy dominates the mud yield stress at high void ratios, e.g. $e > 20$.

Stuck pipe by differential pressure sticking is responsible for costly non-productive time. The offsetting force increases sub-linearly as a power function of the still-time.

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Table 1. Summary of constitutive equations in compressible cake models. Compressibility and Permeability

Compressibility	Power law	$\frac{(1-n)}{(1-n_0)} = \left(1 + \frac{\sigma'}{\sigma_0}\right)^\alpha$	Tiller and Leu 1980, Stamatakis and Tien 1991, Sørensen et al. 1996, Theliander and Fathi-Najafi 1996, Tien and Bai 2003, Johansson and Theliander 2007
		$\frac{n}{n_0} = \left(\frac{\sigma'}{\sigma_0}\right)^{-\alpha}$	Outmans 1963, Holdich 1993, Chenevert and Dewan 2001, Johansson and Theliander 2007
		$\frac{1-n_{avg}}{1-n_{avg0}} = \left(\frac{\sigma}{\sigma_0}\right)^{-\alpha}$	Tiller and Cooper 1962
		$e = e_0 + \left(\frac{\sigma_1}{\sigma' + \sigma_2}\right)^\alpha$	Sherwood and Meeten 1997
		$e = e_0 + \left(\frac{\sigma_1}{\sigma'}\right)^\alpha$	Carrier and Beckman 1984
	Inverse function	$n = n_0 + \frac{\sigma_1}{\sigma' + \sigma_2}$	Johansson and Theliander 2007
	Logarithm	$e = \alpha \log\left(\frac{-\sigma'}{\sigma_0}\right) + \beta$	Smiles 1970
	Modified semi-logarithmic model	$e = e_c - C_c \log\left(\frac{1kPa}{\sigma' + \sigma'_L} + \frac{1kPa}{\sigma'_H}\right)$	Chong and Santamarina 2016

Permeability	Power law	$k = k_0 \left(\frac{\sigma'}{\sigma_0} \right)^{-\beta}$	Tiller and Cooper 1962, Outmans 1963, Holdich 1993, Chenevert and Dewan 2001
		$k = k_0 \left(1 + \frac{\sigma'}{\sigma_0} \right)^{-\beta}$	Tiller and Leu 1980, Stamatakis and Tien 1991, Sørensen et al. 1996, Theliander and Fathi-Najafi 1996, Tien and Bai 2003, Johansson and Theliander 2007
		$k = k_0 \left(\frac{e}{e_0} \right)^{\beta}$	Ren and Santamarina 2017
	Polynomial	$k = k_0 e^3 (1 + e)^{-2}$	Smiles 1970

Table 2. Model parameters

Permeability $k = k_0 \left(\frac{e}{e_0} \right)^\beta$	k_0	5.6×10^{-5} mD
	e_0	1
	b	3
Compressibility $e = e_c - C_c \log \left(\frac{1kPa}{\sigma'_1 + \sigma'_L} + \frac{1kPa}{\sigma'_H} \right)$ $\sigma'_H = 10^{(e_c - e_H)/C_c} kPa$ $\sigma'_L = \frac{\sigma'_H}{10^{(e_L - e_H)/C_c} - 1}$	e_c	23.5
	e_L	36
	e_H	0.6
	C_c	9.5

Figure 1. (a) Schematic representation of the piston model. (b) Infinitesimal element in the Lagrangian system.

Figure 2. Constitutive equations of sodium bentonite. (a) Effective stress vs. void ratio. (b) Permeability vs. void ratio. (Data from: Sharma and Zongming 1991, Sherwood et al. 1991, Sherwood and Meeten 1997, Sivapullaiah, Sridharan et al. 2000)

Figure 3. Impact of environmental factors on the bentonite fabric: dispersed and aggregated clay platelets.

Figure 4. Environmental factor: ionic concentration. (a) The influence of the salt concentration on the liquid limit of sodium bentonite. Data from: Schmitz et al., 2003. (b) Odometer curves for bentonite reconstituted with and immersed in NaCl solutions at various concentrations. (Data from: Di Maio et al., 2004)

Figure 5. Viscosity and filtration loss properties at 4% bentonite as a function of the amount of exchangeable calcium. Filtration loss in API tests. (Data from: Williams et al. 1953).

Figure 6. Cake formation: the influences of filtration time. Void ratio profiles at 50s, 200s, 800s and 3,200s for a 2MPa filtration pressure. Constitutive parameters are in Table 2.

Figure 7. Cake formation: the effect of filtration pressure. Void ratio contours versus wellbore depth presented in terms of filtration pressure. Filtration time=2.8hr at all depths

Figure 8. Cake formation: the effect of time. (a) Case: from 400m to 600m depth and rate of penetration ROP=20m/hr. (b) Increasing mudcake thickness ($e=10$) with increasing still-time from 0hr to 24hr.

Figure 9. The influence of the liquid viscosity on the filtration behavior. (a) Void ratio profile. (b) Cumulative filtration volume. Fluid viscosities between 0.3cp and 100cp, filtration pressure=1MPa and filtration time=1hr.

Figure 10. The influence of permeability on the filtration response. (a) Void ratio distribution. (b) Cumulative filtration volume. Reference permeability $k_0 = 1 \times 10^{-5}$ mD, 5×10^{-5} mD and 1×10^{-4} mD, filtration pressure = 1 MPa and filtration time = 1 hr.

Figure 11. Thixotropic behavior of bentonite suspension with void ratio $e=$ 19, 23, 32 and 49. Solid lines are the model prediction (Equation 12) and symbols are measured values. (Data from: El Mohtar and Yoon 201

Figure 12. Yield Strength. Effective stress dependency in the cake, and thixotropic time-dependency in the mud.

Figure 13. Residual cake thickness after cement invasion. Black lines: shear stress imposed by the invading cement. Blue line: cake-mud shear strength (refer to Figure 12).

Figure 14. “Stuck pipe” by differential pressure sticking. (a) Fluid pressure against the casing for a 1MPa filtration pressure. (b) The effect of the still-time t on the total force against the pipe T .

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Highlights

- The Lagrangian mudcake growth model in this study involves wide stress-range constitutive models for compressibility and permeability, avoid non-natural assumptions in moving boundary type solutions and readily accommodates environmental effects.
- Mud cake thickness is more sensitive to time than to filtration pressure, therefore, time controls the non-uniform distribution of mudcake thickness during drilling.
- Long filtration time, high permeability, high salinity, high in-situ temperature and low viscosity exacerbate fluid loss and give rise to thick filter cakes.
- The mud yield stress is both effective stress and time-dependent. Thixotropy dominates at high void ratio, e.g. $e > 20$.
- The offsetting force that causes differential pressure sticking increases sub-linearly as a power function of the still-time.