Multi-parameter elastic full waveform inversion in the presence of azimuthally rotated orthorhombic anisotropy: Application to 9C land data

Ju-Won Oh* and Tariq Alkhalifah, King Abdullah University of Science and Technology

Summary

To examine the feasibility of elastic full waveform inversion (FWI) for azimuthally rotated orthorhombic (rORT) media, we analyze the sensitivity of the 9-component (9C) land data set acquired on the surface on each of the ORT parameters. The trade-off analysis supports that the parameter set that includes deviation parameters offers the best choice for a 9C data set. Compared to the data from an explosive source, using the 9C land data, ORT parameters show different trade-off patterns for the different source and receiver components. For this reason, finding an optimal component considering trade-offs is another important issue to better recover subsurface rotated orthorhombic anisotropy.

Introduction

The elastic orthorhombic assumption (Tsvankin, 1997), which takes into account the horizontal layering and vertical fractures, is one of the most practical Earth models. Under the orthorhombic assumption, finding the azimuthal variations of anisotropic parameters along fast and slow axes and the rotation angle of the symmetry planes is important to increase the convergence rate of multi-parameter FWI. To reduce the trade-off in multi-parameter FWI (Operto et al., 2013), finding an optimal parameterization is crucial. Alkhalifah and Plessix (2014) compared parameterizations for acoustic vertical transversely isotropic (VTI) FWI. Then, Masmoudi and Alkhalifah (2016) extended this investigation to acoustic ORT FWI and showed a new parameterization by employing deviation parameters to define differences between fast and slow axes. For elastic FWI, although finding an optimal parameterization also has been actively studied from isotropic (Tarantola, 1986; Köhn et al., 2012; Prioux et al., 2013; Operto et al., 2013) to TI approximations (He and Plessix, 2016; Kamath and Tsvankin, 2016; Pan et al., 2016), the studies on elastic ORT media are relatively rare. Oh and Alkhalifah (2016) suggested a new elastic ORT parameterization, which provides the opportunity to apply a multi-stage FWI strategy by decoupling the scattering potentials of the model parameters corresponding to high-symmetry and low-symmetry anisotropy as well as between acoustic and elastic media. For this ORT parameterization, resolution analysis is also conducted (Kazei and Alkhalifah, 2017). Oh and Alkhalifah (2017) further extended this parameter set to rORT media and examined the feasibility of inverting for the rotation angle of a symmetric plane assuming

marine streamer data. However, using the marine data, all source and receiver components are coupled and the seismic source does not generate S-waves thus, the coupling between parameters are hard to be detached. On the other hand, 9C 3D seismic data (Simmons and Backus, 2001) provides the opportunity to recover more ORT parameters with less trade-off by capturing more wave modes than conventional data (Rusmanugroho and McMechan, 2012).

In this study, we examine the feasibility of the elastic ORT FWI to 9C land data considering a rotation in the orthorhombic anisotropy (Tsvankin and Grechka, 2011). First, we analyze the trade-off among parameters considering different wave modes and different source and receiver components, and use such analysis to build up an optimal FWI strategy. Finally, the proposed multi-stage FWI is applied on synthetic 9C land data set.

Inverse problem in elastic rORT media

The elastic wave equation in rORT media can be expressed with particle displacements (u_i), stress (σ_{ij}), strain (ϵ_{ij}) and seismic source (f_i) as follows:

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \partial_j \sigma_{ij} + f_i \quad (1)$$

and

$$\sigma_{ij} = C_{ijkl}^{\text{rORT}} \epsilon_{kl} \quad (2)$$

In Voigt notation ($C_{ijkl} \rightarrow C_{IJ}$), the stiffness matrix can be expressed as the rotation of the orthorhombic stiffness matrix (C_{IJ}^{ORT} ; Tsvankin, 1997) around the vertical direction (z) using bond transformation matrix ($\mathbf{M}_z(\phi)$; Ivanov and Stova, 2016) as follows:

$$\tilde{C}_{IJ}^{\text{rORT}} = \mathbf{M}_z(\phi) \mathbf{C}_{IJ}^{\text{ORT}} \mathbf{M}_z^T(\phi), \quad (3)$$

where

$$\tilde{C}_{IJ}^{\text{rORT}} = \begin{pmatrix} \tilde{C}_{11} & \tilde{C}_{12} & \tilde{C}_{13} & 0 & 0 & \tilde{C}_{16} \\ \tilde{C}_{12} & \tilde{C}_{22} & \tilde{C}_{23} & 0 & 0 & \tilde{C}_{26} \\ \tilde{C}_{13} & \tilde{C}_{23} & \tilde{C}_{33} & 0 & 0 & \tilde{C}_{36} \\ 0 & 0 & 0 & \tilde{C}_{44} & \tilde{C}_{45} & 0 \\ 0 & 0 & 0 & \tilde{C}_{45} & \tilde{C}_{55} & 0 \\ \tilde{C}_{16} & \tilde{C}_{26} & \tilde{C}_{36} & 0 & 0 & \tilde{C}_{66} \end{pmatrix} \quad (4)$$

and the azimuth angle (ϕ) is defined as the angle measured from x -axis in a counter-clockwise direction. The goal of FWI is finding a solution, which minimizes the objective function that measures the misfit between modeled (u) and observed (d) data. In the least squares sense, the objective function can be expressed by

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$$E(\mathbf{p}) = \int [d(t) - u(\mathbf{p}, t)]^2 dt. \quad (5)$$

The gradient direction for an arbitrary parameter, p , is given by

$$\nabla_p E = \int u_{PDW}^p(\mathbf{p}, t) [d(t) - u(\mathbf{p}, t)] dt. \quad (6)$$

Here, u_{PDW} and σ_{PDW} are the partial derivative wavefield in terms of displacement and stress, respectively, that are the solution of following wave equation.

$$\rho \frac{\partial^2 u_{PDW,i}^p}{\partial t^2} = \partial_j \sigma_{PDW,ij}^p + \partial_j \left(\frac{\partial \tilde{C}_{ijkl}}{\partial p} \varepsilon_{kl} \right) \quad (7)$$

We ignore density in this abstract. The last term is the virtual source (Pratt et al., 1998) acting like a point moment tensor source induced by the model parameter perturbation (Oh and Alkhalifah, 2016). The gradient direction for any parameter (p) can be obtained by the linear combination of 13 monoclinic C_{IJ} parameters as follows:

$$\nabla_p E = \sum_{IJ} \frac{\partial \tilde{C}_{IJ}}{\partial p} (\nabla_{\tilde{C}_{IJ}} E) \quad (8)$$

Optimal parameterization

To define rORT media, Oh and Alkhalifah (2017) suggested a parameterization given by the following parameters:

$$\left(v_{p1}, v_{s1}, \varepsilon_1, \eta_1, \gamma_1, \delta_3, \varepsilon_D, \eta_D, \gamma_D, \phi \right). \quad (9)$$

The main advantage of this parameterization is the hierarchical parameter update demonstrated in the radiation pattern analysis. Figure 1 shows the geometry of source (red dot), receiver (green dot) and reflection point (yellow dot) for an rORT layer interbedded by isotropic layers.

Table 1: Wave modes associated with different seismic data acquisition options (after Hardage et al., 2011).

Data acquisition option	source	receiver	Captured modes	
			Isotropic	Anisotropic
9C	XYZ	XYZ	P-P, P-SV, SV-SV, SV-P, SH-SH	P-P, P-SV ₁ , P-SV ₂ , SV ₁ -SV ₁ , SV ₂ -SV ₂ , SV ₁ -P, SV ₂ -P, SH ₁ -SH ₁ , SH ₂ -SH ₂
4C	Z or A	XYZH	P-P, P-SV	P-P, P-SV ₁ , P-SV ₂
3C	Z	XYZ	P-P, P-SV	P-P, P-SV ₁ , P-SV ₂
1C	Z	Z	P-P	P-P

The radiation patterns in Figure 2 are calculated in terms of the opening angle following the Snell's law for a horizontal reflector (Gholami et al., 2013; Oh and Alkhalifah, 2016). Because of the limited space, we only display P-P and SH-SH modes. This parameterization offers an optimal configuration for orthorhombic anisotropy. As Oh and Alkhalifah (2017) showed, we can build up the subsurface model from isotropic (v_{p1} and v_{s1}) to VTI ($\varepsilon_1, \eta_1, \gamma_1$) regardless of the azimuthal rotation of the orthorhombic

anisotropy. Then, the deviation parameters ($\varepsilon_D, \eta_D, \gamma_D, \delta_3$) and the azimuth angle (ϕ) can be inverted. In this study, focusing on land seismic data, we compare FWI performance of this parameterization for the data from an explosive source and 9C land data. Because we can capture various wave modes in 9C land survey (Table 1), we have the opportunity to recover more ORT parameters. To do that, we need to choose an optimal FWI strategy for 9C land data to mitigate the trade-off among parameters.

Figure 1: The geometry of a 3-layered model: L_1 and L_3 are isotropic backgrounds and L_2 is the target rORT layer. The blue line indicates a source-receiver line. The red, green and yellow dots denote the source, receiver and perturbation points used for radiation pattern analysis. For the 9C seismic survey, each source (S) and receiver (R) has 3 components along inline (I), crossline (X) and vertical (Z) directions. θ_0 is the opening angle.

Figure 2: The reflection based radiation pattern of P-P and SH-SH modes from a horizontal reflector (Oh and Alkhalifah, 2016). We assume an isotropic background model and isotropically radiated incidence wave. ϕ_{sr} is the azimuth angle of the source-receiver line (blue line in Figure 1), which is not related to the rotation angle of fractures (ϕ), and the angles in each plot indicate the opening angle (θ_0) in Figure 1. Yellow area denotes the resolution limit as a result offset-to-depth ratio (4) for the doughnut model.

Trade-off analysis

To analyze trade-off between the parameters for different source types, we calculate the gradient direction for a

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simple model called the doughnut model, in which each ORT parameter perturbation occurs at a different piece of the doughnut (Figure 3). Uniformly spaced 400 sources (3-10Hz) are located at 200 m in depth and 3C receivers are located at all nodal points at the same depth. For each source, only receivers within 4 km offset are activated thus the offset-to-depth ratio at a target doughnut is about 4. For this ratio, the maximum opening angle is about 126° , which indicates that the yellow area in the radiation pattern (Figure 2) is not available in this example. The benefit of this model is that we can clearly distinguish the trade-off among ORT parameters in a given acquisition geometry (source type and offset-to-depth ratio). For this case, we compare the trade-off patterns for different source types. Here, for simplicity, we assume that orthorhombic media is not rotated. However, the influences of azimuth angle on the radiation pattern and gradient directions are well described in Oh and Alkhalifah (2017).

Figure 3: A doughnut model for the trade-off analysis. The anomaly has 9 different parameter perturbations and each parameter perturbation occurs at a different piece.

Figure 4: The gradient directions for 9 orthorhombic parameters from explosive sources and 3C recordings for the doughnut model. The horizontal slice is chosen at the top of the anomaly and 4 vertical slices are chosen along different azimuth angles in Figure 3. For visualization, only the area within black dashed lines is displayed.

Figure 5: Same as Figure 4 but from (a) vertical, (b) inline and (c) crossline sources with 3C recordings.

Figure 4 shows the gradient direction from the explosive source at each slice. Due to the limited space, only v_{p1} , v_{s1} , ϵ_1 and ϵ_D , following Oh and Alkhalifah (2017) reference as “ideal” parameters, are displayed. For each parameter, the arrow indicates the true location of that parameter while the parameters having trade-off are written in blue color. The black, white and red colors for each arrow indicate that the influence of that parameter is strong, medium and weak, respectively, compared to trade-off imprints. In Figure 4, because the explosive source only generates P-waves, the parameter’s trade-off are mainly constrained by only P-P

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and P-SV modes (Table 1). For this reason, the ideal parameters suffer from trade-off for the given offset-to-depth ratio (4). Figure 5 shows the gradients of the ideal parameters obtained from the 9C land data. As shown in Table 1, seismic data from the vertical source also captures mainly P-P and P-SV modes. However, because most of the energy is actually propagating in the vertical direction, the parameters that require wide opening angle like ε_D are weakly resolved. As Oh and Alkhalifah (2017) showed, the resolution of ε_D is the crucial to the success of inverting for the azimuth angle. For this reason, using only the vertical source, we do not expect to recover crucial subsurface ORT features. Figures 5b and 5c show the gradients from the horizontal sources. As the radiation pattern (Figure 2) shows, the parameters, v_{s1} , η_1 , η_D and γ_D , have strong trade-off between them. The trade-off is hard to decouple. However, for the inline source (Figure 5b), $[v_{s1}, \eta_1]$ and $[\eta_D, \gamma_D]$ are decoupled because the latter parameters are sensitive to wide azimuth data only. For the crossline source (Figure 5c), although the trade-off is more severe, ε_D has a strong gradient (sensitivity). However, ε_D suffers from a strong trade-off with γ_1 . Because the parameter γ_1 is only sensitive to SH-SH mode (Oh and Alkhalifah, 2016), such a trade-off between ε_D and γ_1 is the product of SH-SH correlation. This indicates that, if we properly decouple ε_D and γ_1 by isolating the SH-SH mode in the data, the parameter ε_D can be recovered using the crossline source (or inline source when the rotation is large). From this initial ε_D , ϕ can be estimated, being recovered with ε_D simultaneously, as Oh and Alkhalifah (2017) showed.

Figure 6: The true model. Note that other parameters are set to 0.

Numerical example: Synthetic 9C land data

To test the multi-parameter FWI of the 9C land data on a more complex 3D model, we conduct FWI considering a 3D model with a high velocity mud-filled channel (Figure 6). The true azimuth angle is designed to be less than $\pm 45^\circ$ so that fast axis is still closer to crossline direction. Because recovering ORT parameters requires wide-offset data, a huge 3D elastic ORT simulation is inevitable. In this study, the FWI is conducted by MPI-based time-domain elastic FWI code using a 4th-order finite difference on a staggered grid (Graves, 1996), in which the modeling process is

parallelized over shots and subdomains. The gradient direction is computed in the frequency domain (3-10Hz) after taking discrete Fourier transforms of the forward and backward wavefields on the fly (Sirgue et al., 2008). To reduce the complexity, we consider only the ideal parameters in Figure 6 with the other parameters set to 0. For an initial model, we use a smoothed VTI version of the model. We start FWI by inverting for v_{p1} , v_{s1} and ε_1 using the data from the vertical source. In Figure 5, the inline source is crucial to separate v_{p1} and v_{s1} . However, because we do not consider the η and γ parameters in this example, the vertical source is good enough to build v_{p1} and v_{s1} with reduced trade-off (Figure 5a). Then, we run FWI for ε_D and ϕ by matching the data from the crossline source, which is the most sensitive component to ε_D under our assumption ($\phi < \pm 45^\circ$). Thanks to the multi-stage FWI, subsurface rORT features are well estimated (Figure 7). However, depending on the true azimuth angle, a proper horizontal source component should be chosen. We will discuss our observations in more detail when we present it.

Figure 7: Inverted model with multi-stage approach.

Conclusions

We analyze the sensitivity of elastic orthorhombic parameters to 9C land data acquired on the surface. A simple doughnut-shaped model is designed to investigate the trade-off among parameters for the various source components. The trade-off analysis shows that 9C land data has the potential to resolve subsurface ORT anisotropy with less trade-off between the parameters. An efficient implementation is needed through a hierarchical strategy for only the parameters that the 9C data is sensitive to.

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