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Automatic Wave Equation Migration Velocity Analysis by Focusing Subsurface Virtual Sources

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Running head: **WEMVA by Focusing Virtual Sources**

ABSTRACT

Macro velocity model building is important for subsequent pre-stack depth migration and full waveform inversion. Wave equation migration velocity analysis (WEMVA) utilizes the band-limited waveform to invert for the velocity. Normally, inversion would be implemented by focusing the subsurface offset common image gathers (SOCIGs). We re-examine this concept with a different perspective : In subsurface offset domain , using extended Born modeling, the recorded data can be considered as invariant with respect to the perturbation of the position of the virtual sources and velocity at the same time. A linear system connecting the perturbation of the position of those virtual sources and velocity is derived and solved subsequently by Conjugate Gradient method. In theory, the perturbation of the position of the virtual sources is given by the Rytov approximation. Thus, compared to the Born approximation, it relaxes the dependency on amplitude and makes the proposed

method more applicable for real data. We demonstrate the effectiveness of the approach by applying the proposed method on both isotropic and anisotropic VTI synthetic data. A real dataset example verifies the robustness of the proposed method.

INTRODUCTION

Macro velocity model building is important in seismic data processing. Prestack depth migration relies on a smooth kinematically accurate velocity model to image the subsurface and Full Waveform Inversion needs a correct background velocity to avoid cycle-skipping and to converge as well (Virieux and Operto, 2009; Alkhalifah, 2016a). Although the seismic migration method has evolved from relying on ray based methods such as Kirchhoff migration to waveform based method like wave equation migration and reverse time migration (Etgen et al., 2009), many of the methods for estimating the macro velocity model in the industry are still based on ray tracing (Jones et al., 2008).

Wave equation migration velocity analysis utilizes the wavefiled as a carrier for velocity information. Although a few variations of the WEMVA methods have been proposed over the years, they share the same main features of back propagating the residual images for velocity updating and the difference lies in how to compute such residual images. The residual image can be constructed in the image domain by a penalty function (Shen and Symes, 2008), warping (Perrone et al., 2014) or horizontal concentration (Shen and Symes, 2015). Among these methods, Differential Semblance Optimization (DSO) (Symes, 2008) is the most popular, as it relies on expanding the image space and penalizing energy residing in this nonphysical extension. However, the gradient of the DSO operator is severely contaminated by artifacts. Recent developments (Hou and Symes, 2015, 2017; Chauris and Cocher, 2017) demonstrate that an inversion rather than migration for the reflectivity can reduce such artifacts. This demonstrates that the DSO approach is sensitive to the amplitude of the resulting image, requiring "true amplitude" extended imaging.

Anisotropic media assumptions provide more realistic representations of the subsurface

where complex geological environment exists. Successfully building the anisotropic model is essential to properly image the subsurface in such media. The vertical transverse isotropic (VTI) (Alkhalifah, 1998) model is representative of the first-order anisotropy present in the subsurface. Several authors have used WEMVA in VTI media to build an anisotropic model based on the DSO method (Li et al., 2014; Weibull and Arntsen, 2014).

In this paper, we re-examine WEMVA method with a different perspective. We specifically represent the energy in the extended image as virtual sources in the subsurface offset domain, and by considering the recorded data to be stationary, we build a linear system connecting perturbations in the position of the virtual sources from zero offset to perturbations in the background velocity model. The derived linear system can be efficiently solved using conjugate gradient methods. The proposed approach does not compute the residual image in the image domain as other WEMVA methods. Instead it has a demigration step induced by perturbing the subsurface offset virtual sources (SOVS), and thus, the residuals are actually in the data space. By proper parameterization of the anisotropic parameters (Alkhalifah, 2016b) combined with an appropriate preconditioning of the inversion process, we also extend the proposed method to update the anisotropic parameters of VTI media.

In the following, we first discuss the theory and then show how to incorporate anisotropic parameters in the inversion. For both isotropic and anisotropic migration velocity analysis, we present synthetic and real data examples to verify the proposed method.

THEORY

Isotropic WEMVA

The extended imaging condition (Sava and Fomel, 2003) provides the subsurface offset common image gathers (SOCIGs) $I(\mathbf{x}, \mathbf{h})$ as follows

$$I(\mathbf{x}, \mathbf{h}) = \int d\mathbf{x}_s d\omega S^*(\mathbf{x} - \mathbf{h}; \mathbf{x}_s, \omega) R(\mathbf{x} + \mathbf{h}; \mathbf{x}_s, \omega), \quad (1)$$

where $*$ represent the complex conjugate operation, $\mathbf{x} = (x, y, z)$ is Cartesian coordinates, $\mathbf{x}_s$ denotes location of the sources of the seismic experiments, and ω is the frequency. The functions S and R correspond to the source and receiver wavefields, respectively. The subsurface half-offset is given by $\mathbf{h}$: for 2D, it has one component and corresponds to a shift in the x -direction, $\mathbf{h} = (h_x, 0, 0)$; while for 3D, it will contain two components with respect to both shifts in x and y directions: $\mathbf{h} = (h_x, h_y, 0)$.

Considering the adjoint of equation (1), we obtain the extended Born modeling or demigration (Hou and Symes, 2015, 2017; Chauris and Cocher, 2017) as :

$$d(\mathbf{x}_r; \mathbf{x}_s, \omega) = \int d\mathbf{x} d\mathbf{h} G(\mathbf{x} - \mathbf{h}; \mathbf{x}_s, \omega) I(\mathbf{x}, \mathbf{h}) G(\mathbf{x}_r; \mathbf{x} + \mathbf{h}, \omega), \quad (2)$$

where $G(\mathbf{x}_1; \mathbf{x}_2, \omega)$ denotes the Greens function from position $\mathbf{x}_2$ to $\mathbf{x}_1$, and $d(\mathbf{x}_r; \mathbf{x}_s, \omega)$ denotes the record at receiver position $\mathbf{x}_r$ due to source at position $\mathbf{x}_s$. Equation (2) can be regarded as a virtual source $f_l(\mathbf{x}, \mathbf{h}, \omega)$ at location $\mathbf{x} - \mathbf{h}$ with source function

$$f_l(\mathbf{x}, \mathbf{h}, \omega) = G(\mathbf{x} - \mathbf{h}; \mathbf{x}_s, \omega) I(\mathbf{x}, \mathbf{h}), \quad (3)$$

where l is the position for the virtual source

$$l(\mathbf{x}, \mathbf{h}) = \mathbf{x} - \mathbf{h}. \quad (4)$$

The introduction of virtual sources in subsurface offset and perturbation of its position constitutes the essential parts of the proposed method. Alkhalifah (2010) shows how the perturbation wavefield relates to the perturbation of source position, in Appendix A , we have a review of the relevant theory.

The record $d(\mathbf{x}_r; \mathbf{x}_s, \omega)$ can be considered invariant with respect to perturbations in the position of the virtual sources and an equivalent perturbation in the velocity model. Considering a perturbation in equation (2) for the velocity $v(\mathbf{x})$ and virtual source position $l(\mathbf{x}, \mathbf{h})$, we end up with:

$$\begin{aligned} 0 = \delta d(\mathbf{x}_r; \mathbf{x}_s, \omega) &= \delta d_l(\mathbf{x}_r; \mathbf{x}_s, \omega) + \delta d_v(\mathbf{x}_r; \mathbf{x}_s, \omega) \\ &= \int d\mathbf{x} d\mathbf{h} \frac{\partial d(\mathbf{x}_r; \mathbf{x}_s, \omega)}{\partial l(\mathbf{x}, \mathbf{h})} \cdot \delta l(\mathbf{x}, \mathbf{h}) \\ &+ \int d\mathbf{x} d\mathbf{h} \left[\frac{\partial G(\mathbf{x} - \mathbf{h}; \mathbf{x}_s, \omega)}{\partial v(\mathbf{x})} \delta v(\mathbf{x}) I(\mathbf{x}, \mathbf{h}) G(\mathbf{x}_r; \mathbf{x} + \mathbf{h}, \omega) + G(\mathbf{x} - \mathbf{h}; \mathbf{x}_s, \omega) I(\mathbf{x}, \mathbf{h}) \frac{\partial G(\mathbf{x}_r, \mathbf{x} + \mathbf{h}, \omega)}{\partial v(\mathbf{x})} \delta v(\mathbf{x}) \right], \end{aligned} \quad (5)$$

where δd_l and δd_v corresponds to the perturbation of the record with respect to virtual source position and the velocity respectively.

We can rewrite the above equation as

$$\delta d_v = K_v \delta v = -\delta d_l, \quad (6)$$

where K_v is the sensitivity kernel for perturbation in the velocity. In order to use Conjugate Gradient (CG) method (Shewchuk et al., 1994) to solve this equations, we apply the adjoint of the operator K_v and obtain the normal equations for CG

$$K_v^* K_v \delta v = -K_v^* \delta d_l \quad (7)$$

In appendix B, we give detailed derivations of the operator K_v and K_v^* .

For MVA, focusing in the SOCIGs can also be explained as moving the position of the virtual source from $\mathbf{x} - \mathbf{h}$ to $\mathbf{x}$, i.e., the perturbation of the source position is the negative value of the subsurface halt-offset $\mathbf{h}$:

$$\delta l(\mathbf{x}, \mathbf{h}) = -\mathbf{h}. \quad (8)$$

Here we assumed that when the velocity is changing toward to the correct one, the position of the virtual source shifts toward zero offset, and like MVA, we expect smoothed velocity perturbations, and assume a smooth background. However for a more complex medium like having a small Gaussian lens with a high velocity perturbation, the behavior in the evolution of the virtual source position can be complicated resulting in the divergence of the approach. According to equation (A-14), δd_l can be computed as

$$\delta d_l = \int d\mathbf{x} d\mathbf{h} f_l(\mathbf{x}, \mathbf{h}, \omega) \delta l \cdot \nabla_{\mathbf{h}} G(\mathbf{x}_r; \mathbf{x} + \mathbf{h}, \omega) \quad (9)$$

Anisotropic WEMVA

Extending the proposed method to VTI media is straightforward. We need to modify equation 7 to include the anisotropic behavior of waves and the additional anisotropic parameters needed to describe the model:

$$\delta d_m(x_r; x_s; \omega) = K_m \delta \mathbf{m} = -\delta d_l. \quad (10)$$

The computation of δd_l is the same as for the isotropic case defined in equation (9). In this study, we use the one-way acoustic anisotropic equation (Alkhalifah, 1997) for wavefield and sensitivity kernel computation. The anisotropic parameters used here are the vertical velocity v_0 , NMO velocity v_{nmo} and η (Alkhalifah, 1998), i.e. $\mathbf{m} = (v_0, v_{nmo}, \eta)$. This parameterization results in reduced trade-off between different parameters (Alkhalifah, 2016b) in MVA.

For the simultaneous inversion of the anisotropic parameters, as they have different scales, we apply a preconditioning to $\delta \mathbf{m}$ in equation (10) as follows:

$$K_m P \delta \mathbf{m} = -\delta d_1, \quad (11)$$

where P properly scales the anisotropic parameters to guarantee all parameters can get updated simultaneously in the inversion process. In both synthetic and real data examples, the scaling parameters are chosen as $P_{v_{nmo}} : P_\eta = 2000 : 1$.

IMPLEMENTATION OF THE METHOD

As the implementation of the isotropic and anisotropic versions of the proposed method are similar, we restrict the discussion to the isotropic WEMVA. Implementation of the proposed method includes inner and outer loop iterations. The outer loop iteration provides the gradient of the velocity perturbation by solving the linear system defined in equation 7. Within every outer loop iteration, inner loop iterations are used in a line search in order to determine the optimal step length, and we use for that the misfit function defined by the DSO approach, i.e. we seek a step length which minimizes the DSO misfit function. Note, that the DSO misfit function is only used for step length optimization. The gradient computation is based on the virtual source method not by adjoint analysis of the DSO misfit function. In the outer loop, and in order to solve the linear system defined by equation 7, we need to perform demigration using equation (9) to obtain the residual data, which is the RHS of linear system. —the linear system is solved using conjugate gradient methods. We proceed with the inner and outer loops until a predefined maximum iterations number is met or the residual data becomes relative small compared to the initial model.

EXAMPLES

In this section, we show the results of applying the proposed approach on synthetic isotropic and anisotropic data. These tests are meant to demonstrate the approach features and limitations. We also show results from field data to demonstrate robustness.

Tests of isotropic WEMVA

The first example corresponds to a layered model. The size of the model is 4 km in depth and 6 km laterally. We use Born modeling to create the data and the maximum offset is 6km. In Figure 1a, the red line is the true velocity. The black line corresponds to a highly smoothed initial velocity. The blue line is the inverted velocity after 10 nonlinear iterations. We do not show here that actually 2-3 iterations would give a relatively good result and flatten the gathers for this synthetic example, however more iterations improves the inverted model. Note that the inverted velocity has reasonably high resolution and is generally consistent with the true model, which is mainly due to the large number of reflectors in the model in which a Gauss-Newton solution of the linear system would boost the high frequency component. Figures 1b and 1c are the SOCIGs for the initial and the inverted velocity model, respectively. From this result, it is clear that after the WEMVA updating, SOCIGs becomes well focused.

The second example is a marine real dataset from offshore Australia. The offset range is from 160 m to 8200 m. The initial velocity is given in Figure 2a. During the inversion, we low pass filtered the record data up to 30Hz and the we use a 15Hz peak frequency Ricker wavelet for the source. The image and the angle domain common image gathers (ADCIG) are shown in Figures 3a and 4a, respectively. We can see that the ADCIGs has residual

moveouts (RMOs) indicating the inaccuracy of the initial velocity and the image is not well focused as well. Figure 2b shows the inverted velocity after 2 nonlinear iterations. Figure 3b and 4b are the image and ADCIGs produced with the inverted velocity. Compared to the image produced using initial velocity, the resulting image from our method is far more focused and the ADCIGs is reasonably flat in most regions.

Tests of anisotropic WEMVA

Like the isotropic case, we first test the approach on an anisotropic synthetic layered model. The model size is 7.5 km in depth and 21 km laterally. The maximum offset is 10km. In the inversion, we set $v_0 = v_{nmo}$ and invert for v_{nmo} and η , simultaneously. Figures 5a and 5b show the result for the inverted anisotropic parameters after 20 nonlinear iterations. From these results, we see that v_{nmo} is well resolved and η captures the main features in the true model. Figures 6a and 6b are the SOCIGs for the initial and inverted models, respectively. As expected, the gathers are far more focused to the near subsurface offset after our anisotropic WEMVA.

The second example corresponds to the Volve OBC field data set. The maximum offset is about 5km. During the inversion, we set $v_0 = v_{nmo}$ and invert for v_{nmo} and η . Figures 7 and 8 are the initial and inverted anisotropic parameters using 2 nonlinear iterations. Figures ?? and ?? are the image using the initial and inverted models, respectively. It is obvious that the image is better focused using the inverted model, and this, in particular, is clear in the area around 2 km depth. When we look at the angle domain common image gathers (ADCIGs), we see better flattening of the image gathers using the inverted model (Figure ??) compared with the initial model (Figure ??), especially for the shallow part.

For the deeper part, we may see some RMO for example around 3km. We expect it is due to the relative small offset to depth ratios there.

CONCLUSIONS

We proposed a method for macro velocity updating by focusing the subsurface offset virtual sources. Unlike other WEMVA approaches, the proposed method does not require computing the residual image and instead it utilizes a demigration step to compute the residual in data domain. A linear system of equations relating the residual data to velocity perturbation is constructed and a conjugate gradient method is utilized to obtain the solution. The advantage of the proposed method is provided by two factors: the measure of focussing is provided by the virtual source representation rather than penalizing the amplitude, which renders our approach to be extended image amplitude insensitive. In other words, and in theory, the DSO method is based on the Born approximation while the virtual source method is based on the Rytov approximation, which can relax the requirement for true amplitude imaging. The second factor is that we include a demigration step and the residual is measured in the data domain. This would provide more robustness for application on real data. For example, we can apply denoising and demultiple in the image domain prior to demigration to obtain a cleaner residual to extract the gradient. As shown in the real data example, we also extend this method to anisotropic media, specifically VTI. We use a parameterization that provides reduced trade off between the different parameters and use preconditioning to scale the updates of the anisotropic parameters properly, as they are updated simultaneously. Both synthetic and real data examples demonstrate the effectiveness and robustness of the proposed method.

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APPENDIX A

WAVEFIELD PERTURBATION FOR SOURCE PERTURBATION

The acoustic wave equation in frequency domain can be expressed as

$$-\frac{\omega^2}{v^2(\mathbf{x})}u(\mathbf{x},\omega) = \nabla^2 u(\mathbf{x},\omega) + f(\omega)\delta(\mathbf{x} - \mathbf{x}_s), \quad (\text{A-1})$$

where ω is the frequency, $f(\omega)$ is source wavelet, $v(\mathbf{x})$ is velocity and $u(\mathbf{x},\omega)$ is the wavefield.

A perturbation $\delta\mathbf{x}_s$ in the source position leads to wavefield perturbation δu . We denote the wavefield computed with source at location $\mathbf{x}_s + \delta\mathbf{x}_s$ as $u_s(\mathbf{x},\omega)$ and we have

$$u_s(\mathbf{x},\omega) = u(\mathbf{x},\omega) + \delta u(\mathbf{x},\omega). \quad (\text{A-2})$$

Alkhalifah (2010) pointed out that the waves generated at the location $\mathbf{x}_s + \delta\mathbf{x}_s$ with velocity model $v(\mathbf{x})$ and received at receiver $\mathbf{x}_r$ are identical to the waves generated at location $\mathbf{x}_s$ with velocity model $v(\mathbf{x} + \delta\mathbf{x}_s)$ and received at $\mathbf{x}_r - \delta\mathbf{x}_s$, thus considering the wavefield

$q(\mathbf{x}, \omega)$ calculated by

$$-\frac{\omega^2}{v^2(\mathbf{x} + \delta\mathbf{x}_s)}q(\mathbf{x}, \omega) = \nabla^2 q(\mathbf{x}, \omega) + f(\omega)\delta(\mathbf{x} - \mathbf{x}_s). \quad (\text{A-3})$$

It can be related to $u_s(\mathbf{x}, \omega)$ by

$$u_s(\mathbf{x}, \omega) = q(\mathbf{x} - \delta\mathbf{x}_s, \omega) \approx q(\mathbf{x}, \omega) - \delta\mathbf{x}_s \cdot \nabla q(\mathbf{x}, \omega) \approx q(\mathbf{x}, \omega) - \delta\mathbf{x}_s \cdot \nabla u(\mathbf{x}, \omega). \quad (\text{A-4})$$

Hence we have:

$$q(\mathbf{x}, \omega) = u_s(\mathbf{x}, \omega) + \delta\mathbf{x}_s \cdot \nabla u(\mathbf{x}, \omega) \approx u(\mathbf{x}, \omega) + \delta u(\mathbf{x}, \omega) + \delta\mathbf{x}_s \cdot \nabla u(\mathbf{x}, \omega). \quad (\text{A-5})$$

Inserting $v(\mathbf{x} + \delta\mathbf{x}_s) \approx v(\mathbf{x}) + \delta\mathbf{x}_s \cdot \nabla v(\mathbf{x})$ in to equation (A-3) and using Taylor expansion, we obtain

$$-\frac{\omega^2}{v^2(\mathbf{x})}q(\mathbf{x}, \omega) = \nabla^2 q(\mathbf{x}, \omega) + f(\omega)\delta(\mathbf{x} - \mathbf{x}_s) - \frac{2\omega^2\delta\mathbf{x}_s \cdot \nabla v(\mathbf{x})}{v^3(\mathbf{x})}q(\mathbf{x}, \omega). \quad (\text{A-6})$$

Substitution of equation (A-5) into equation (A-6) and then subtracting with equation (A-1), we obtain the perturbation of the wavefield $\delta u(\mathbf{x})$

$$-\frac{\omega^2}{v^2(\mathbf{x})}\delta u(\mathbf{x}, \omega) = \nabla^2 \delta u(\mathbf{x}, \omega) + \delta\mathbf{x}_s \cdot \left[\nabla \nabla^2 u(\mathbf{x}, \omega) + \frac{\omega^2}{v^2(\mathbf{x})} \nabla u(\mathbf{x}, \omega) - \frac{2\omega^2 \nabla v(\mathbf{x})}{v^3(\mathbf{x})} u(\mathbf{x}, \omega) \right]. \quad (\text{A-7})$$

It can also be expressed using sensitivity kernel

$$\delta u(\mathbf{x}_r, \mathbf{x}_s, \omega) = \int \delta\mathbf{x}_s \cdot K(\mathbf{x}, \omega, \mathbf{x}_r, \mathbf{x}_s) d\mathbf{x}, \quad (\text{A-8})$$

where

$$K(\mathbf{x}, \omega, \mathbf{x}_r, \mathbf{x}_s) = \frac{\partial u(\mathbf{x}_r, \mathbf{x}_s, \omega)}{\partial \mathbf{x}_s} = K_1(\mathbf{x}, \omega, \mathbf{x}_r, \mathbf{x}_s) + K_2(\mathbf{x}, \omega, \mathbf{x}_r, \mathbf{x}_s), \quad (\text{A-9})$$

and

$$K_1(\mathbf{x}, \omega, \mathbf{x}_r, \mathbf{x}_s) = \left[\nabla \nabla^2 u(\mathbf{x}, \omega) + \frac{\omega^2}{v^2(\mathbf{x})} \nabla u(\mathbf{x}, \omega) \right] G(\mathbf{x}_r; \mathbf{x}, \omega), \quad (\text{A-10})$$

$$K_2(\mathbf{x}, \omega, \mathbf{x}_r, \mathbf{x}_s) = -\frac{2\omega^2 \nabla v(\mathbf{x})}{v^3(\mathbf{x})} u(\mathbf{x}, \omega) G(\mathbf{x}_r; \mathbf{x}, \omega), \quad (\text{A-11})$$

where K_1 is for the spatial variation of the wavefield and it accounts for the geometrical perturbation of the source position while K_2 mainly accounts for the effects for the spatial variation of the velocity around the source. For MVA, the velocity is assumed to be smooth, we can safely omit the part of K_2 in the demigration process.

Further considering the term in the parentheses for K_1 , it can be approximated as

$$\nabla \nabla^2 u(\mathbf{x}, \omega) + \frac{\omega^2}{v^2(\mathbf{x})} \nabla u(\mathbf{x}, \omega) \approx -f(\omega) \nabla \delta(\mathbf{x} - \mathbf{x}_s). \quad (\text{A-12})$$

The resulted wavefield perturbation can be calculated as

$$\delta u(\mathbf{x}_r, \mathbf{x}_s, \omega) = - \int \delta \mathbf{x}_s \cdot f(\omega) \nabla \delta(\mathbf{x} - \mathbf{x}_s) G(\mathbf{x}_r, \mathbf{x}, \omega) d\mathbf{x}. \quad (\text{A-13})$$

Using the shifting property of the Dirac delta function, we have

$$\delta u(\mathbf{x}_r, \mathbf{x}_s, \omega) = f(\omega) \delta \mathbf{x}_s \cdot \nabla_{\mathbf{x}_s} G(\mathbf{x}_r, \mathbf{x}_s, \omega). \quad (\text{A-14})$$

APPENDIX B

WAVEFIELD PERTURBATION FOR VELOCITY PERTURBATION

This appendix is supplement for explanation of the velocity sensitivity term of K_v in equation 7. By Born approximation, we rewrite the second term of equation 5 as

$$\begin{aligned} \delta d_v(\mathbf{x}_r; \mathbf{x}_s, \omega) &= \int d\mathbf{x} d\mathbf{h} K_v(\mathbf{x}, \mathbf{h}, \omega, \mathbf{x}_r, \mathbf{x}_s) \delta v(\mathbf{x}) \\ &= \int d\mathbf{x} d\mathbf{h} \left[\frac{-\omega^2 G(\mathbf{x} - \mathbf{h}; \mathbf{x}_s, \omega)}{v^3(\mathbf{x})} I(\mathbf{x}, \mathbf{h}) G(\mathbf{x}_r; \mathbf{x} + \mathbf{h}, \omega) \right. \\ &\quad \left. + G(\mathbf{x} - \mathbf{h}; \mathbf{x}_s, \omega) I(\mathbf{x}, \mathbf{h}) \frac{-\omega^2 G(\mathbf{x}_r; \mathbf{x} + \mathbf{h}, \omega)}{v^3(\mathbf{x})} \right] \delta v(\mathbf{x}) \end{aligned} \quad (\text{B-1})$$

By adjoint analysis, we can have can obtain the gradient for the velocity perturbation given the residual data :

$$\begin{aligned}\delta\tilde{v}(\mathbf{x}) &= \int d\mathbf{h} K_v^*(\mathbf{h}, \omega, \mathbf{x}_r, \mathbf{x}_s) \delta d_v(\mathbf{x}_r; \mathbf{x}_s, \omega) \\ &= \int d\mathbf{h} \left[\frac{-\omega^2 G^*(\mathbf{x} - \mathbf{h}, \mathbf{x}_s, \omega)}{v^3(\mathbf{x})} I(\mathbf{x}, \mathbf{h}) G^*(\mathbf{x}_r; \mathbf{x} + \mathbf{h}, \omega) \right. \\ &\quad \left. + G^*(\mathbf{x} - \mathbf{h}; \mathbf{x}_s, \omega) I(\mathbf{x}, \mathbf{h}) \frac{-\omega^2 G^*(\mathbf{x}_r; \mathbf{x} + \mathbf{h}, \omega)}{v^3(\mathbf{x})} \right] \delta d_v(\mathbf{x}_r; \mathbf{x}_s, \omega)\end{aligned}\tag{B-2}$$

Using reciprocity of the Green function $G(\mathbf{a}; \mathbf{b}, \omega) = G^*(\mathbf{b}, \mathbf{a}, \omega)$, we can reformulate equation B-2 as

$$\begin{aligned}\delta\tilde{v}(\mathbf{x}) &= \int d\mathbf{h} K_v^*(\mathbf{h}, \omega, \mathbf{x}_r, \mathbf{x}_s) \delta d_v(\mathbf{x}_r; \mathbf{x}_s, \omega) \\ &= \int d\mathbf{h} \left[\frac{-\omega^2 G^*(\mathbf{x} - \mathbf{h}, \mathbf{x}_s, \omega)}{v^3(\mathbf{x})} I(\mathbf{x}, \mathbf{h}) G(\mathbf{x} + \mathbf{h}; \mathbf{x}_r, \omega) \right. \\ &\quad \left. + G^*(\mathbf{x} - \mathbf{h}; \mathbf{x}_s, \omega) I(\mathbf{x}, \mathbf{h}) \frac{-\omega^2 G(\mathbf{x} + \mathbf{h}; \mathbf{x}_r, \omega)}{v^3(\mathbf{x})} \right] \delta d_v(\mathbf{x}_r; \mathbf{x}_s, \omega)\end{aligned}\tag{B-3}$$

With equation B-1 and B-3, we can evaluate the sensitivity kernel K_v and its adjoint K_v^* .

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1a. WEMVA result of the layered model: (a) velocity;

197x356mm (300 x 300 DPI)

1b. WEMVA result of the layered model:
(b) SOCIGs using the initial velocity;

197x356mm (300 x 300 DPI)

1c. WEMVA result of the layered model:
(c) SOCIGs using the inverted velocity.

197x356mm (300 x 300 DPI)

2a. WEMVA result of the marine dataset
(a) initial velocity;

111x83mm (300 x 300 DPI)

2b. WEMVA result of the marine dataset
(b) inverted velocity.

111x83mm (300 x 300 DPI)

3a. WEMVA result of the marine dataset
(a) image obtained using the initial velocity;

111x82mm (300 x 300 DPI)

3b. WEMVA result of the marine dataset
(b) image obtained using the inverted velocity.

111x83mm (300 x 300 DPI)

4a. WEMVA result of the marine dataset
(a) ADCIGs using the initial velocity;

110x83mm (300 x 300 DPI)

4b. WEMVA result of the marine dataset
(b) ADCIGs using the inverted velocity.

110x83mm (300 x 300 DPI)

5a. Anisotropic WEMVA result of the layered model:
(a) NMO velocity;

317x635mm (300 x 300 DPI)

5b. Anisotropic WEMVA result of the layered model:
(b) η .

317x635mm (300 x 300 DPI)

6a. Anisotropic WEMVA result of the layered model:
 (a) SOCIGs using the initial model;

317x635mm (300 x 300 DPI)

6b. Anisotropic WEMVA result of the layered model:
(b) SOCIGs using the inverted model.

317x635mm (300 x 300 DPI)

7a. Anisotropic WEMVA result of the Volve data:
 (a) initial NMO velocity;

111x83mm (300 x 300 DPI)

7b. Anisotropic WEMVA result of the Volve data:
(b) inverted NMO velocity.

111x83mm (300 x 300 DPI)

8a. Anisotropic WEMVA result of the Volve data:
 (a) initial η ;

111x83mm (300 x 300 DPI)

8b. Anisotropic WEMVA result of the Volve data:
(b) inverted η .

111x83mm (300 x 300 DPI)

9a. Anisotropic WEMVA result of the Volve data:
(a) image obtained using the initial model ;

111x83mm (300 x 300 DPI)

9b. Anisotropic WEMVA result of the Volve data:
(b) image obtained using the inverted model.

111x83mm (300 x 300 DPI)

10a. Anisotropic WEMVA result of the Volve data:
(a) ADCIGs using the initial model;

111x83mm (300 x 300 DPI)

10b. Anisotropic WEMVA result of the Volve data:
(b) ADCIGs using the inverted model.

111x83mm (300 x 300 DPI)