Waveform inversion in acoustic orthorhombic media with a practical set of parameters

Nabil Masmoudi* and Tariq Alkhalifah, King Abdullah University of Science and Technology

SUMMARY

Full-waveform inversion (FWI) in anisotropic media is overall challenging, mainly because of the large computational cost, especially in 3D, and the potential trade-offs between the model parameters needed to describe such a media. We propose an efficient 3D FWI implementation for orthorhombic anisotropy under the acoustic assumption. Our modeling is based on solving the pseudo-differential orthorhombic wave equation split into a differential operator and a scalar one. The modeling is computationally efficient and free of shear wave artifacts. Using the adjoint state method, we derive the gradients with respect to a practical set of parameters describing the acoustic orthorhombic model, made of one velocity and five dimensionless parameters. This parameterization allows us to use a multi-stage model inversion strategy based on the continuity of the scattering potential of the parameters as we go from higher symmetry anisotropy to lower ones. We apply the proposed approach on a modified SEG-EAGE overthrust synthetic model. The quality of the inverted model suggest that we may recover only 4 parameters, with different resolution scales depending on the scattering potential of these parameters.

INTRODUCTION

The objective of full-waveform inversion (FWI) is to recover a high-resolution model that is capable of matching the observed seismic data, trace by trace, through repetitive modeling and a local optimization technique (Lailly, 1983; Tarantola, 1984). Over the last three decades, most FWI methods were designed to recover only P-wave velocity because of the high computational cost. The recent progress in high-performance computing and the improvement in data acquisition resulted in the extension of FWI to 2D and 3D acoustic and elastic vertical transverse isotropic (VTI) media, e.g. (Operto et al., 2014; Wu and Alkhalifah, 2016). This extension has also resulted in challenges related to the trade-off between the model parameters in our inversion process.

The orthorhombic model is usually regarded as the most practical realistic approximation of the subsurface, as it combines both the anisotropy admitted by the natural, mostly horizontal layering of the Earth (due to gravity), as well as the vertical aligned fractures, usually found in fractured reservoirs (Schoenberg and Helbig, 1997; Tsvankin, 1997; Bakulin et al., 2000). For seismic data exhibiting azimuthal anisotropy, we expect FWI to perform better with an orthorhombic model representation. Recently, some approaches of waveform inversion for elastic and acoustic orthorhombic models have been proposed (Albertin et al., 2016; Oh and Alkhalifah, 2016; Wang and Tsvankin, 2016).

Here, we focus on acoustic orthorhombic models, described by six parameters (Tsvankin, 1997; Alkhalifah, 2003). These

parameters include the P-wave vertical velocity V_{P0} , the parameters ϵ_1 and δ_1 defined in the (x,z) vertical plane, ϵ_2 and δ_2 defined in the (y,z) vertical plane, and δ_3 defined in the (x,y) horizontal plane. The trade-off between the model parameters and their resolution limits, due mainly to the limited data coverage and the different influence that parameters exert on seismic data, have motivated several studies investigating optimal parameterization set for our inversion schemes (Operto et al., 2013; Gholami et al., 2013; Alkhalifah and Plessix, 2014; Alkhalifah, 2016). Most of these studies rely on the analysis of the radiation (scattering) patterns based on the Born approximation (Wu and Aki, 1985; Panning et al., 2009). In this study, we use the parameter set of Masmoudi and Alkhalifah (2016a), built around a central parameter: the horizontal velocity v_{h1} in the (x,z) plane, two VTI parameters ϵ_1 and η_1 in the (x,z) plane, two deviations parameters $\epsilon_d = (\epsilon_2 - \epsilon_1)/(1 + 2\epsilon_1)$ and $\eta_d = (\eta_2 - \eta_1)/(1 + 2\eta_1)$, and finally the parameter δ_3 . Here, η_1 and η_2 are the anellipticity parameters defined respectively in the (x,z) and (y,z) planes. The main features of this parameterization are discussed in Alkhalifah et al. (2016).

The efficiency of wave simulators is a vital ingredient in FWI. Several methods have been proposed to model P-wave propagation in orthorhombic media, among them solving coupled-systems of equations (Fowler and King, 2011; Cheng and Kang, 2014) and the mixed domain wavefield extrapolators (Fowler and Lapilli, 2012; Fomel et al., 2013). One common issue in the coupled system approaches is the existence of shear wave artifacts in the simulated wavefield. Alkhalifah (2000) propose a pseudo-acoustic wave equation which provides accurate simulation of the kinematics of P-wave propagation. Xu and Zhou (2014) propose to solve this pseudo-acoustic wave equation by decomposing the pseudo-differential operator into a scalar and a differential operator. It has been shown that such an approach can yield accurate simulation of P-wave kinematics and a wavefield free of shear wave artifacts. This new approach has been successfully applied to VTI migration (Mu et al., 2015) and inversion (Le et al., 2015).

Here, we first apply Xu and Zhou (2014) decomposition method to Alkhalifah (2003) orthorhombic wave equation. Then, we derive the gradients with respect to the model parameters using the adjoint state method. Finally, we test our approach in inverting for the parameters of a modified orthorhombic SEG-EAGE overthrust model, and analyze the resolution of the inverted parameters.

FORWARD MODELING

The acoustic wave equation (Alkhalifah, 2000) for anisotropic media satisfies a linear pseudo-differential equation, which we can write in the following form:

$$\left[\omega^2 - v_{h1}^2 \phi(\mathbf{x}, \mathbf{k}) \right] u(\mathbf{k}, \omega) = 0, \quad (1)$$

Waveform inversion in acoustic orthorhombic media

where $u(\mathbf{k}, \omega)$ is the pressure wavefield, ω is the angular frequency, $\mathbf{k}$ is the magnitude of the wave vector, v_{h1} is the horizontal velocity in the $[x, z]$ plane and ϕ is the pseudo-differential phase operator. In orthorhombic media, the phase operator can be obtained from the dispersion relation using the Christoffel equation (Alkhalifah, 2003). This yields a cubic polynomial, with one of its roots (specifically, the biggest root) corresponding to P-waves. The general form of the polynomial is given below (Song and Alkhalifah, 2013) with the parameterization of Masmoudi and Alkhalifah (2016a):

$$-\phi^3 + a\phi^2 + b\phi + c = 0, \quad (2)$$

where

$$\begin{aligned} a &= k_x^2 + (1 + 2\varepsilon_d)k_y^2 + \frac{1}{1 + 2\varepsilon_1}k_z^2, \\ b &= ((1 + 2\delta_3) - (1 + 2\varepsilon_d))k_x^2k_y^2 - \frac{2\eta_1}{(1 + 2\eta_1)(1 + 2\varepsilon_1)}k_x^2k_z^2 \\ &\quad - \frac{(1 + 2\varepsilon_d)((1 + 2\eta_d)(1 + 2\eta_1) - 1)}{(1 + 2\eta_d)(1 + 2\eta_1)(1 + 2\varepsilon_1)}k_y^2k_z^2, \\ c &= -\frac{k_x^2k_y^2k_z^2}{(1 + 2\varepsilon_1)(1 + 2\eta_1)} \left((1 + 2\eta_1)(1 + 2\delta_3) - \right. \\ &\quad \left. 2\sqrt{\frac{(1 + 2\varepsilon_d)(1 + 2\delta_3)}{1 + 2\eta_d}} + (1 + 2\varepsilon_d) \left(-2\eta_1 + \frac{1}{1 + 2\eta_d} \right) \right). \end{aligned} \quad (3)$$

To solve equation 1 in the time-space domain, we propose similarly to Xu and Zhou (2014) to decompose the pseudo-differential operator into a Laplacian operator ∇_E^2 and a scalar operator $S(\mathbf{x})$ as follows:

$$\partial_{tt}u(\mathbf{x}, t) - v_{h1}^2 S(\mathbf{x}) \nabla_E^2 u(\mathbf{x}, t) = 0, \quad (4)$$

where

$$\begin{cases} \nabla_E^2 u(\mathbf{x}, t) = \partial_{xx}u + (1 + 2\varepsilon_d)\partial_{yy}u + \frac{1}{1 + 2\varepsilon_1}\partial_{zz}u, \\ S(\mathbf{x}) = \frac{\phi(\mathbf{x}, \mathbf{n})}{n_x^2 + (1 + 2\varepsilon_d)n_y^2 + \frac{1}{1 + 2\varepsilon_1}n_z^2}, \end{cases} \quad (5)$$

where ∂_{tt} is the second-order time derivative, ∂_{xx} , ∂_{yy} and ∂_{zz} are the second-order space derivatives, n_x , n_y and n_z are the components of the unit vector $\mathbf{n}$ of phase direction $\mathbf{n} = \mathbf{k}/|\mathbf{k}|$. To compute the scalar operator in the space domain, we approximate the phase direction $\mathbf{n}$ from the components of $\nabla u/|\nabla u|$ (Xu and Zhou, 2014). In our decomposition, the ellipticity of the wavefield is taken into account in the Laplacian operator, while the anellipticity is handled in the scalar operator. This elliptic decomposition yields better handling of amplitudes and a more stable solution (Xu et al., 2015).

We compare the accuracy of the wave equation 4 with the low-rank approximation (Song and Alkhalifah, 2013), as well as with the traveltimes from the eikonal solution (Waheed et al., 2014). Figures 1(a) and 1(b) show wavefield snapshots in an orthorhombic medium with constant parameters ($V_{P0} = 2000$ m/s, $\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.4$, $\delta_1 = 0.1$, $\delta_2 = 0.15$, $\delta_3 = 0.05$), which correspond to ($v_{h1} \approx 2366$, $\varepsilon_1 = 0.2$, $\varepsilon_d \approx 0.143$, $\eta_1 \approx 0.0833$, $\eta_d \approx 0.093$, $\delta_3 = 0.05$) in the parameterization considered here. We note that the new wavefield solution matches the kinematics of the low-rank solution, as confirmed by comparison with the traveltimes from the eikonal equation. Obviously, the new

wavefield solution is free of shear wave artifacts. Figure 1(c) shows the corresponding S operator. One can notice that S corrects for the anellipticity of the wavefield, since its maximum influence resides at 45° propagation direction and is equal to 1 at the principal axes of symmetry.

Figure 1: Wavefield snapshots computed with the low-rank method (a), the new modeling (equations 4 and 5) (b) and the corresponding S operator (c). The red dashed contours represent the eikonal solution.

COMPUTING THE GRADIENTS

The objective of FWI is to minimize a certain misfit function. We consider here the classic L2-norm data difference defined as:

$$\chi = \frac{1}{2} \sum_r \int_0^T \|u(\mathbf{x}_r, t) - d(\mathbf{x}_r, t)\|^2 dt, \quad (6)$$

where $\mathbf{x}_r$ are the receivers' locations, $d(\mathbf{x}_r, t)$ is the observed data and T is total time. We derive the adjoint wavefield and the gradients of the misfit function 6 using the adjoint state method (Liu and Tromp, 2006; Plessix, 2006). In this case, the adjoint wavefield λ satisfies the following wave equation:

$$\begin{aligned} \frac{1}{v_{h1}^2} \partial_{tt}\lambda(\mathbf{x}, t) - S(\mathbf{x}) \nabla_E^2 \lambda(\mathbf{x}, t) - \nabla_E^2 u(\mathbf{x}, t) \nabla \lambda(\mathbf{x}, t) \left(\frac{\partial S}{\partial \nabla u} \right)^T = \\ \sum_r [u(\mathbf{x}, T - t) - d(\mathbf{x}_r, T - t)] \delta(\mathbf{x} - \mathbf{x}_r), \end{aligned} \quad (7)$$

with the boundary conditions:

$$\lambda(\mathbf{x}, T) = 0, \quad \partial_t \lambda(\mathbf{x}, T) = 0, \quad \lambda|_{\partial\Omega} = 0, \quad (8)$$

where $\delta(\mathbf{x} - \mathbf{x}_r)$ is the Dirac delta function. The gradients with respect to the model parameters v_{h1} , ε_1 , ε_d , η_1 , η_d and δ_3 are:

Waveform inversion in acoustic orthorhombic media

$$\begin{aligned}\frac{\partial \chi}{\partial v_{h1}} &= \frac{2}{v_{h1}^3} \int_0^T \lambda(\mathbf{x}, T-t) \left[\frac{\partial^2 u}{\partial t^2} \right] dt, \\ \frac{\partial \chi}{\partial \varepsilon_1} &= -2 \int_0^T \lambda(\mathbf{x}, T-t) \left[S \frac{\partial^2 u}{\partial z^2} + \frac{\partial S}{\partial \varepsilon_1} \nabla_E^2 u \right] dt, \\ \frac{\partial \chi}{\partial \varepsilon_d} &= 2 \int_0^T \lambda(\mathbf{x}, T-t) \left[S \frac{\partial^2 u}{\partial y^2} + \frac{\partial S}{\partial \varepsilon_d} \nabla_E^2 u \right] dt, \\ \frac{\partial \chi}{\partial \eta_1} &= \int_0^T \lambda(\mathbf{x}, T-t) \left[\frac{\partial S}{\partial \eta_1} \nabla_E^2 u \right] dt, \\ \frac{\partial \chi}{\partial \eta_d} &= \int_0^T \lambda(\mathbf{x}, T-t) \left[\frac{\partial S}{\partial \eta_d} \nabla_E^2 u \right] dt, \\ \frac{\partial \chi}{\partial \delta_3} &= \int_0^T \lambda(\mathbf{x}, T-t) \left[\frac{\partial S}{\partial \delta_3} \nabla_E^2 u \right] dt.\end{aligned}\quad (9)$$

The partial derivatives of S involved in equations 7 and 9 are not easy to obtain from the cubic polynomial 2. Therefore, we approximate its solution using Taylor expansion. Specifically, by assuming the anisotropy parameters are relatively small, we expand the solution of ϕ in terms of ε_1 , ε_d , η_1 , η_d and δ_3 as follows:

$$\phi \approx \phi_0 + \phi_{\varepsilon_d} \varepsilon_d + \phi_{\varepsilon_1} \varepsilon_1 + \phi_{\eta_1} \eta_1 + \phi_{\eta_d} \eta_d + \phi_{\delta_3} \delta_3, \quad (10)$$

where ϕ_{ε_1} , ϕ_{ε_d} , ϕ_{η_1} , ϕ_{η_d} and ϕ_{δ_3} are the coefficients in this expansion. By replacing equation 10 into polynomial 2, we solve for the coefficients of the trial solutions corresponding to the P-wave phase velocity, and obtain an approximation of ϕ :

$$\begin{aligned}\phi \approx & 1 + 2k_y^2(k_y^2 + k_z^2)\varepsilon_d - 2k_z^2\varepsilon_1 - 2k_z^2(k_x^2 + k_y^2)\eta_1 \\ & + 2k_y^2k_z^2\eta_d + 2k_x^2k_y^2\delta_3.\end{aligned}\quad (11)$$

We use a preconditioned non-linear conjugate gradient method to update the model. The preconditioner is the diagonal of the approximate Hessian $\mathbf{H}_a$ (Shin et al., 2001), referred as the pseudo-Hessian:

$$p_i = \frac{1}{\text{diag}(\mathbf{H}_a)} g_i, \quad (12)$$

where g_i and p_i are respectively the conjugate and the preconditioned conjugate gradients corresponding to the i -th parameter. We estimate the steplengths using a second-order approximation of the objective function (Hu et al., 2011). In the multi-parameter case, this requires solving a linear system of equations of the form:

$$\left[(F p_i)^T (F p_j) \right] \alpha_j = -g_i^T p_i, \quad (13)$$

where F is the Frechet derivative and α represents the vector of steplengths. The effect of the Frechet derivative on the gradients is computed using a first-order finite difference approximation (Pica et al., 1990), and requires one extra modeling for each parameter.

NUMERICAL EXAMPLES

We test our inversion algorithm on a modified version of an SEG-EAGE orthorhombic overthrust model (see Figure 2). The

Figure 2: Orthorhombic model parameters obtained from the SEG-EAGE overthrust model.

inversion domain is 8km by 6km laterally and 4km in depth. The domain is discretized using a 25m grid spacing in all three dimensions. We employ 60 sources distributed on the surface of the inversion domain in three lines at $y=\{1.5\text{km}, 3\text{km}, 4.5\text{km}\}$. We record the data on the surface from receivers placed on all nodal points of the inversion domain. We invert data filtered between 2Hz to 8Hz using the starting model parameters shown in Figure 3.

We use a multi-stage model inversion strategy: we update first the *isotropic* model given by v_{h1} (20 iterations), then the *VTI* model by including η_1 and ε_1 (5 iterations), and finally the *orthorhombic* model by including ε_d , η_d and δ_3 (5 iterations). This inversion strategy takes advantage of the practical parameterization that allows for a continuity in the scattering potential of the model parameters as we move from higher symmetry anisotropy to lower ones (Masmoudi and Alkhalifah, 2016b). Figure 4 shows the inverted model. Vertical profiles located at $(x=4\text{km}, y=3\text{km})$ comparing initial, true and inverted v_{h1} , ε_1 , ε_d and δ_3 are shown in Figure 5. Our inversion strategy leads to a very well recovered horizontal velocity. ε_1 which affects the data mainly at narrow opening angles is generally well recovered here, as well. In general, ε_1 can help in fitting the data amplitudes at near offsets. Therefore, when real data is inverted, ε_1 should help in fitting the mismatch between amplitudes, due to an acoustic approximation. Moreover, ε_d and δ_3 , which affect large offsets data in the cross line and in the 45° -source-to-receiver-azimuth directions, are only recovered at the shallow part (up to 1km depth). This is due to the limited offset range (used here) where these parameters exert the most influence. Finally, η_1 and η_d are not recovered, as predicted, by some previous studies pin-pointing the weak influence of η

Waveform inversion in acoustic orthorhombic media

on surface seismic data as a scatterer in this parameterization (Alkhalifah, 2016; Alkhalifah and Guitton, 2016). However, η is still important in defining the wave propagation and should be extracted from migration velocity analysis (MVA) methods.

Figure 3: Initial model parameters for the inversion.

Figure 4: Inverted model parameters.

Figure 5: Vertical profiles located at $(x=4\text{km}, y=3\text{km})$ comparing initial, true and inverted v_{h1} , ϵ_1 , ϵ_d and δ_3 .

CONCLUSION

We proposed a waveform inversion strategy to update the model parameters of an acoustic orthorhombic medium. Decomposing the pseudo-differential wave equation into two solvable terms yielded an efficient wavefield simulation. The choice of a practical parameterization allowed us to invert the main parameters the data are sensitive to, specifically 4 parameters: v_{h1} , ϵ_1 , ϵ_d and δ_3 . The resolution of the inverted parameters is consistent with their scattering potential given by their radiation patterns. The model update was based on preconditioned non-linear conjugate gradient and steplengths computed from a second-order approximation of the objective function. Additional gradient preconditioning such as the truncated newton might help improve the quality of the inverted model.

ACKNOWLEDGMENTS

We would like to thank KAUST for financial support and SWAG members for many useful discussions. For computer time, this research used the resources of the Supercomputing Laboratory in KAUST.

EDITED REFERENCES

Note: This reference list is a copyedited version of the reference list submitted by the author. Reference lists for the 2017 SEG Technical Program Expanded Abstracts have been copyedited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

REFERENCES

- Albertin, U., P. Shen, A. Sekar, T. Johnsen, C. Wu, K. Nihei, and K. Bube, 2016, 3D orthorhombic elastic full-waveform inversion in the reflection domain from hydrophone data: 86th Annual International Meeting, SEG, Expanded Abstracts, 1094–1098, <http://doi.org/10.1190/segam2016-13866375.1>.
- Alkhalifah, T., 2000, An acoustic wave equation for anisotropic media: *Geophysics*, **65**, 1239–1250, <http://doi.org/10.1190/1.1444815>.
- Alkhalifah, T., 2003, An acoustic wave equation for orthorhombic anisotropy: *Geophysics*, **68**, 1169–1172, <http://doi.org/10.1190/1.1598109>.
- Alkhalifah, T., 2016, Research note: Insights into the data dependency on anisotropy: an inversion prospective: *Geophysical Prospecting*, **64**, 505–513, <http://doi.org/10.1111/gpr.2016.64.issue-2>.
- Alkhalifah, T., and A. Guitton, 2016, An optimal parameterization for full waveform inversion in anisotropic media: 78th Annual International Conference and Exhibition, EAGE, Extended Abstracts, <http://doi.org/10.3997/2214-4609.201601192>.
- Alkhalifah, T., N. Masmoudi, and J.-W. Oh, 2016, A recipe for practical full-waveform inversion in orthorhombic anisotropy: *The Leading Edge*, **35**, 1076–1083, <http://doi.org/10.1190/tle35121076.1>.
- Alkhalifah, T., and R. E. Plessix, 2014, A recipe for practical full-waveform inversion in anisotropic media: An analytical parameter resolution study: *Geophysics*, **79**, no. 3, R91–R101, <http://doi.org/10.1190/geo2013-0366.1>.
- Bakulin, A., V. Grechka, and I. Tsvankin, 2000, Estimation of fracture parameters from reflection seismic data-Part II: Fractured models with orthorhombic symmetry: *Geophysics*, **65**, 1803, <http://doi.org/10.1190/1.1444864>.
- Cheng, J., and W. Kang, 2014, Simulating propagation of separated wave modes in general anisotropic media, Part I: qP-wave propagators: *Geophysics*, **79**, no. 1, C1–C18, <http://doi.org/10.1190/geo2012-0504.1>.
- Fomel, S., L. Ying, and X. Song, 2013, Seismic wave extrapolation using lowrank symbol approximation: *Geophysical Prospecting*, **61**, 526–536, <http://doi.org/10.1111/gpr.2013.61.issue-3>.
- Fowler, P. J., and R. King, 2011, Modeling and reverse time migration of orthorhombic pseudoacoustic P waves: 81st Annual International Meeting, SEG, Expanded Abstracts, 190–195, <http://doi.org/10.1190/1.3627580>.
- Fowler, P. J., and C. Lapilli, 2012, Generalized pseudospectral methods for orthorhombic modeling and reverse-time migration: 82nd Annual International Meeting, SEG, Expanded Abstracts, 1–5, <http://doi.org/10.1190/segam2012-0708.1>.
- Gholami, Y., R. Brossier, S. Operto, A. Ribodetti, and J. Virieux, 2013, Which parameterization is suitable for acoustic vertical transverse isotropic media full waveform inversion? Part 1: Sensitivity and trade-off analysis: *Geophysics*, **78**, no. 2, R81–R105, <http://doi.org/10.1190/geo2012-0204.1>.
- Hu, W., A. Abubakar, T. Habashy, and J. Liu, 2011, Preconditioned non-linear conjugate gradient method for frequency domain full-waveform seismic inversion: *Geophysical Prospecting*, **59**, 477–491, <http://doi.org/10.1111/gpr.2011.59.issue-3>.

- Lailly, P., 1983, The seismic inverse problem as a sequence of before stack migrations: Conference on Inverse Scattering, Theory and Application, Society of Industrial and Applied Mathematics, Expanded Abstracts, 206–220.
- Le, H., B. Biondi, R. G. Clapp, and S. A. Levin, 2015, Using a nonlinear acoustic wave equation for anisotropic inversion: 85th Annual International Meeting, SEG, Expanded Abstracts, 467–471, <http://doi.org/10.1190/segam2015-5868499.1>.
- Liu, Q., and J. Tromp, 2006, Finite-frequency kernels based on adjoint methods: Bulletin of the Seismological Society of America, **96**, 2383–2397, <http://doi.org/10.1785/0120060041>.
- Masmoudi, N., and T. Alkhalifah, 2016a, A new parameterization for waveform inversion in acoustic orthorhombic media: Geophysics, **81**, no. 4, R157–R171, <http://doi.org/10.1190/geo2015-0635.1>.
- Masmoudi, N., and T. Alkhalifah, 2016b, Scattering potential of acoustic orthorhombic parametrization - An inversion perspective: 78th Annual International Conference and Exhibition, EAGE, Extended Abstracts, <http://doi.org/10.3997/2214-4609.201601010>.
- Mu, J., B. Tang, S. Xu, H. Zhou, and A. DeNosaquo, 2015, Quasi-p wave reverse time migration on seam dataset: 85th Annual International Meeting, SEG, Expanded Abstracts, 3991–3995, <http://doi.org/10.1190/segam2015-5870250.1>.
- Oh, J.-W., and T. Alkhalifah, 2016, 3D elastic-orthorhombic anisotropic full-waveform inversion: Application to field OBC data: 86th Annual International Meeting, SEG, Expanded Abstracts, 1206–1210, <http://doi.org/10.1190/segam2016-13862511.1>.
- Operto, S., R. Brossier, L. Combe, L. Mtivier, A. Ribodetti, and J. Virieux, 2014, Computationally efficient three-dimensional acoustic finite-difference frequency-domain seismic modeling in vertical transversely isotropic media with sparse direct solver: Geophysics, **79**, no. 5, T257–T275, <http://doi.org/10.1190/geo2013-0478.1>.
- Operto, S., Y. Gholami, V. Prieux, A. Ribodetti, R. Brossier, L. Metivier, and J. Virieux, 2013, A guided tour of multiparameter full-waveform inversion with multicomponent data: From theory to practice: The Leading Edge, **32**, 1040–1054, <http://doi.org/10.1190/tle32091040.1>.
- Panning, M. P., Y. Capdeville, and B. A. Romanowicz, 2009, Seismic waveform modelling in a 3-D Earth using the Born approximation: Potential shortcomings and a remedy: Geophysical Journal International, **177**, 161–178, <http://doi.org/10.1111/gji.2009.177.issue-1>.
- Pica, A., J. P. Diet, and A. Tarantola, 1990, Nonlinear inversion of seismic reflection data in a laterally invariant medium: Geophysics, **55**, 284–292, <http://doi.org/10.1190/1.1442836>.
- Plessix, R.-E., 2006, A review of the adjoint-state method for computing the gradient of a functional with geophysical applications: Geophysical Journal International, **167**, 495–503, <http://doi.org/10.1111/gji.2006.167.issue-2>.
- Schoenberg, M., and K. Helbig, 1997, Orthorhombic media: Modeling elastic wave behavior in a vertically fractured earth: Geophysics, **62**, 1954–1974, <http://doi.org/10.1190/1.1444297>.
- Shin, C., S. Jang, and D. J. Min, 2001, Improved amplitude preservation for prestack depth migration by inverse scattering theory: Geophysical Prospecting, **49**, 592–606, <http://doi.org/10.1046/j.1365-2478.2001.00279.x>.
- Song, X., and T. Alkhalifah, 2013, Modeling of pseudoacoustic p-waves in orthorhombic media with a low-rank approximation: Geophysics, **78**, no. 4, C33–C40, <http://doi.org/10.1190/geo2012-0144.1>.
- Tarantola, A., 1984, Inversion of seismic reflection data in the acoustic approximation: Geophysics, **49**, 1259–1266, <http://doi.org/10.1190/1.1441754>.
- Tsvankin, I., 1997, Anisotropic parameters and P-wave velocity for orthorhombic media: Geophysics, **62**, 1292–1309, <http://doi.org/10.1190/1.1444231>.

- Waheed, U., C. E. Yarman, and G. Flagg, 2014, An iterative fast sweeping based eikonal solver for tilted orthorhombic media: 84th Annual International Meeting, SEG, Expanded Abstracts, 480–485, <http://doi.org/10.1190/segam2014-0846.1>.
- Wang, H., and I. Tsvankin, 2016, Feasibility of waveform inversion in acoustic orthorhombic media: 86th Annual International Meeting, SEG, Expanded Abstracts, 311–316, <http://doi.org/10.1190/segam2016-13965913.1>.
- Wu, R., and K. Aki, 1985, Scattering characteristics of elastic waves by an elastic heterogeneity: *Geophysics*, **50**, 582–595, <http://doi.org/10.1190/1.1441934>.
- Wu, Z., and T. Alkhalifah, 2016, Waveform inversion for acoustic VTI media in frequency domain: 86th Annual International Meeting, SEG, Expanded Abstracts, 1184–1189, <http://doi.org/10.1190/segam2016-13867221.1>.
- Xu, S., B. Tang, J. Mu, and H. Zhou, 2015, Elliptic decomposition of quasi-P wave equation: 77th Annual International Conference and Exhibition, EAGE, Extended Abstracts, <http://doi.org/10.3997/2214-4609.201413134>.
- Xu, S., and H. Zhou, 2014, Accurate simulations of pure quasip-waves in complex anisotropic media: *Geophysics*, **79**, no. 6, T341–T348, <http://doi.org/10.1190/geo2014-0242.1>.