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ELECTROSTATICALLY TUNABLE NANOMECHANICAL SHALLOW ARCHES

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ABSTRACT

We report an analytical and experimental study on the tunability of in-plane doubly-clamped nanomechanical arches under varied DC bias conditions at room temperature. For this purpose, silicon based shallow arches are fabricated using standard e-beam lithography and surface nanomachining of a highly conductive device layer on a silicon-on-insulator (SOI) wafer. The experimental results show good agreement with the analytical results with a maximum tunability of 108.14% for 180 nm thick arch with a transduction gap of 1 μm between the beam and the driving/sensing electrodes. The high tunability of shallow arches paves the ways for highly tunable band pass filtering applications in high frequency range.

INTRODUCTION

Micro/Nanoelectromechanical systems (MEMS/NEMS) have received considerable attention in the recent decade for their interesting advantages for several applications, such as mass/gas sensing [1,2], memory [3], logic [4], filtering [5], communications [6], gyroscopes [7], energy harvesting [8], and signal processing [9]. Tunability of MEMS/NEMS structures is

highly desirable since it gives the flexibility of on-demand frequency selection for communication systems.

Many approaches have been presented to tune the resonance frequency of MEMS resonators by controlling the stiffness of the MEMS resonators with applied axial loads. Several approaches have been investigated to apply axial loads and tune the resonance frequency of a resonator, such as electrostatic [10], electromagnetic [11], and electrothermal [12]. A recent study has demonstrated large tunability of MEMS clamped-clamped straight micro beam resonators by controlling its axial load electrostatically [10] and electrothermally [12].

Since the early work of Nathanson [13], many researches have focused on the tunability of MEMS resonators by electrostatic means with applied DC bias voltage. One major drawback of the electrostatic tuning lies in its limitation of pull-in instability that leads to the collapse of the resonating structure with the stationary electrode especially when a relatively higher DC voltage is applied [14]. Typically, the electrostatic force is used to induce the softening effect in straight clamped-clamped beam that results a decrease in the resonance frequency of the MEMS structure [15], except few studies where they show that the resonance frequency could be

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tuned for higher values while increasing the DC voltage [16]. In a recent work, we have shown that a careful choice of geometric parameters, mainly the ratio between the transduction gap and the thickness, can result in considerable frequency tuning of an electrostatically driven clamped-clamped straight micro beam. Sazonova *et al.* [17] demonstrated experimentally the tunability (increase) of the resonance frequencies at nano scale for Carbon Nano Tube (CNT) resonators by increasing the DC bias voltage, which is considered a distinct feature of CNTs due to their high stiffness.

In this current work, we present an investigation into the tunability of NEMS shallow arches and study the effect of geometric parameters, mainly thickness of the arch and the transduction gap between the beam and the stationary electrodes under DC bias voltages. Further, the application of these arches, based on our analytical results, can be extended to tunable band pass filters for high frequency applications.

PROBLEM FORMULATION

The device under consideration, Fig. 1, consists of an initially curved clamped-clamped nano arch, which is curved due to the residual stresses from the fabrication process, actuated by a DC polarization voltage V_{DC} and an AC harmonic voltage of amplitude V_{AC} and frequency $\hat{\Omega}$, and is subjected to a viscous damping of coefficient $\hat{c}$. The initial shape $\hat{w}_0(x)$ is governed by (1) [18].

$$\hat{w}_0(\hat{x}) = -\frac{1}{2} \hat{b}_0 \left(1 - \cos(2\pi \frac{\hat{x}}{l}) \right) \quad (1)$$

Fig. 1: Schematic of an electrostatically actuated clamped-clamped shallow arch.

The nonlinear Euler-Bernoulli equation of motion governing the transverse deflection of the arch $\hat{w}(\hat{x}, \hat{t})$ is written as follows [18]:

$$\begin{aligned} \rho b h \frac{\partial^2 \hat{w}}{\partial \hat{t}^2} + \hat{c} \frac{\partial \hat{w}}{\partial \hat{t}} + E I \frac{\partial^4 \hat{w}}{\partial \hat{x}^4} = & \left(\frac{\partial^2 \hat{w}}{\partial \hat{x}^2} + \frac{d^2 \hat{w}_0}{d \hat{x}^2} \right) \\ & \cdot \left[\hat{N} + \frac{E A}{2 l} \int_0^l \left(\left(\frac{\partial \hat{w}}{\partial \hat{x}} \right)^2 + 2 \frac{\partial \hat{w}}{\partial \hat{x}} \frac{d \hat{w}_0}{d \hat{x}} \right) d \hat{x} \right] \\ & + \frac{1}{2} \epsilon b \frac{(V_{DC} + V_{AC} \cos(\hat{\Omega} \hat{t}))^2}{(d - \hat{w} - \hat{w}_0)^2} + \frac{1}{2} \epsilon b \frac{(V_{DC} - V_s)^2}{(d + \hat{w} + \hat{w}_0)^2} \end{aligned} \quad (2)$$

The nano arch is subjected to the following boundary conditions:

$$\hat{w}(0, \hat{t}) = \hat{w}(l, \hat{t}) = 0 \text{ and } \frac{\partial \hat{w}}{\partial \hat{x}} \bigg|_{(0, \hat{t})} = \frac{\partial \hat{w}}{\partial \hat{x}} \bigg|_{(l, \hat{t})} = 0 \quad (3)$$

where $\hat{x}$ is the position along the nano arch, $\hat{t}$ is time, l , b and h are, respectively, the length, depth, and thickness of the nano arch, ρ is the material density, $I = b h^3 / 12$ is the moment of inertia of the rectangular cross-section of area $A = b h$, E is Young's modulus, N is the residual axial load, d is the gap between the nano arch and the stationary electrode, and ϵ is the dielectric constant of the medium. The last term represents the electrostatic force from the sensing electrode. V_s is the DC voltage applied to the sensing electrode, which is taken in this work equal to zero.

For convenience, we introduce the nondimensional variables as below

$$\begin{aligned} w = \frac{\hat{w}}{d}; x = \frac{\hat{x}}{l}; t = \frac{\hat{t}}{T}; b_0 = \frac{\hat{b}_0}{d} \\ \text{and } w_0 = \frac{\hat{w}_0}{d} = \frac{1}{2} b_0 (1 - \cos(2\pi x)) \end{aligned} \quad (4)$$

where $T = \sqrt{\rho b h l^4 / E I}$ is a time scale. Substituting (4) into (2) and (3), we obtain the nondimensional equation of motion of the beam

$$\begin{aligned} \frac{\partial^2 w}{\partial t^2} + c \frac{\partial w}{\partial t} + \frac{\partial^4 w}{\partial x^4} = & \left(\frac{\partial^2 w}{\partial x^2} + \frac{d^2 w_0}{d x^2} \right) \\ & \cdot \left[N + \alpha_1 \int_0^1 \left(\left(\frac{\partial w}{\partial x} \right)^2 + 2 \frac{\partial w}{\partial x} \frac{d w_0}{d x} \right) dx \right] \\ & + \alpha_2 \frac{(V_{DC} + V_{AC} \cos(\Omega t))^2}{(1 - w - w_0)^2} - \alpha_2 \frac{V_{DC}^2}{(1 + w + w_0)^2} \end{aligned} \quad (5)$$

subjected to the nondimensional boundary conditions

$$w(0, t) = w(1, t) = 0 \text{ and } \frac{\partial w}{\partial x} \bigg|_{(0, t)} = \frac{\partial w}{\partial x} \bigg|_{(1, t)} = 0 \quad (6)$$

The nondimensional (except for α_2) parameters appearing in (5) are defined as below

$$\alpha_1 = 6 \left(\frac{d}{h} \right)^2; \alpha_2 = \frac{6 \epsilon l^4}{E h^3 d^3}; N = \frac{l^2}{E I} \hat{N}; c = \frac{l^4}{E I T} \hat{c} \text{ and } \Omega = T \hat{\Omega} \quad (7)$$

The deflection of the nano arch under the electrostatic force is split into a static deflection $w_s(x)$ due to V_{DC} and a small dynamic deflection as below

$$w(x, t) = w_s(x) + w_d(x, t) \quad (8)$$

To determine the static deflection $w_s(x)$ we set the time derivatives as well as the AC force equal to zero, which yields

$$\begin{aligned} \frac{d^4 w_s}{dx^4} = & \left(\frac{d^2 w_s}{dx^2} + \frac{d^2 w_0}{dx^2} \right) \left[N + \alpha_1 \int_0^1 \left(\frac{dw_s}{dx} \right)^2 \right. \\ & \left. + 2 \frac{dw_s}{dx} \frac{dw_0}{dx} \right] dx + \alpha_2 \frac{V_{DC}^2}{(1 - w_s - w_0)^2} \\ & - \alpha_2 \frac{V_{DC}^2}{(1 + w_s + w_0)^2} \end{aligned} \quad (9)$$

with the associated boundary conditions

$$w_s(0) = w_s(1) = 0 \text{ and } \left. \frac{dw_s}{dx} \right|_{x=0} = \left. \frac{dw_s}{dx} \right|_{x=1} = 0 \quad (10)$$

The linearized equation of motion describing the small dynamic behavior of the shallow arch around the deflected shape is derived by substituting equation (8) into equation (6) and dropping the terms representing the equilibrium position, equations (9), and the nonlinear terms. These yield

$$\begin{aligned} \frac{\partial^2 w_d}{\partial t^2} + \frac{\partial^4 w_d}{\partial x^4} = & \left[N + \alpha_1 \int_0^1 \left(\frac{dw_s}{dx} \right)^2 + 2 \frac{dw_s}{dx} \frac{dw_0}{dx} \right] dx \frac{\partial^2 w_d}{\partial x^2} \\ & + 2\alpha_1 \int_0^1 \left(\frac{dw_s}{dx} + \frac{dw_0}{dx} \right) \frac{\partial w_d}{\partial x} dx \left(\frac{d^2 w_s}{dx^2} + \frac{d^2 w_0}{dx^2} \right) \\ & + 2\alpha_2 \frac{(V_{DC} + V_{AC} \cos(\Omega t))^2}{(1 - w_s - w_0)^3} w_d + 2\alpha_2 \frac{V_{DC}^2}{(1 + w_s + w_0)^3} w_d \end{aligned} \quad (11)$$

with the associated boundary conditions

$$w_d(0, t) = w_d(1, t) = 0 \text{ and } \left. \frac{\partial w_d}{\partial x} \right|_{x=0, t} = \left. \frac{\partial w_d}{\partial x} \right|_{x=1, t} = 0 \quad (12)$$

To solve the static problem and equation (9), we refer to the Galerkin procedure in which we use the undamped linear mode shapes of a straight micro beam as basis functions. Therefore the deflection is expressed as

$$w_s(x) = \sum_{i=0}^n k_i \kappa_i(x) \quad (13)$$

where k_i ($i=0, 1, 2, \dots, n$) denotes the nondimensional modal coordinates and $\kappa_i(x)$ ($i=0, 1, 2, \dots, n$) denotes the undamped mode shapes of the unactuated shallow arch governed by

$$\frac{d^4 \kappa_i}{dx^4} - \omega_i^2 \kappa_i = 2\alpha_1 \frac{d^2 w_0}{dx^2} \int_0^1 \frac{d\kappa_i}{dx} \frac{dw_0}{dx} dx \quad (14)$$

with the associated boundary conditions:

$$\kappa_i(0) = \kappa_i(1) = 0 \text{ and } \left. \frac{d\kappa_i}{dx} \right|_{x=0} = \left. \frac{d\kappa_i}{dx} \right|_{x=1} = 0 \quad (15)$$

where ω_i represents the i^{th} natural frequency of the nano arch.

To determine the variation of the natural frequency of the nano arch under the DC polarization voltage, we resort also to the Galerkin discretization to solve the eigenvalue problem [18]. Toward this, we let:

$$w_d(x, t) = \sum_{i=0}^n q_i(t) \kappa_i(x) \quad (16)$$

where $q_i(t)$ ($i=0, 1, 2, \dots, n$) denotes the nondimensional modal coordinates and $\kappa_i(x)$ ($i=0, 1, 2, \dots, n$) denotes the unactuated shallow arch governed by equation (14).

Then, we substitute equation (16) into equation (11), multiplying the outcome by the mode shape κ_j and integrating over the arch domain (from 0 to 1), which yields the below equation [18]

$$\begin{aligned} \ddot{q}_j = & \left[2\alpha_1 \int_0^1 \kappa_j \frac{\partial^2 w_s}{\partial x^2} dx \int_0^1 \left(\sum_{i=0}^n q_i \kappa_i \frac{\partial w_s}{\partial x} \right) dx \right] + \left[\alpha_1 \int_0^1 \left(\frac{\partial w_s}{\partial x} \right)^2 dx \right] \\ & \cdot \int_0^1 \kappa_j \left(\sum_{i=0}^n q_i \kappa_i \right) dx + \int_0^1 \kappa_j \frac{2\alpha_2 V_{DC}^2}{(1 - w_s - w_0)^3} \left(\sum_{i=0}^n q_i \kappa_i \right) dx \\ & + \int_0^1 \kappa_j \frac{2\alpha_2 V_{DC}^2}{(1 + w_s + w_0)^3} \left(\sum_{i=0}^n q_i \kappa_i \right) dx \end{aligned} \quad (17)$$

Then, we compute the Jacobin of the system for each value of V_{DC} and find the corresponding resonance frequency by taking the square root of its eigenvalues.

FABRICATION AND MEASUREMENTS

The shallow arches with gap to thickness ratio of more than 4 are fabricated from a <100> highly- conductive boron doped Si device layer of silicon-on-insulator (SOI) wafer. The fabrication process employ standard e-beam lithography and surface nanomachining techniques. The main fabrication steps are illustrated in Fig. 2. The fabrication starts with cleaning of SOI wafer by dipping it in a piranha solution, heated at 120 °C, followed by a deionized (DI) water rinse to remove the organic residues. The native oxide is removed by using 1% HF solution until the surface turns to hydrophobic followed by a deionized (DI) water rinse and spin drying, Fig. 2a. The thickness of the device layer is reduced from 2 μm to 850 nm by thermal oxidation of the SOI wafer, Fig 2b, and then removing the thermally grown oxide using buffered oxide etch (BOE), Fig 2c. In order to achieve dimensions in nanometers range the e-beam lithography is employed to pattern 950 PMMA A4 positive tone e-beam resist, Fig. 2d. Reactive ion etching (RIE) is performed at -10 °C to separate the nano beam, anchoring pads, and the RF electrodes from the rest of wafer surface. This entire wafer surface will serve as ground during the characterization of fabricated devices, Fig. 2e. Finally, the HF vapor etch is done to have free standing nano beams by removing the underneath ~1 μm box layer, Fig. 2f. The beams are approximately 15 μm long (l), 850 nm deep (b), 130 nm and 180 nm thick (h) with 850 nm and 1 μm transduction gaps (d) between the beam and the driving/sensing electrodes.

Fig. 3 shows an SEM image of a shallow arch fabricated using the fabrication process flow of Fig. 2. The beam is approximately 15 μm long, 130 nm thick, and 850 nm deep with a transduction gap of 1 μm between the arch and the

driving/sensing electrodes. Note that the beam is slightly curved due to the residual stress in the device layer. Therefore, thinner beams are prone to relatively higher initial curvature than the thicker beams after HF vapor release. The observed initial rise ranges from ~ 100 nm to ~ 50 nm for beams of thickness 130 nm and 180 nm, respectively.

Fig. 2: Process flow for nano arches (a) Substrate cleaning and native oxide removal. (b) Thermal oxidation to thin down the device layer. (c) BOE treatment to remove the thermally grown oxide. (d) Patterning of e-beam resist (950 PMMA A4). (e) RIE to fabricate the nano arches and to separate the entire device structure from the ground plane. (f) HF vapor etching for free standing nano arches.

Fig. 3: An SEM image of an arch having a length (l) of 15 μm , depth (b) of 850 nm, and thickness (h) of 130 nm with a transduction gaps (d) of 1 μm between the beam and the driving/sensing electrodes.

The electrical characterization of the fabricated devices is performed in an ST-500 JANIS probe station, Fig. 4a, capable of measuring the arch response under controlled vacuum and varied chuck temperature conditions. The S_{21} transmission parameter, frequency response, is measured through Agilent E5071C network analyzer. Fig. 4b shows the photograph of the

nano arch under test inside the probe station at room temperature.

(a)

(b)

Fig. 4: (a) Photograph of ST-500 JANIS probe station having 2 RF and 4 DC probe arms. (b) Snapshot of a nano arch under test.

Fig. 5 shows schematic of the driving and sensing configuration of the resonator in its two-port. The arch is excited by applying a DC bias voltage on it and an ac signal from one of the port of the network analyzer (Agilent E5071C) to one of its electrodes, named as driving electrode. The motional current is sensed through the other electrode, named as sensing electrode, and is fed into a low-noise amplifier (LNA) with its output coupled to the other port of the network analyzer.

Fig. 5: A schematic of the measurement setup in its two-port driving and sensing configuration with DC polarization voltage applied to the nano arch.

RESULTS AND DISCUSSION

Fig. 6a and Fig. 6b show the variation of the resonance frequency of two nano arches of 15 μm length, 850 nm depth, and 130 nm thickness with the transduction gaps of 850 nm and 1 μm between the beam and the driving/sensing electrodes for varied DC polarization voltage. The figures compare the analytical and experimental data. The analytical results are obtained by considering the effective values as in Table I, which are slightly different from the designed nominal values due to the fabrication tolerances. Note that all the analytical results presented in this paper are obtained while considering the Young's modulus of 169 GPa and Poisson's ratio of 0.27 for silicon based nano beams.

Length (μm)	depth (nm)	Thickness (nm)	Gap (nm)	Initial rise (nm)
15	850	70	850	122
15	850	120	850	116
15	850	70	1000	123
15	850	120	1000	97.3

Table I. Effective dimensions used for obtaining the analytical results.

The experimental results show an increase in the resonance frequency of the arches with an increase in the DC bias voltage. This is due to the increase in the stiffness of the arch with the increasing axial force due to the applied voltage. The arches shows tunability of 77.12% and 85.51% under DC biased conditions with 850 nm and 1 μm transduction gap between the beam and the driving/sensing electrodes, respectively. A closer look towards the tunability of the two arches under study with gaps of 850 nm and 1 μm reveals an increase of 56.29% and 66.07%, respectively for DC bias changes from 40 V to 110 V. This is attributed to the dominating effect of mid-plane stretching over the softening effect of the electrostatic force for large gaps, as previously found in [10] and [16].

Similarly, the resonance frequencies of other two nano arches of 15 μm long, 850 nm deep, and 180 nm thick with transduction gaps of 850 nm and 1 μm are found to increase with increasing DC polarization voltages, Fig. 7a and Fig. 7b. The increase in resonance frequency of these shallow arches is also due to the increase in the stiffness of the arch with increasing axial force due to applied DC voltage showing a tunability of 73.05% and 108.14% with 850 nm and 1 μm transduction gap between the beam and the driving/sensing electrodes, respectively. Tunability of 34.71% and 45.14% are achieved for the arches with 850 nm and 1 μm transduction gap, respectively, with DC bias changes from 40 V to 110 V again reflects the dominating effect of mid-plane stretching over electrostatic force for large gaps.

Therefore, thicker arches can achieve higher tunability compared to the thin arches due to the relatively higher pull-in voltages. However, the thinner beams show higher tunability per unit increase in the DC polarization voltage due to the combined effect of increasing arch stiffness and dominating effect of mid-plane stretching over electrostatic force for large

gaps. The practical realization of these arches open up the possibility to use them for tunable band pass filtering for TV tuner applications with the flexibility of adjustable bandwidth by coupling the nanomechanical arches.

Fig. 6: The DC bias voltage versus resonance frequency for shallow arches of 15 μm length, 850 nm depth and 130 nm thickness with a transduction gap of (a) 850 nm (b) 1 μm .

Fig. 7: The DC bias voltage versus resonance frequency for shallow arches of 15 μm length, 850 nm depth and 180 nm thickness with a transduction gap of (a) 850 nm (b) 1 μm .

CONCLUSIONS

In conclusions, an analytical and experimental study is presented on the tunability of in-plane doubly-clamped nanomechanical arches under varied DC bias conditions at room temperature. The experimental results show good agreement with that of the analytical ones for the arches with carefully designed gap to thickness ratios. Therefore, the arches show high tunability by electrostatic actuation. The high tunability of shallow arch open up the possibilities for tunable band pass filtering application in high frequency range (3-30 MHz).

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