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**Time-domain full waveform inversion of exponentially
damped wavefield using the deconvolution-based objective
function**

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Right Running Head: FWI of exponentially damped wavefield

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ABSTRACT

Full waveform inversion (FWI) suffers from the cycle-skipping problem when the available frequency-band of data is not low enough. We apply an exponential damping to the data to generate artificial low frequencies, which helps FWI avoid cycle skipping. In this case, the least-square misfit function does not properly deal with the exponentially damped wavefield in FWI, because the amplitude of traces decays almost exponentially with increasing offset in a damped wavefield. Thus, we use a deconvolution-based objective function for FWI of the exponentially damped wavefield. The deconvolution filter includes inherently a normalization between the modeled and observed data, thus it can address the unbalanced amplitude of a damped wavefield. We, specifically, normalize the modeled data with the observed data in the frequency-domain to estimate the deconvolution filter and selectively choose a frequency-band for normalization that mainly includes the artificial low frequencies. We calculate the gradient of the objective function using the adjoint-state method. The synthetic and benchmark data examples show that our FWI algorithm generates a convergent long wavelength structure without low frequency information in the recorded data.

INTRODUCTION

Full waveform inversion (FWI) is developed to obtain an accurate and detailed subsurface model. FWI, however, still suffers from the cycle-skipping problem, which usually leads FWI to local minima solutions. The multi-scale approach (Bunks et al., 1995) is utilized to mitigate the cycle-skipping problem, where FWI starts from a low enough frequency and gradually moves to higher frequency. Low enough frequency has a sufficiently long cycle, thus it can prevent FWI from cycle skipping even for a poor initial velocity model. In seismic data, however, the available frequencies are usually not low enough, because of the physical limitation of our acquisition instruments, environmental noise, and other related phenomena. A starting model close enough to the true model can also help FWI avoid the cycle-skipping problem, but obtaining a good starting model is not a trivial task and requires elaborate techniques such as the travel-time tomography, migration velocity analysis, and etc.

A lot of research in FWI has been devoted to solving the cycle-skipping problem in the case of the lack in low frequency information. Choi and Alkhalifah (2013, 2015) implicitly and explicitly unwrapped the phase in the frequency domain and inverted the unwrapped phases to avoid the cycle-skipping problem. Warner and Guasch (2014, 2016) calculated the deconvolution filter (or matching filter) between the modeled and observed data in the time-domain, then they applied an optimization based on forcing the deconvolution filter to reduce to the Dirac delta function. Wu et al. (2014) estimated an envelope function of the seismic trace along the time-axis and inverted it. The envelope function includes artificial low frequencies, which helps FWI admit long wavelength structure in the update without real low frequency information. On the other hand, Xu et al. (2012) and Wu and Alkhalifah (2015) simulated

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3 reflected waves from the migrated images and inverted the reflected waves to obtain a long
4 wavelength gradient along wave paths without migration isochrones. There are also many other
5 FWI methods developed to mitigate the cycle-skipping problem for the case of a lack in low
6 frequency information in the data.
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9 In this paper, we propose a time-domain FWI with an exponential damping applied to the data
10 along the time-axis. In this case, we mute energy (possibly noise) prior to the first arrival before
11 applying the exponential damping. Such damping amplifies the first arrival event and attenuates
12 the later arrivals. Some strong to moderate exponential damping isolates near the first arrival
13 energy and such energy yields artificial low frequencies. Since the damping process along the
14 time-axis is expressed as a convolution in the frequency domain, it theoretically generates
15 artificial low frequencies. The artificial low frequencies resulting from applying an exponential
16 damping are kinematically controlled by the early arrivals, thus their phase information closely
17 represents the kinematics of the early arrivals. We invert these artificial low frequency
18 components to help FWI avoid the cycle-skipping problem especially in the case of lack of real
19 low frequencies.
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Exponential damping has been applied in frequency-domain FWI (Min and Shin, 2006;
Brossier et al., 2009; Shin and Cha, 2009; Choi and Alkhalifah, 2015), since it is easy to
incorporate it in frequency-domain modeling by considering complex angular frequencies.
However, frequency-domain modeling with a complex angular frequency provides only the
damped Green function, whereas the application of an exponential damping to seismic data along
the time-axis attenuates the convolution of a Green function and source wavelet. The damped
Green function and source wavelet do not reproduce the damped convolution of a Green function
and source wavelet. We explain this in detail in Appendix A. Therefore, if the exponential

damping is moderate or weak, the frequency-domain FWI cannot theoretically deal with the exponentially damped wavefield. In the case of a strong exponential damping, the phase of the damped wavefield becomes close to that of the damped Green function, thus the phase of the damped wavefield can be inverted through frequency-domain FWI with a strong damping. In this study, on the other hand, we apply an exponential damping to both the modeled and observed data along the time-axis, thus we don't encounter the above issue.

We apply a stationary exponential damping to data, in which the damping starts from zero time regardless of source-receiver offset. If we use a nonstationary exponential damping, the adjoint-state method for FWI is difficult to implement. Applying a stationary exponential damping, the energy (or amplitude) of trace is almost exponentially decreasing as the offset increases, thus in a shot-gather, most of the energy is focused at the very near offset and early time. In this case, the least-square misfit function for FWI does not work because of the unbalanced energy of traces as a function of offset. In the Laplace- or frequency-domain FWI, the logarithmic or phase-only objective function has been utilized to handle the exponentially damped wavefield, but utilizing these objective function in time-domain FWI is not straightforward. In this study, we use a deconvolution-based objective function to properly deal with the exponentially damped wavefield in time-domain FWI. Since the deconvolution filter includes the normalization process between the modeled and observed data, it can properly address the unbalanced energy of traces in the damped wavefields.

Luo and Sava (2011) proposed a deconvolution-based objective function for the frequency-domain FWI. They calculated the deconvolution between the modeled and observed data at each frequency with an additional axis of time-lag (or phase). They attenuated the deconvolution value at the non-zero time-lag to make the modeled data converge to the true data. Warner and

Guasch (2014, 2016) calculated the deconvolution filter in the time-domain with a supplementary optimization and pushed its values at the non-zero time-lag to zero. In both cases, the normalization included in the deconvolution filter boosts up weak amplitudes of low-frequency components as long as it exists, which makes FWI less sensitive to the cycle-skipping problem. However, both FWI methods still require low enough frequency information for a successful inversion. In this paper, we focus on the potential of the normalization included in the deconvolution filter to balance the energy of traces in the damped wavefield. Since an exponential damping generates artificial low frequencies, the deconvolution-based objective function incorporating the damping can mitigate the cycle-skipping problem better than the pure deconvolution-based objective function, especially in the case of lack of low frequency information.

We use Fourier transform to estimate the time-domain deconvolution filter. We first calculate the Fourier transform of the modeled and observed data, normalize the transformed modeled data by the observed one, and then take the inverse Fourier transform of the normalized data. For the normalization, we selectively choose a frequency band that mainly includes artificial low frequencies. We apply a weighting function to the deconvolution filter and construct the objective function as an L2 norm of the weighted deconvolution filter. We calculate the gradient of the objective function using the adjoint-state technique, which is similar to that of Warner and Guasch (2014, 2016).

In the numerical examples, we apply our FWI algorithm to the Marmousi synthetic data and the SEG2014 benchmark dataset. In the Marmousi synthetic data example, we filtered out low frequencies of data below 5 Hz. The SEG2014 bench mark dataset lacks low frequency

components below 3.5 Hz. Examples demonstrate that our FWI algorithm generates a convergent long wavelength model without low frequency information in the recorded data.

THEORY

We first describe the features of the exponentially damped wavefield with frequency-domain amplitude and phase spectra, and then we introduce the objective function based on the deconvolution filter and its gradient expression.

Exponentially damped wavefield

Applying an exponential damping to seismic data along the time-axis is expressed as

$$\hat{d}(t) = d(t) \exp(-\alpha t), \quad (1)$$

where t is the time variable, $d(t)$ is the original seismic wavefield (or observed data), $\hat{d}(t)$ is the exponentially damped wavefield, and α is a damping factor. Applying an exponential damping in equation 1 is expressed as a convolution in the frequency-domain, thus it can theoretically generate artificial low frequencies depending on the frequency range of the seismic trace and the exponential function (Ha and Shin, 2012).

We plot 4 Hz cosine functions with various damping factors and their amplitude spectra in Figure 1. The original cosine function (without damping) has amplitude only at 4 Hz in the spectra, whereas the exponential damping applied to the cosine function generates the wide-band artificial frequencies. The higher the damping factor, the stronger the amplitude of the artificial frequencies (Figure 1b). We focus on the artificial low frequencies for FWI to construct a long wavelength structure.

We display a shot-gather seismogram and its damped versions with various damping factors in Figure 2. We filter out low frequencies below 5 Hz and mute the signals prior to the first arrival in the original seismogram (Figure 2a). In the damped seismograms in Figure 2 (left side of each plot), the energy (or amplitude) of traces are almost exponentially decreasing with increasing offset and most of the energy is focused at very near offset and early time. The higher the damping factor, the faster the energy of the trace decays as the offset distance increases. Therefore, if we apply an exponential damping, the least-square misfit function for FWI does not work because of the unbalanced energy of the traces.

We normalize each trace of seismograms by its maximum value to highlight the features of the damped seismograms (right side of each plot in Figure 2). The exponential damping relatively amplifies the early arrivals and attenuates later arrivals. The higher the damping factor, the stronger the amplification of the first arrivals. With an extremely strong damping, only the first arrival events remain (right side of Figure 2d). This amplification (or isolation) of the first arrival energy generates the artificial low frequency components, which actually have similar kinematic behavior to that of the first arrival event.

Figure 3 shows the amplitude spectra of the original and damped seismograms with various damping factors at different offset distances. Each amplitude spectrum is normalized by its maximum value for better comparison. Since we mute the signals prior to the first arrivals in the original seismogram in Figure 2a, its amplitude spectra still show some energy below 5 Hz (Figure 3a). We note that an exponential damping significantly generates the artificial low frequency components below 5 Hz (Figures 3b ~ 3d). As the damping factor is increased, the amplitude of the artificial low frequency gets larger. If the available minimum frequency of the data is low (e.g. 3 or 5 Hz), a weak damping (e.g. 4 s^{-1}) might be enough to generate the

necessary artificial low frequencies, whereas if the minimum frequency of the data is quite high (e.g. 15 or 20 Hz), we need a strong damping to obtain the desired low frequencies.

Figure 4 shows the phase spectra according to the tested damping factors. The phase spectra of the original filtered seismogram show severe distortions below 3 Hz (Figure 4a), whereas the exponential damping eliminates such distortions in the phase spectra (Figures 4b ~ 4d). The phase spectra with a damping factor of 32 s^{-1} (Figure 4d) represent the kinematics of the first arrival events, because the damped seismogram with such a damping factor (Figure 2d) includes almost only the first arrival events. The phase spectra with damping factors of 4 and 8 s^{-1} show similar patterns with that of a damping factor of 32 s^{-1} in the frequency range of $0 \sim 3 \text{ Hz}$, whereas they include non-linear patterns (residues) above 3 Hz, which is related to the later arrivals.

If the chosen damping factor is high (e.g. 8 or 32 s^{-1}) and the first arrival time is late, amplitude of all events in the damped wavefield could be smaller than the computing precision, and thus the normalization described in the next subsection cannot be applied. If we use a high level precision in computing the deconvolution filter for damping factors of 8 and 32 s^{-1} , the computational cost will be higher. A damping factor of 4 s^{-1} might be strong enough to generate the artificial low frequencies we need and low enough to avoid the issue of computing precision. We invert these artificial low frequency components through FWI to generate a long wavelength gradient.

Deconvolution-based objective function

The least-square misfit function does not work for the exponentially damped wavefield in FWI, since the energy (or amplitude) of the traces decreases almost exponentially with increasing offset as shown in Figure 2. As an alternative, we suggest using the deconvolution-based objective function for FWI of the exponentially damped wavefield. Since the deconvolution filter includes the normalization process between the modeled and observed data, it can properly deal with the unbalanced energy of the traces in the damped wavefields. Warner and Guasch (2014, 2016) calculated the deconvolution filter in the time-domain, but we estimate it in the frequency-domain and selectively choose a frequency-band for normalization to mainly include the artificial low frequencies.

To calculate the deconvolution filter, we take the Fourier transform of the damped observed and modeled wavefields:

$$\tilde{d}(\omega) = FFT[\hat{d}(t)] \quad \text{and} \quad \tilde{u}(\omega) = FFT[\hat{u}(t)], \quad (2)$$

where $FFT[]$ stands for the Fourier transform operation, $\hat{d}(t)$ and $\hat{u}(t)$ are the damped observed and modeled wavefields, respectively, and $\tilde{d}(\omega)$ and $\tilde{u}(\omega)$ are the corresponding Fourier transformed wavefields. We prefer normalizing $\tilde{u}(\omega)$ by $\tilde{d}(\omega)$ rather than normalizing $\tilde{d}(\omega)$ by $\tilde{u}(\omega)$, because the previous one admits simpler gradient derivation based on the adjoint-state technique. We normalize $\tilde{u}(\omega)$ by $\tilde{d}(\omega)$ as

$$\tilde{f}(\omega) = \begin{cases} \frac{\tilde{u}(\omega)}{\tilde{d}(\omega)}, & \text{when } |\tilde{d}(\omega)| \geq \varepsilon \\ 1, & \text{when } |\tilde{d}(\omega)| < \varepsilon \end{cases}, \quad (3)$$

where $\tilde{f}(\omega)$ is the deconvolution filter in the frequency-domain, and ε is a threshold. We further discuss the normalizing method in equation 3 and the role of the threshold (ε) in the Discussion section. We perform the normalization in a frequency range of $0 \sim \omega_M$, where we selectively choose ω_M to mainly include the artificial low frequency components coming from an exponential damping. We take the inverse Fourier transform of $\tilde{f}(\omega)$ in equation 3 to get the time-domain deconvolution filter:

$$f(t) = FFT^{-1}[\tilde{f}(\omega)], \quad (4)$$

where $f(t)$ is the deconvolution filter in the time-domain and $FFT^{-1}[\]$ means the inverse Fourier transform. Since we choose a frequency range of $0 \sim \omega_M$, the sampling rate of $f(t)$ is $1/2\omega_M$. The deconvolution filter in the time-domain becomes close to the Dirac delta function when the modeled data converges to the observed data.

We construct the objective function for FWI using the deconvolution filter:

$$E = \sum_{shot, receiver} \left[\frac{1}{2} \int_0^T \{f(t)w(t)\}^2 dt \right], \quad (5)$$

where $w(t)$ is a weighting function with a sampling rate of $1/2\omega_M$. Since we want to suppress the non-zero lag values of $f(t)$ through minimizing the above objective function, we set $w(t)$ as a linearly increasing function in t away from $t=0$. If we set $w(t)$ as a linearly decreasing function, we must maximize the objective function for $f(t)$ to force it to be the Dirac delta function.

By following Warner and Guasch (2014, 2016), we obtain the gradient expression by taking the derivative of the objective function with respect to the k th model parameter p_k :

$$\frac{\partial E}{\partial p_k} = \sum_{shot, receiver} \left[\int_0^T \left\{ \frac{\partial u(t)}{\partial p_k} r(t) \right\} dt \right], \quad (6)$$

where $u(t)$ is the original modeled wavefield,

$$r(t) = FFT^{-1} \left[\frac{FFT[w^2(t)f(t)]}{\tilde{d}^*(\omega)} \right] \cdot \exp(-\alpha t), \quad (7)$$

and the superscript $*$ indicates the complex conjugate. Prior to applying the final inverse Fourier transform in equation 7, we forcibly add zeros from ω_M to the Nyquist frequency of the original sampling rate (zero padding) and then compute the inverse Fourier transform, which makes the sampling rate of the adjoint source in equation 7 consistent with that of the numerical modeling. The only difference between the proposed FWI and conventional FWI is the adjoint-source expressed in equation 7. We describe in detail the derivation of the adjoint-source in the Appendix B. To calculate the gradient, we back-propagate the new residual in equation 7 and then estimate the zero-lag correlation of the forward-propagated and back-propagated wavefields. We describe the detailed work flow in Table 1.

NUMERICAL EXAMPLES

Marmousi synthetic data example

We generate the synthetic data from the Marmousi model shown in Figure 5a using an acoustic finite-difference modeling technique considering constant density. We filter out low frequencies of the synthetic data below 5 Hz and mute non-zero values (such as tails resulting from filtering and some noise) prior to the first arrivals. The representative shot-gather and its amplitude and phase spectra are shown in Figures 2a, 3a, and 4a, respectively. We place 200 shots with an interval of 40 m at a depth of 10 m and 900 receivers with an interval of 10 m at the same depth of shots. We assume the source wavelet is known. We use the nonlinear conjugate-gradient method for optimization and apply Gaussian smoothing to the calculated gradient (Virieux and Operto, 2009). The starting model for the inversion has linearly increasing velocities with depth (Figure 5b).

We first apply FWI using the least-square misfit and deconvolution-based objective functions to the synthetic data without damping (a damping factor of 0 s^{-1}). The frequency band for the deconvolution is $0 \sim 3 \text{ Hz}$. Since we pick and mute the signals prior to the first arrivals, the synthetic data without damping still has some energy below 3 Hz as shown in Figures 3a and 4a. Therefore, the inverted models of both objective functions show some important structures even without damping but do not converge to the true one and include noise-like images resulting from the distortions in phase spectra shown in Figure 4a (Figures 5c and 5d).

Next, we apply an exponential damping with a damping factor of 4 s^{-1} to the synthetic data (see the last paragraph of the subsection ‘Exponentially damped wavefield’) and perform the same

two FWIs. Except for the damped wavefield, all the elements of the FWI are the same as the previous example. In the inverted models in Figures 5e and 5f, we note that the least-square misfit function updates only the near surface (Figure 5f), since most of the energy of the damped seismogram is focused on the near offset and early time as shown in Figure 2b. On the other hand, the inverted model of the deconvolution-based objective function shows a long wavelength structure of the true model (Figure 5e), which demonstrates the validity of the deconvolution-based objective function for the exponentially damped wavefield. The histories of both objective functions over iterations in Figure 6 also show that our FWI for a damping factor of 4 s^{-1} converges, whereas FWI using the least-square misfit does not.

We plot the deconvolution filters estimated before and after FWI for a damping factor of 4 s^{-1} . The deconvolution filter has some energy at non-zero time-lags before the inversion (Figure 7a), whereas it converges to the delta function after the inversion (Figure 7b), which shows the convergence of the proposed FWI.

We sequentially reduce the damping factor to 2 s^{-1} and increase the frequency band for the deconvolution filter to the range of $0 \sim 5 \text{ Hz}$, and perform the proposed FWI starting from the previously inverted model in Figure 5e. The inversion result shows more detailed structures (Figure 8a). Finally, we perform a subsequent FWI without damping. The frequency band for the subsequent FWI is $5 \sim 16 \text{ Hz}$ and the starting model is the inverted model in Figure 8a. Since, in this case, the new starting model in Figure 8a is close enough to the true model and the deconvolution-based objective function is more expensive to implement than the least-square misfit function, we use the least-square misfit function for the final subsequent FWI without damping. For comparison, we also perform subsequent FWIs starting from the inverted models in Figures 5f and 5c. The subsequent FWI starting from the inverted model in Figure 8a yields a

good convergent result with high resolution which is compatible with the true model (Figure 8b), whereas FWIs starting from the inverted models in Figures 5f and 5c result in poor convergence due to the cycle-skipping problem (Figures 8c and 8d).

The examples of Marmousi synthetic data demonstrate that the artificial low frequencies resulting from an exponential damping can be used to generate a long wavelength structure through FWI with a help of the deconvolution-based objective function.

SEG2014 benchmark data example

The benchmark dataset is a 2D marine isotropic elastic synthetic data that includes noise. The true model is not yet released and only a single velocity profile of the true model at a well log was provided. An initial velocity model was also provided. We apply the proposed FWI based on the acoustic modeling to the benchmark dataset. The benchmark dataset has a maximum offset of 8 km and a recording time of 8 sec. We estimate the source wavelet from the undamped wavefield by following Pratt (1999)'s method. We use the finite-difference modeling method with 2nd order in time and 4th order approximations in space and apply the free-surface boundary condition on the top of the model and the PML boundary condition on the rest of the sides of the model. A representative shot gather of the benchmark dataset is shown in Figure 14 (left side of figure). Since the data are contaminated with noise, we perform a first-arrival picking and mute the energy prior to the first arrival to allow us to apply an exponential damping. Figure 9 shows the amplitude spectra of the data according to damping factors. We note that the frequency information below 3.5 Hz is not available without damping (Figure 9a), whereas an exponential damping generates the artificial low frequencies (Figures 9b ~ 9d). We estimate the

source wavelet through the inversion. Since it is a computational burden to deal with all of the shot gathers available, we select every third shot gather; thus, only 346 shot gathers are treated in this example. The shot interval of the selected shot gathers is 75 m and the receiver interval is 25 m.

In the application of the proposed FWI, we progressively reduce the damping factor from 4, 2, to 1 s^{-1} and, at the same time, increase the frequency band for the deconvolution from $0 \sim 3$, $0 \sim 5$, to $0 \sim 12$ Hz. We perform 30 iterations of updates for each damping factor. Figures 10a and 10b show the provided initial model and the inverted model obtained from the proposed FWI algorithm. The proposed FWI generates long wavelength structure even without low frequency information (Figure 10b). Eventually, we apply a subsequent FWI using the least-square misfit function without damping. The frequency band for the subsequent FWI is $3 \sim 35$ Hz. The subsequent FWI starts from both the provided initial and the inverted model in Figure 10b. When the inversion starts from the provided initial model, the inverted model converges to a wrong velocity model showing a lot of local minima features (Figure 10c), whereas the one starting from the result of the proposed FWI yields a much improved velocity model with high resolution (Figure 10d). Since we apply an acoustic-based FWI algorithm to the elastic data, there are still room for improvements in the final inverted model in Figure 10d. We compare the depth profiles of the inverted models and the given true velocity profile (Figure 11). The true velocity profile is located at a distance of 24.375 km. We note from Figure 11 that, even though the given initial model is far from the true one, the proposed FWI provides a convergent velocity model and the subsequent FWI generates more detailed subsurface structures.

We perform reverse-time migration using the inverted models of the subsequent FWI in Figure 10. The reverse-time migration with the final inverted model in Figure 10d provides a much

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3 better image than that with the inverted model in Figure 10c (Figure 12). The low velocity zone
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5 at distances between 13 and 14 km and at depth of 2.5 km in the inverted model is not recovered
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7 well, which causes a pull-down effects for reflections under the low-velocity zone in the
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9 migration image (Figure 12). We need further study to define the low velocity zone well. The
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11 angle-domain common image gathers in Figure 13 demonstrate, through its flatness, that the
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13 inverted model in Figure 10d is more accurate than the inverted model in Figure 10c. We also
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15 compare the modeled seismograms obtained from the initial and final inverted model in Figure
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17 10d with the observed seismogram. The modeled seismogram from the final inverted model
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19 shows similar kinematic features with the observed seismogram (Figure 14).

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22 The benchmark data example demonstrates that our FWI algorithm generates a convergent
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24 long wavelength model using the generated artificial low frequencies coming from the
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DISCUSSION

In our study, we use a stationary exponential damping function ($e(t) = \exp(-\alpha t)$), in which the damping starts from zero time regardless of the source-receiver offset. If we use a dynamic (nonstationary) exponential damping ($e(x,t)$) (e.g. damping starts from the picked first arrival time), we face two problems:

First, applying the adjoint-state method for FWI using a dynamic exponential damping is not straightforward. If we use a stationary exponential damping, we can calculate the Born modeled data (partial derivative wavefield) with a clear damping intensity at every grid point of the model. (We actually don't need to directly calculate the Born modeled data in the adjoint-state method, but it is related to the theoretical foundation of the adjoint-state method.) However, if we use a dynamic exponential damping, we cannot evaluate the Born modeled wavefield with a damping intensity at every grid point of the model, because we cannot extend the receiver-space damping function ($e(x,t)$) to whole model-space.

Second, when we use a dynamic exponential damping ($e(x,t)$), we must apply the same dynamic exponential damping to both observed and modeled data for consistency. Since the picked traveltimes of the observed and modeled data are different from each other, applying a dynamic exponential damping based on the observed data to the modeled data could miss some of the kinematic information embedded in the modeled data and vice versa.

Another issue is the normalizing approach for the calculation of the deconvolution filter in equation 3. Instead of the normalizing in equation 3, we can multiply both denominator and numerator by the complex conjugate of the denominator and add a threshold (ε) to the

denominator part (smooth transition). If the threshold is very small, the result of equation 3 is almost the same as that of the smooth transition method. However, in this case, the calculations of deconvolution filter in both methods could be unstable because the division process is very sensitive to extremely small values in the denominator. If the threshold is not small, the deconvolution filter of the smooth transition method after inverse Fourier transform does not become a Dirac delta function even when the modeled data is very close to the observed data. On the other hand, the deconvolution filter of equation 3 should admit a Dirac delta function when the modeled data is close to the observed data. As the threshold becomes larger, the discontinuity in equation 3 (Jump between 1 and the normalized value) pushes the deconvolution filter (after inverse Fourier transform) more to a Dirac delta function, whereas the smooth transition method pushes it less to the Dirac delta function. The normalizing strategy in equation 3 is more appropriate to the proposed FWI method than the smooth transition method, since we want to push the deconvolution filter to admit a Dirac delta function through optimization. In our FWI algorithm, we empirically choose a large value for the threshold, such as 20 ~ 30 % of the maximum value in the chosen frequency band ($0 \sim \omega_M$), in the early stage of the inversion and then progressively reduce the value as the inversion proceeds, such as 10, 5, and 1 % of the maximum value. Choosing a large value for the threshold in equation 3 means that the deconvolution process considers only dominant frequencies in the chosen frequency band ($0 \sim \omega_M$) depending on a value of threshold, which makes the deconvolution process stable.

CONCLUSIONS

We propose a time-domain FWI of exponentially damped wavefields using the deconvolution-based objective function. The artificial low frequencies coming from the exponential damping help FWI avoid the cycle-skipping problem. However, the least-square misfit objective function does not properly deal with the exponentially damped wavefield, because the amplitude of traces in the shot gather decreases almost exponentially with increasing offset. We suggest using the deconvolution-based objective function to invert the exponentially damped wavefield. Since the deconvolution filter includes a normalization process between the modeled and observed data, it can properly address the exponentially decreasing energy of the damped wavefield along the offset axis. To calculate the deconvolution filter, we normalize the modeled data by the observed data in the frequency-domain and take the inverse Fourier transform of it. We also selectively choose a frequency-band for normalization to mainly include the artificial low frequencies. Numerical examples demonstrate that FWI of the exponentially damped wavefield using the deconvolution-based objective function generates a convergent result without the need for real low-frequency information.

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APPENDIX A: Frequency-domain damped Green function and wavefield

Applying an exponential damping to seismic data along the time-axis is expressed as

$$\hat{d}(t) = \exp(-\alpha t)d(t) = \exp(-\alpha t) \int_0^t g(\tau)s(t-\tau)d\tau, \quad (A1)$$

where $g(t)$ is a Green function, $s(t)$ is a source wavelet. On the other hand, the frequency-domain modeling with a complex angular frequency provides only the damped Green function. Applying inverse Fourier transform, the damped Green function and source wavelet reproduce the wavefield:

$$\bar{d}(t) = [\exp(-\alpha t)g(t)] * s(t) = \int_0^t \exp(-\alpha \tau)g(\tau)s(t-\tau)d\tau, \quad (A2)$$

where “*” indicates the convolution process. We note that the result of an exponential damping along the time-axis (equation A1) is different from that obtained from the frequency-domain modeling (equation A2). Therefore, the frequency-domain FWI with an exponential damping is not appropriate to deal with the time-domain damped wavefield. Meanwhile, the proposed FWI doesn't encounter this issue, since we apply an exponential damping to both observed and modeled data in the time-domain and invert the damped wavefield through the time-domain FWI.

On the other hand, if the damping factor (α) is extremely strong, $\hat{d}(t)$ in equation A1 becomes close to the weighted delta function, $a\delta(t-t_1)$, where t_1 is the first arrival time and a is the amplitude. With a strong damping, the damped Green function ($\exp(-\alpha t)g(t)$) in equation A2 is also close to the delta function, $b\delta(t-t_1)$, where b is the amplitude. In this case, the phase

of the time-domain damped wavefield can be properly inverted through the frequency-domain
FWI using an exponential damping.

APPENDIX B: Adjoint-source of the deconvolution-based objective function

In order to easily derive the adjoint-source, we consider the discretized time, frequency, and discrete Fourier transform. The deconvolution-based objective function in equation 5 is rewritten in matrix-vector form as

$$E = \sum_{shot, receiver} \frac{1}{2} \|\mathbf{W}(t)\mathbf{f}(t)\|_2^2, \quad (B1)$$

where $\mathbf{W}(t)$ is a diagonal matrix which diagonal elements are corresponding to the sampled elements of weighting function $w(t)$ and $\mathbf{f}(t)$ is a vector expression of discretized deconvolution filter $f(t)$. The gradient expression is obtained by taking the derivative of the objective function in equation B1 with respect to the k th model parameter p_k :

$$\frac{\partial E}{\partial p_k} = \sum_{shot, receiver} \left(\frac{\partial \mathbf{f}(t)}{\partial p_k} \right)^T (\mathbf{W}^2 \mathbf{f}(t)), \quad (B2)$$

where $\mathbf{W}^2 = \mathbf{W}^T \mathbf{W}$ and the superscript ' T ' represents the transpose.

On the other hand, based on the discrete Fourier transform, we can express the Fourier and inverse Fourier transform in matrix-vector form (Oppenheim et al., 1999) as

$$\tilde{\mathbf{x}}(\omega) = \mathbf{M}\mathbf{x}(t) \text{ and } \mathbf{x}(t) = \frac{1}{N} \mathbf{M}^* \tilde{\mathbf{x}}(\omega), \quad (B3)$$

where $\mathbf{x}(t)$ and $\tilde{\mathbf{x}}(\omega)$ are discretized time- and frequency-domain signal with N elements, $\mathbf{M}$ is the Fourier transform operator which is symmetric complex matrix ($\mathbf{M}^T = \mathbf{M}$), and the superscript '*' represents the complex conjugate. From equations 2, 3, 4, and B3, $\mathbf{f}(t)$ is expressed as

$$\mathbf{f}(t) = \frac{1}{N} \mathbf{M}^* \tilde{\mathbf{f}}(\omega) = \frac{1}{N} \mathbf{M}^* (\mathbf{D}\tilde{\mathbf{u}}), \quad (B4)$$

where $\tilde{\mathbf{f}}(\omega)$ is the frequency-domain deconvolution filter, $\tilde{\mathbf{u}}$ is the Fourier transformed of the exponentially damped modeled wavefield ($\tilde{\mathbf{u}}(\omega) = \mathbf{M}\hat{\mathbf{u}}(t)$) and $\mathbf{D}$ is a diagonal matrix which diagonal elements are the reciprocal of the corresponding element of $\tilde{\mathbf{d}}$, the Fourier transformed of the exponentially damped observed wavefield. Substituting equation B4 to equation B2, we obtain

$$\begin{aligned} \frac{\partial E}{\partial p_k} &= \sum_{shot, receiver} \text{Re} \left\{ \left(\frac{\partial \tilde{\mathbf{u}}(\omega)}{\partial p_k} \right)^T \left(\frac{1}{N} \mathbf{D} \mathbf{M}^* (\mathbf{W}^2 \mathbf{f}) \right) \right\} \\ &= \sum_{shot, receiver} \text{Re} \left\{ \left(\frac{\partial \hat{\mathbf{u}}(t)}{\partial p_k} \right)^T \left(\frac{1}{N} \mathbf{M} \mathbf{D} \mathbf{M}^* (\mathbf{W}^2 \mathbf{f}) \right) \right\}. \end{aligned} \quad (\text{B5})$$

Since we consider only real value of the gradient in equation B5, taking the complex conjugate gives the same result:

$$\begin{aligned} \frac{\partial E}{\partial p_k} &= \sum_{shot, receiver} \text{Re} \left\{ \left(\frac{\partial \hat{\mathbf{u}}(t)}{\partial p_k} \right)^T \left(\frac{1}{N} \mathbf{M} \mathbf{D} \mathbf{M}^* (\mathbf{W}^2 \mathbf{f}) \right)^* \right\} \\ &= \sum_{shot, receiver} \text{Re} \left\{ \left(\frac{\partial \hat{\mathbf{u}}(t)}{\partial p_k} \right)^T \left(\frac{1}{N} \mathbf{M}^* \mathbf{D}^* \mathbf{M} (\mathbf{W}^2 \mathbf{f}) \right) \right\}. \end{aligned} \quad (\text{B6})$$

The exponentially damped model wavefield $\hat{\mathbf{u}}(t)$ is rewritten as $\hat{\mathbf{u}}(t) = \mathbf{C}\mathbf{u}(t)$, where $\mathbf{u}(t)$ is the original modeled wavefield and $\mathbf{C}$ is a diagonal matrix which diagonal elements represent the exponential damping. Finally equation B6 is expressed as

$$\frac{\partial E}{\partial p_k} = \sum_{shot, receiver} \text{Re} \left\{ \left(\frac{\partial \mathbf{u}(t)}{\partial p_k} \right)^T \left(\frac{1}{N} \mathbf{C} \mathbf{M}^* \mathbf{D}^* \mathbf{M} (\mathbf{W}^2 \mathbf{f}) \right) \right\}, \quad (\text{B7})$$

where the adjoint-source for the calculation of the gradient is $\frac{1}{N} \mathbf{C} \mathbf{M}^* \mathbf{D}^* \mathbf{M} (\mathbf{W}^2 \mathbf{f})$ which is the same with equation 7.

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Table 1. The algorithm of FWI of exponentially damped wavefield using the deconvolution-based objective function.

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- Figure 2. The (a) synthetic seismogram filtered below 5 Hz and its damped versions with damping factors of (b) 4, (c) 8, and (d) 32 s⁻¹. The left side of each figure is the original damped version and the right side shows the normalized damped version where each trace is normalized by its maximum value. The signals prior to the first arrival are muted in the synthetic data in (a).
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- Figure 4. The phase spectra of the (a) synthetic seismogram and its damped versions with damping factors of (b) 4, (c) 8, and (d) 32 s⁻¹ in Figure 2.
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- Figure 8. The inverted models obtained from (a) FWI using the deconvolution-based objective function with a reduced damping factor of 2 s⁻¹ for a frequency band of 0 ~ 5 Hz starting from the velocity model in Figure 5e and the subsequent FWI using the least-square misfit function without damping for a frequency band of 5 ~ 16 Hz starting from (b) Figure 8a, (c) Figure 5f, and (d) Figure 5c.
- Figure 9. The amplitude spectra of the benchmark data at different offset distances (a) without damping and with damping factors of (b) 4, (c) 8, and (d) 16 s⁻¹.
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Figure 12. The reverse-time migration images using the subsequent FWI results in (a) Figure 10c and (b) Figure 10d.

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Figure 14. Comparison of the observed seismogram (left part) and modeled seismogram (right part) (a) before and (b) after inversion.

Input: starting velocity model & observed data after muting

for (damping factor from high to low and, at the same time, ω_M from low to high and Gaussian smoothing factor from strong to weak)

1. Apply an exponential damping to observed data & Fourier transform of it $\rightarrow \tilde{d}(\omega)$

repeat

2. Generate modeled data from velocity model
3. Apply an exponential damping to modeled data & Fourier transform of it $\rightarrow \tilde{u}(\omega)$
4. Normalize $\tilde{u}(\omega)$ by $\tilde{d}(\omega)$ using equation 3 in a frequency range of $0 \sim \omega_M$ & Inverse Fourier transform of it $\rightarrow f(t)$ with a sampling rate of $1/(2\omega_M)$
5. Multiply the squared weighting function ($w^2(t)$) to $f(t)$ & Fourier transform of it & Divide it by $\tilde{d}^*(\omega)$ (equation 7)
6. Add zeros from ω_M to the Nyquist frequency of the original sampling rate & Inverse Fourier transform of it & Calculate the adjoint source in equation 7
7. Calculate the gradient using the adjoint source & Apply Gaussian smoothing to the gradient
8. Update velocity model using the smoothed gradient

until satisfying stop criterion

end (for)

Table 1. The algorithm of FWI of exponentially damped wavefield using the deconvolution-based objective function.

Figure 1a. The (a) 4 Hz cosine functions with various damping factors and (b) their amplitude spectra. The black line, red dot, and blue cross stand for no damping and damping factors of 4 and 8 s-1, respectively.

85x40mm (300 x 300 DPI)

Figure 1b. The (a) 4 Hz cosine functions with various damping factors and (b) their amplitude spectra. The black line, red dot, and blue cross stand for no damping and damping factors of 4 and 8 s⁻¹, respectively.

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Figure 2a. The (a) synthetic seismogram filtered below 5 Hz and its damped versions with damping factors of (b) 4, (c) 8, and (d) 32 s⁻¹. The left side of each figure is the original damped version and the right side shows the normalized damped version where each trace is normalized by its maximum value. The signals prior to the first arrival are muted in the synthetic data in (a).

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Figure 2b. The (a) synthetic seismogram filtered below 5 Hz and its damped versions with damping factors of (b) 4, (c) 8, and (d) 32 s⁻¹. The left side of each figure is the original damped version and the right side shows the normalized damped version where each trace is normalized by its maximum value. The signals prior to the first arrival are muted in the synthetic data in (a).

179x109mm (300 x 300 DPI)

Figure 2c. The (a) synthetic seismogram filtered below 5 Hz and its damped versions with damping factors of (b) 4, (c) 8, and (d) 32 s⁻¹. The left side of each figure is the original damped version and the right side shows the normalized damped version where each trace is normalized by its maximum value. The signals prior to the first arrival are muted in the synthetic data in (a).

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Figure 2d. The (a) synthetic seismogram filtered below 5 Hz and its damped versions with damping factors of (b) 4, (c) 8, and (d) 32 s⁻¹. The left side of each figure is the original damped version and the right side shows the normalized damped version where each trace is normalized by its maximum value. The signals prior to the first arrival are muted in the synthetic data in (a).

179x109mm (300 x 300 DPI)

Figure 3a. The amplitude spectra of the synthetic seismograms (a) without damping and with damping factors of (b) 4, (c) 8, and (d) 32 s⁻¹ in Figure 2 at different offset distances. The spectrum of each trace is normalized by its maximum value.

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Figure 3b. The amplitude spectra of the synthetic seismograms (a) without damping and with damping factors of (b) 4, (c) 8, and (d) 32 s⁻¹ in Figure 2 at different offset distances. The spectrum of each trace is normalized by its maximum value.

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Figure 3c. The amplitude spectra of the synthetic seismograms (a) without damping and with damping factors of (b) 4, (c) 8, and (d) 32 s⁻¹ in Figure 2 at different offset distances. The spectrum of each trace is normalized by its maximum value.

87x42mm (300 x 300 DPI)

Figure 3d. The amplitude spectra of the synthetic seismograms (a) without damping and with damping factors of (b) 4, (c) 8, and (d) 32 s⁻¹ in Figure 2 at different offset distances. The spectrum of each trace is normalized by its maximum value.

87x42mm (300 x 300 DPI)

Figure 4a. The phase spectra of the (a) synthetic seismogram and its damped versions with damping factors of (b) 4, (c) 8, and (d) 32 s⁻¹ in Figure 2.

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Figure 4b. The phase spectra of the (a) synthetic seismogram and its damped versions with damping factors of (b) 4, (c) 8, and (d) 32 s⁻¹ in Figure 2.

154x101mm (200 x 200 DPI)

Figure 4c. The phase spectra of the (a) synthetic seismogram and its damped versions with damping factors of (b) 4, (c) 8, and (d) 32 s⁻¹ in Figure 2.

154x101mm (200 x 200 DPI)

Figure 4d. The phase spectra of the (a) synthetic seismogram and its damped versions with damping factors of (b) 4, (c) 8, and (d) 32 s⁻¹ in Figure 2.

154x101mm (200 x 200 DPI)

Figure 5a. The (a) Marmousi model, (b) starting model of which velocity is increasing with depth, and inverted models for a frequency band of 0 ~ 3 Hz without damping using the (c) deconvolution-based and (d) least-square misfit objective function and with a damping factor of 4 s-1 using the (e) deconvolution-based and (f) least-square misfit objective function.

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Figure 5b. The (a) Marmousi model, (b) starting model of which velocity is increasing with depth, and inverted models for a frequency band of 0 ~ 3 Hz without damping using the (c) deconvolution-based and (d) least-square misfit objective function and with a damping factor of 4 s⁻¹ using the (e) deconvolution-based and (f) least-square misfit objective function.

179x68mm (200 x 200 DPI)

Figure 5c. The (a) Marmousi model, (b) starting model of which velocity is increasing with depth, and inverted models for a frequency band of 0 ~ 3 Hz without damping using the (c) deconvolution-based and (d) least-square misfit objective function and with a damping factor of 4 s⁻¹ using the (e) deconvolution-based and (f) least-square misfit objective function.

179x68mm (200 x 200 DPI)

Figure 5d. The (a) Marmousi model, (b) starting model of which velocity is increasing with depth, and inverted models for a frequency band of 0 ~ 3 Hz without damping using the (c) deconvolution-based and (d) least-square misfit objective function and with a damping factor of 4 s⁻¹ using the (e) deconvolution-based and (f) least-square misfit objective function.

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Figure 5e. The (a) Marmousi model, (b) starting model of which velocity is increasing with depth, and inverted models for a frequency band of 0 ~ 3 Hz without damping using the (c) deconvolution-based and (d) least-square misfit objective function and with a damping factor of 4 s⁻¹ using the (e) deconvolution-based and (f) least-square misfit objective function.

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Figure 5f. The (a) Marmousi model, (b) starting model of which velocity is increasing with depth, and inverted models for a frequency band of 0 ~ 3 Hz without damping using the (c) deconvolution-based and (d) least-square misfit objective function and with a damping factor of 4 s⁻¹ using the (e) deconvolution-based and (f) least-square misfit objective function.

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Figure 6. The histories of the deconvolution and least-square misfit objective functions through iterations for a damping factor of 4 s⁻¹. All values of the objective functions are normalized by the value of the first iteration.

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Figure 7a. The estimated deconvolution filters at different offset distances for a damping factor of 4 s⁻¹ (a) before and (b) after applying the proposed FWI.

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Figure 7b. The estimated deconvolution filters at different offset distances for a damping factor of 4 s-1 (a) before and (b) after applying the proposed FWI.

85x40mm (300 x 300 DPI)

Figure 8a. The inverted models obtained from (a) FWI using the deconvolution-based objective function with a reduced damping factor of 2 s⁻¹ for a frequency band of 0 ~ 5 Hz starting from the velocity model in Figure 5e and the subsequent FWI using the least-square misfit function without damping for a frequency band of 5 ~ 16 Hz starting from (b) Figure 8a, (c) Figure 5f, and (d) Figure 5c.

179x68mm (200 x 200 DPI)

Figure 8b. The inverted models obtained from (a) FWI using the deconvolution-based objective function with a reduced damping factor of 2 s⁻¹ for a frequency band of 0 ~ 5 Hz starting from the velocity model in Figure 5e and the subsequent FWI using the least-square misfit function without damping for a frequency band of 5 ~ 16 Hz starting from (b) Figure 8a, (c) Figure 5f, and (d) Figure 5c.

179x68mm (200 x 200 DPI)

Figure 8c. The inverted models obtained from (a) FWI using the deconvolution-based objective function with a reduced damping factor of 2 s⁻¹ for a frequency band of 0 ~ 5 Hz starting from the velocity model in Figure 5e and the subsequent FWI using the least-square misfit function without damping for a frequency band of 5 ~ 16 Hz starting from (b) Figure 8a, (c) Figure 5f, and (d) Figure 5c.

179x68mm (200 x 200 DPI)

Figure 8d. The inverted models obtained from (a) FWI using the deconvolution-based objective function with a reduced damping factor of 2 s⁻¹ for a frequency band of 0 ~ 5 Hz starting from the velocity model in Figure 5e and the subsequent FWI using the least-square misfit function without damping for a frequency band of 5 ~ 16 Hz starting from (b) Figure 8a, (c) Figure 5f, and (d) Figure 5c.

179x68mm (200 x 200 DPI)

Figure 9a. The amplitude spectra of the benchmark data at different offset distances (a) without damping and with damping factors of (b) 4, (c) 8, and (d) 16 s⁻¹.

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Figure 9b. The amplitude spectra of the benchmark data at different offset distances (a) without damping and with damping factors of (b) 4, (c) 8, and (d) 16 s-1.

87x42mm (300 x 300 DPI)

Figure 9c. The amplitude spectra of the benchmark data at different offset distances (a) without damping and with damping factors of (b) 4, (c) 8, and (d) 16 s⁻¹.

87x42mm (300 x 300 DPI)

Figure 9d. The amplitude spectra of the benchmark data at different offset distances (a) without damping and with damping factors of (b) 4, (c) 8, and (d) 16 s⁻¹.

87x42mm (300 x 300 DPI)

Figure 10a. The (a) given initial model, (b) inverted model obtained from the proposed FWI, and inverted models obtained from the subsequent FWI using the least-square misfit function without damping starting from the (c) given initial model and (d) inversion result of the proposed FWI in Figure 10b.

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Figure 10b. The (a) given initial model, (b) inverted model obtained from the proposed FWI, and inverted models obtained from the subsequent FWI using the least-square misfit function without damping starting from the (c) given initial model and (d) inversion result of the proposed FWI in Figure 10b.

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Figure 10c. The (a) given initial model, (b) inverted model obtained from the proposed FWI, and inverted models obtained from the subsequent FWI using the least-square misfit function without damping starting from the (c) given initial model and (d) inversion result of the proposed FWI in Figure 10b.

179x76mm (200 x 200 DPI)

Figure 10d. The (a) given initial model, (b) inverted model obtained from the proposed FWI, and inverted models obtained from the subsequent FWI using the least-square misfit function without damping starting from the (c) given initial model and (d) inversion result of the proposed FWI in Figure 10b.

179x76mm (200 x 200 DPI)

Figure 11. Comparison of depth profiles of the inverted models and given true one.

100x55mm (300 x 300 DPI)

Figure 12a. The reverse-time migration images using the subsequent FWI results in (a) Figure 10c and (b) Figure 10d.

922x437mm (150 x 150 DPI)

Figure 12b. The reverse-time migration images using the subsequent FWI results in (a) Figure 10c and (b) Figure 10d.

922x437mm (150 x 150 DPI)

Figure 13a. The angle-domain common image gathers using the subsequent FWI results in (a) Figure 10c and (b) Figure 10d.

727x496mm (150 x 150 DPI)

Figure 13b. The angle-domain common image gathers using the subsequent FWI results in (a) Figure 10c and (b) Figure 10d.

727x496mm (150 x 150 DPI)

Figure 14a. Comparison of the observed seismogram (left part) and modeled seismogram (right part) (a) before and (b) after inversion.

205x127mm (300 x 300 DPI)

Figure 14b. Comparison of the observed seismogram (left part) and modeled seismogram (right part) (a) before and (b) after inversion.

205x127mm (300 x 300 DPI)