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A Simple Physics-Based Model Predicts Oil Production from Thousands of Horizontal Wells in Shales

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Summary

Over the last six years, crude oil production from shales¹ and ultra-deep GOM² in the United States has accounted for most of the net increase of global oil production. Therefore, it is important to have a good predictive model of oil production and ultimate recovery in shale wells. Here we introduce a simple model of producing oil and solution gas from the horizontal hydrofractured wells. This model is consistent with the basic physics and geometry of the extraction process. We then apply our model thousands of wells in the Eagle Ford shale.

Given well geometry, we obtain a one-dimensional nonlinear pressure diffusion equation that governs flow of mostly oil and solution gas. In principle, solutions of this equation depend on many parameters, but in practice and within a given oil shale, all but three can be fixed at typical values, leading to a nonlinear diffusion problem we linearize and solve exactly with a scaling “type” curve. After the initial 1-3 months of the generally unstable production, the scaled production rate declines as one over the square root of time early on and later it declines exponentially. The three governing parameters are the mean cumulative gas-oil ratio, GOR, the mass of saturated oil in place, $\mathcal{M}$, and the characteristic time of pressure interference between each pair of consecutive hydrofractures, τ . This time depends on the effective formation permeability to oil, porosity, oil saturation, and the overall reservoir compressibility. GOR influences ultimate recovery, while the other two parameters determine where on the master curve production from a given well falls, depending on the $\mathcal{M}$, and how it stretches or shrinks, depending on the τ . The distribution of τ also provides constraints on infill well locations. We implemented our automatic fitting procedure on a PC.

In February 2017, there were 13,057 physical oil wells in the Eagle Ford shale. However, there were only 4,734 unallocated well records because of the peculiar reporting requirements in Texas, explained in the paper. This means that up to 71 physical wells can be reported as one unallocated lease production record in the Eagle Ford. Since we are only interested in black oil horizontal wells, we have selected 2,611 wells with at least 6 months of oil production, GOR less than 2500 scf/stb and liquid gravity less than 40⁰ API. In practice, we match the production data for each well to a dimensionless

¹Strictly speaking, the calcareous and silicious mudrock reservoirs in the Bakken, Eagle Ford, Permian, etc.

²Oil & gas production from federal leases on the continental shelf in the Gulf of Mexico, Tainter and Patzek (2011).

master curve with the recovery factor, $RF = N_p/\mathcal{M}$, as the y-axis and the dimensionless time, t/τ , as the x-axis. The match relies on adjusting the unknown parameters $\mathcal{M}$ and τ . Here N_p is cumulative mass production of oil and t is elapsed time on production in months. 429 selected wells were still in the early time flow regime with $t/\tau < 1$. In the remaining 2,182 wells, hydrofractures started pressure interfering in less than 46 months on production. Our scaling of production of the 2,611 black oil wells and back allocating production to the corresponding physical wells gives the total EUR close to 440 million bbl of oil.

Compared with reservoir simulation, the scaling curve method is more robust for thousands of wells and several orders of magnitude faster. Our method is especially useful for shale plays with limited access to reservoir data and it allows one to predict EURs of individual wells on a small laptop. Because we base our approach on the essential physics of oil recovery and on hydrofractured well geometry, the proposed method is highly predictive, while the popular hyperbolic decline curve analysis is not. To our knowledge, the proposed method is new.

KEYWORDS: Eagle Ford; black oil; EUR forecast; predictive robust model; fieldwide prediction

Preliminaries

The numbers of active oil and gas wells in the Eagle Ford are summarized in **Table 1**. Oil production in the Eagle Ford is an important part of shale oil production in the United States that has controlled global oil supply and prices over the last six years, see **Fig. 1**. Therefore, strategically, it is of utmost importance to have predictive models of shale oil production that go beyond the *ad hoc* hyperbolic decline curves and statistical studies. Here we attempt to satisfy parts of this strategic requirement.

To fix ideas, let's consider a typical black oil well in the Eagle Ford play, Texas. The typical horizontal well length is 1500 m (5000 ft), there are 12 stages with 4 clusters each, and we assume for the time being that there is one equivalent hydrofracture per stage with the tip-to-tip length, $2L = 400$ m; and height, H , equal to the formation thickness. The hydrofractures are spaced every $2d = 100$ m. For a cartoon of an idealized horizontal well, see **Fig. 2**.

After *Gong* (2013), we choose a well in the north-central part of the play, labeled as “PR3” in his Table 3.2 and others. PR3 is the largest oil-producing part of the Eagle Ford with the area of 6500 km². A typical depth of the formation is 2400 m (7900 ft). The average formation thickness is $H = 27$ m (88 ft), the porosity is $\phi = 8.6\%$, and the connate water saturation is $S_{wc} = 33\%$. The estimated initial formation pressure is $p_i = 36.7$ MPa (5600 psia), the oil gravity is 41⁰ API, and the typical dissolved gas oil ratio is 180 sm³ of gas per m³ of stock tank oil (~ 1000 scf/stb). The downhole well pressure (fracture pressure) is $p_f = 10.5$ MPa (1500 psi) and the wellhead tubing pressure is $P_{wh} = 2$ MPa (300 psi). With the average temperature gradient of 3.6⁰C/100 m (2⁰F/100 ft) shown in **Fig. 3**, the reservoir temperature is $T = 110^0\text{C}$ (230⁰F). Average bubble point pressure in a PVT cell is $p_b \approx 30$ MPa (4300 psi). Average permeability of the calcite-rich matrix is $k \approx 0.1$ μ darcy. All qualifying black oil wells identified in the Eagle Ford are shown in **Fig. 4**. The cumulative GOR at the end of production history of each well is a circle plotted in **Fig. 5**. These GORs are divided into three classes, $\text{GOR} \in [0, 300], (300, 900], (900, 2500]$, identified in blue, green, and yellow. If the median cumulative production of stock tank oil in the first class is 1, then the remaining medians are 2.09 and 3.97; the produced mostly dissolved gas contributes significant mass. The corresponding cumulative mass produced by each well is shown in **Fig. 6**.

Table 1: Statistics of Eagle Ford oil wells accessed by DrillingInfo in Feb. 2017

Number	Unallocated	Allocated
Oil wells	4,734	13,057
Gas wells	6,108	6,108
Total wells	11,004	19,356

The total compressibility of the pore space filled with oil, water and gas is the sum of the saturation weighted isothermal fluid compressibilities, c_o , c_w and c_g , and the pore space compressibility, c_ϕ :

$$c_{\text{fluid}} = \left(\frac{1}{\rho_{\text{fluid}}} \frac{\partial \rho_{\text{fluid}}}{\partial p} \right)_T, \quad \text{fluid} = o, w, g \quad c_\phi = \left(\frac{1}{\phi} \frac{\partial \phi}{\partial p} \right)_{T, p_c} \quad (1)$$

where the pore space compressibility is measured under constant confining pressure, $p_c = \frac{1}{3}(S_1 + S_2 + S_3)$, equal to the trace of the total earth stress tensor, S , see **Fig. A2**. In particular, see *Patzek et al.* (2014),

$$c_g \approx \frac{1}{p} \quad (2)$$

If only oil flows when reservoir pressure decreases by δp , the total production at downhole conditions is equal to the expansions of all fluids plus shrinkage of the pore space:

$$\begin{aligned} \delta V_t &= \delta V_o \delta V_w + \delta V_g + \delta V_\phi \\ &= c_o V_o \delta p + c_w V_w \delta p + c_g V_g \delta p + c_\phi V_\phi \delta p = \phi V \underbrace{(S_o c_o + S_w c_w + S_g c_g + c_\phi)}_{=c_t} \delta p \\ \text{RF} &= \frac{\delta V_t}{\phi V S_{oi}} = \frac{c_t}{S_{oi}} \delta p, \quad \delta p \equiv p_i - \langle p \rangle \end{aligned} \quad (3)$$

where S_{oi} is the initial oil saturation and RF is the recovery factor, calculated as cumulative oil production divided by the volume of oil initially in the pore space. To further fix ideas, a typical compressibility of light oil above the bubble point is $15 \times 10^{-6} \text{ psi}^{-1}$, water compressibility is $3 \times 10^{-6} \text{ psi}^{-1}$, and that of shale is $1 \times 10^{-6} \text{ psi}^{-1}$.

A plausible argument can be made that gas that evolves in the pore space after reservoir pressure falls below bubble point, remains trapped until its saturation builds above a high value, say, $S_{gc} = 15 - 20\%$, *Honarpour et al.* (2004). Because of interfacial tension and small radii, the nanometer scale gas bubbles are at a significantly higher pressure than oil. Therefore, reservoir pressure needs to drop below bubble point by ~ 700 psi to form these bubbles and sustain them, see *Honarpour et al.* (2004). Since ultimate oil recovery in a shale is of the order of 10 saturation percent, flowing gas develops only in the near-fracture regions where average reservoir pressure falls down to just above 1500 psi, and the saturation change of oil is larger than 15 – 20%. This means that for a long time and in most of the pore space, gas remains dissolved in oil in the regions of the reservoir above the adjusted bubble point pressure, P'_b . Below P'_b , typical compressibility of trapped gas bubbles is shown in **Fig. 7**.

Once bubbles of trapped gas develop, their compressibility dominates the system compressibility and enhances oil recovery. At $\langle S_g \rangle \approx 0.02$, gas compressibility is as large as all other compressibilities combined: $S_g c_g \approx S_o c_o + S_w c_w + c_\phi$, see **Fig. 7**.

Below bubble point, let's assume that

$$\begin{aligned} S_g &= a + bp, \quad p_f \leq p < p'_b, \quad p'_b = p_b - \Delta p_b(\bar{r}) \\ b &= -\frac{S_{g,\max}}{p_b - p_f}, \quad a = -bp'_b \\ \Delta V_g &= \int_{p_b}^p c_g S_g dp = \int_{p'_b}^p \left(\frac{a}{p} + b \right) dp = a \ln(p/p'_b) + b(p - p'_b) \end{aligned} \quad (4)$$

where p_b is the unconfined oil saturation pressure or bubble point pressure; Δp_b , a function of the mean pore radius, $\bar{r}$, is the lowering of bubble point in nanopores; and ΔV_g is an increase of gas volume due to increasing gas saturation and gas compressibility as average reservoir pressure falls from $\langle p \rangle = p'_b$ to $\langle p \rangle \geq p_f$. **Fig. 8** shows the results for $S_{g,\max} = 0.15, 0.10$, and 0.04. P_x is defined as a number such that there is an $x\%$ likelihood that true RF exceeds P_x . P_{50} is the median of the

distribution with equal likelihoods, while P_{10} denotes a 10% probability that true RF will be larger than P_{10} . The horizontal lines in Fig. 8 are the P_{50} and P_{10} recovery factors in a set of numerical simulations by *Gong* (2013) for a typical horizontal well in area P3 of the Eagle Ford. With all simulation parameters at their expected values listed above, Fig. 8 implies that there is a 10% probability of exceeding $\langle S_g \rangle = 0.04$ even if reservoir pressure is uniformly depleted by 4,100 psi to $p_f = 1500$ psia. Similarly, $\langle S_g \rangle < 10\%$ if the average reservoir pressure is $\langle p \rangle \geq 2000$ psia and $\langle S_g \rangle < 15\%$ if $\langle p \rangle \geq 1800$ psia at well abandonment. We are not implying that near the hydrofractures gas saturation cannot build up to values above S_{gc} . It does, and gas in the highly fractured reservoir volume that surrounds each hydrofracture will flow producing a higher GOR at wellhead than the initial GOR. The GOR's in the identified black oil wells that correspond to their latest production times are shown in Fig. 5.

In summary, we assume that production of black oil can be modeled with a mildly nonlinear pressure diffusion equation as long as reservoir regions in which gas flows are limited in space and time.

One-dimensional transient flow of black oil above or just below the bubble pressure

Given the simplified picture of oil flow to a hydrofractured horizontal well depicted in Fig. 2, and the preliminary volumetric analysis above, a one dimensional pressure diffusion equation should be sufficient to model the oil and solution gas flow. After replacing the reference state, “0”, introduced in **Appendix A** with the initial state of the reservoir, “i”, the 1D version of Eqs. (A14) and (A15) is

$$\frac{\partial p}{\partial t} = \alpha(p) \frac{\partial^2 p}{\partial x^2}, \quad (5)$$

$$\alpha(p) = \left(\frac{k}{\phi c_t \mu_o} \right) \left(\frac{\rho_o}{\rho_{\text{fluid},i}} \right) \quad (6)$$

Just below the bubble point, we modify the total system compressibility as follows:

$$\begin{aligned} c_t &= S_{oi}c_o + S_{wc}c_w + S_gc_g + c_\phi = \underbrace{c_{\text{others}}}_{=\text{constant}} + S_gc_g \\ S_gc_g &\approx \frac{a}{p} + b \text{ for } p < p'_b \\ S_g &= 0 \text{ for } p \geq p'_b, \end{aligned} \quad (7)$$

see Eq. (4) and Figs. 7 and 8. With this extension, the pressure diffusivity coefficient, α , is constant above bubble point and a nonlinear function of pressure below, see **Fig. 9**.

Production of oil and mostly solution gas from a horizontal well can be described as follows: Find the distribution of oil pressure in the region between each pair of vertical planar hydrofractures located at $\pm d$, with the uniform initial pressure, $p(x, t = 0) = p_i$, $-d < x < d$, and with both hydrofractures held at a constant pressure $p(x = \pm d, t > 0) = p_f$. The thermodynamic pressure, or reservoir pressure, is equal to the oil pressure because gas is either in solution or forms mostly disconnected bubbles. The pressure diffusion Eq. (5) can be recast into a dimensionless using the following variables:

$$\begin{aligned} \tilde{p} &= \frac{p_i - p}{p_i} \\ \tilde{t} &= \frac{\alpha_i}{d^2} t \\ \tilde{x} &= \frac{x}{d}, \end{aligned} \quad (8)$$

where $\alpha_i = \alpha(p_i \geq p_b, T) = \text{const}$. Then Eq. (5) becomes

$$\frac{\partial \tilde{p}}{\partial \tilde{t}} = \frac{\alpha}{\alpha_i} \frac{\partial^2 \tilde{p}}{\partial \tilde{x}^2} \quad (9)$$

subject to the following initial and boundary conditions:

$$\begin{aligned}\tilde{p}(\tilde{x}, \tilde{t} = 0) &= 0 \\ \tilde{p}(\tilde{x} = \pm 1, \tilde{t} > 0) &= \frac{p_i - p_f}{p_i} = \tilde{p}_f\end{aligned}\quad (10)$$

From Eqs. (8) it follows that the dimensional gradient of pressure is

$$\frac{\partial p}{\partial x} = -\frac{p_i}{d} \frac{\partial \tilde{p}}{\partial \tilde{x}} \quad (11)$$

For a horizontal well segment between each two consecutive hydrofractures, the volumetric rate of oil production at the well downhole conditions (p_f, T) is:

$$q(\tilde{t}; y) = -2 \frac{kA_f}{\mu} \left(\frac{\partial p}{\partial x} \right) \Big|_{x=-d} = -2 \frac{kA_f}{\mu} \left(-\frac{p_i}{d} \right) \left(\frac{\partial \tilde{p}}{\partial \tilde{x}} \right) \Big|_{\tilde{x}=-1} \quad (12)$$

where $A_f = 2LH$ is the one-sided area of hydrofracture, and $y = [p_f, p_i, T, k, \mu, A_f]$ is the vector of process parameters.

Let's introduce now the initial mass of oil in place and the mass flow rate of oil at the downhole conditions

$$\begin{aligned}\mathcal{M} &= \rho_{oi} V = \rho_{oi} A_f (2d) \phi S_{oi} \\ \dot{\mathbf{m}} &= \rho_f q.\end{aligned}\quad (13)$$

where ρ_f is the fluid density at the pressure p_f in the hydrofractures. Then

$$\begin{aligned}\dot{\mathbf{m}} &= -\frac{\mathcal{M}}{\tau} \underbrace{\frac{(c_i p_i)}{S_{oi}}}_{\approx 0.1} \underbrace{\left(\frac{\rho_{fluid, i} \rho_f}{\rho_{oi}^2} \right)}_{\approx 1} \left(\frac{\partial \tilde{p}}{\partial \tilde{x}} \right) \Big|_{\tilde{x}=-1} \\ \dot{\mathbf{m}} &\approx -0.1 \frac{\mathcal{M}}{\tau} \left(\frac{\partial \tilde{p}}{\partial \tilde{x}} \right) \Big|_{\tilde{x}=-1} \text{ for average PR3 properties}\end{aligned}\quad (14)$$

Above and just below oil bubble pressure, the initial and boundary value problem given by Eqs. (9) and (10) can be solved immediately using the method of separation of variables, see, e.g., *Carslaw and Jaeger* (1959), p.100. The result is:

$$\frac{\tilde{p}}{\tilde{p}_f} = 1 - \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} e^{-(2n+1)^2 \pi^2 \tilde{t}/4} \cos \left(\frac{(2n+1)\pi \tilde{x}}{2} \right) \quad (15)$$

Because of the exponential factor in each term of the series in Eq. (15), this series converges uniformly for all $x \in (-d, d)$ and all $t > 0$. Therefore, to differentiate or integrate the left-hand side, each term on the right hand side can be differentiated or integrated separately.

The dimensionless pressure gradient at the hydrofracture at $\tilde{x} = -1$ is

$$\frac{\partial \tilde{p}}{\partial \tilde{x}} \Big|_{\tilde{x}=-1} = -2 \tilde{p}_f \sum_{n=0}^{\infty} (-1)^n e^{-(2n+1)^2 \pi^2 \tilde{t}/4} \sin \left(\frac{(2n+1)\pi}{2} \right) \quad (16)$$

Notice that for all n

$$(-1)^n \sin \left(\frac{(2n+1)\pi}{2} \right) = 1 \quad (17)$$

and Eq. (16) reduces to

$$\frac{\partial \tilde{p}}{\partial \tilde{x}} \Big|_{\tilde{x}=-1} = -2 \tilde{p}_f \sum_{n=0}^{\infty} e^{-(2n+1)^2 \pi^2 \tilde{t}/4} \quad (18)$$

$$\begin{aligned}
\dot{\mathbf{m}} &= 2 \frac{\mathcal{M}}{\tau} \frac{c_t p_i}{S_{oi}} \bar{p}_f \sum_{n=0}^{\infty} e^{-(2n+1)^2 \pi^2 \bar{t}/4} \\
&= 2 \frac{\mathcal{M}}{\tau} \frac{c_t}{S_{oi}} (p_i - p_f) \sum_{n=0}^{\infty} e^{-(2n+1)^2 \pi^2 \bar{t}/4}
\end{aligned} \tag{19}$$

The oil recovery factor is

$$\text{RF} = \frac{\tau}{\mathcal{M}} \int_0^{t/\tau} \dot{\mathbf{m}}(t'/\tau) d(t'/\tau) = \frac{c_t}{S_{oi}} (p_i - p_f) \sum_{n=0}^{\infty} \int_0^{\bar{t}} e^{-(2n+1)^2 \pi^2 \bar{t}'/4} d\bar{t}' \tag{20}$$

The term-by-term integration of the series in Eq. (20) yields:

$$\underbrace{\frac{4}{\pi^2} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}}_{1/2} - \frac{4}{\pi^2} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} e^{-(2n+1)^2 \pi^2 \bar{t}/4} \tag{21}$$

After setting $\rho_f/\rho \approx 1$, the oil recovery factor becomes simply

$$\text{RF} = \frac{c_t}{S_{oi}} (p_i - p_f) \left(1 - \frac{8}{\pi^2} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} e^{-(2n+1)^2 \pi^2 \bar{t}/4} \right) \tag{22}$$

This is a satisfying result: If we wait long enough, and average reservoir pressure is still not too far from bubble point, the recovery factor will be equal to the oil recovered due to pore fluid expansion and pore space shrinkage under the overall pressure difference of $p_i - p_f$, see also Eq. (3). If the reservoir parameters and hydrofracture spacing remain unchanged during oil production, this recovery factor is universal.

At stock tank conditions, the mass of saturated oil produced at downhole conditions separates into stock tank (dead) oil and solution gas:

$$\begin{aligned}
\dot{\mathbf{m}} &= \rho_{\text{ST}} q_{\text{ST}} + \rho_{g,\text{ST}} R_s q_{\text{ST}} \\
q_{\text{ST}} &= \frac{\dot{\mathbf{m}}}{\rho_{\text{ST}} + R_s \rho_{g,\text{ST}}} \\
\rho_{\text{ST}} &= 1000 \frac{141.5}{131.5 + API^0} \frac{\text{kg oil}}{\text{sm}^3} \\
\rho_{g,\text{ST}} &= 0.84 \frac{\text{kg gas}}{\text{sm}^3}
\end{aligned} \tag{23}$$

where $p_{\text{ST}} = 1 \text{ atm}$, $T_{\text{ST}} = 288.7 \text{ K}$ (60°F), R_s is the solution gas/oil ratio, and the gas density is that of natural gas. If a horizontal well has N' pairs of fractures, and the first and the last hydrofracture have unshared exterior sides, the production for the well will be $N' + 1 = N$ times higher. Here N is the total number of parallel, vertical hydrofractures in a horizontal well.

The cumulative oil production at stock tank conditions is

$$Q_{\text{ST}} = N \int_0^t q_{\text{ST}}(t') dt' \approx \sum_{i=1}^n q_{\text{field},\text{ST}}(t_i) \Delta t_i = \tau \sum_{i=1}^n q_{\text{field},\text{ST}}(t_i/\tau) \frac{\Delta t_i}{\tau} \tag{24}$$

where t is the elapsed time on production equal to n months, t_i is the i^{th} month on production, and $q_{\text{field},\text{ST}}(t_i)$ is the volume of oil produced in month i by a horizontal well. “Time on production” is defined as the cumulative number of months with nonzero produced oil volumes.

$$\text{RF}(t) = \text{RF}(\tau t) = \frac{\int_0^t \dot{\mathbf{m}}(t') dt'}{\mathcal{M}} \approx \frac{(\rho_{\text{ST}} + R_s \rho_{g,\text{ST}}) \sum_{i=1}^n q_{\text{field},\text{ST}}(t_i) \Delta t_i}{N\mathcal{M}} \quad (25)$$

Finding $\mathcal{M}$ and τ as a Nonlinear Least Squares Optimization Problem

For each suitable well, we subtract Eq. (22) from (25) and take the square of the difference. The result is the following optimization problem: For each qualifying well with $t_{N_{\text{well}}}/\tau \geq 0.6$, see Patzek *et al.* (2013, 2014), find:

$$\min_{\mathcal{M} > 0, \tau > 0} \mathcal{F}(\mathcal{M}, \tau, \dots) = \sum_{j=1}^{N_{\text{well}}} \left[\rho_{\text{ST}} + R_s \rho_{g,\text{ST}} q_{\text{field},\text{ST}}(t_j) \Delta t_j - \frac{c_t(N\mathcal{M})}{S_{oi}} (p_i - p_f) \left(1 - \frac{8}{\pi^2} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} e^{[-(2n+1)^2 \pi^2 t_j / (4\tau)]} \right) \right]^2 \quad (26)$$

where N_{well} is the number of time intervals, here months, a given well was on production. A “qualifying well” is one with at least 6 months on production, and with stable production; we reject wells, whose cumulative oil production plots as a zigzag vs. time on production or plots as an upward pointing parabola vs. the square root of time on production. If the maximum time on production divided by an estimated pressure interference time τ , is larger than one, a qualifying well is in infinite reservoir flow and its cumulative oil production plots as straight line vs the square root of time on production.

Calculating EUR. From the simulations, we obtained the value of τ (months) and $\mathcal{M}$ (ktons) for each allocated lease. Since we have adjusted three Recovery Factor (RF) values for the three different GOR tiers, we can directly calculate the predicted Estimated Ultimate Recovery (EUR) for the particular allocated lease as follows:

$$\text{EUR}_{\text{avg}} = \text{RF} \times \mathcal{M} \text{ (ktons)}$$

Since we have assumed that we have allocated the lease production evenly up to the maximum number of wells, n , we further assume that the EUR for the lease is n times of its average EUR_{avg} :

$$\text{EUR}_{\text{lease}} = n \times \text{EUR}_{\text{avg}} \text{ (ktons)}$$

To compare with the EIA’s results, we may convert back the mass produced in ktons to its corresponding volume in bbls through the following conversion:

$$\begin{aligned} \text{produced volume (boe)} &= \text{mass produced (ktons)} \times 10^6 / 5.615 \frac{\text{cuft}}{\text{bbl}} / (0.3048)^3 \frac{\text{m}^3}{\text{cuft}} \\ &\quad / \left(\rho_o + \rho_g \times \frac{\text{GOR}}{5.615} \right) \frac{\text{kg}}{\text{m}^3} \end{aligned}$$

Results and Discussion

The convolved production records in the Eagle Ford were untangled using an algorithm presented in **Appendix B**, leaving many fewer wells than the number of physical wells. In a forthcoming paper we will propose a few relatively inexpensive ways of remedying this serious deficiency of production reporting that in the long run will hurt operators, who may not know how each well in a lease gathering system performs with time. With infill drilling and subtler ways of operating individual wells, the current gap in reporting production data accurately must be filled.

The key results for the allocated oil wells are presented in **Figs. 10** and **11**. From both figures it follows that higher GOR helps to produce more oil by increasing the system compressibility somewhat. By “pressure interference,” here we mean

only the interference among pairs of the consecutive hydrofractures. This is the intra-well pressure interference. Here, we do not address the inter-well pressure interference, but it may show up as intra-well pressure interference of pad-drilled wells.

There are 428 wells that have not pressure-interfered after up to 44 months on production, and their cumulative production plots as straight lines vs. the square root of time on production. The remaining 2,182 wells have pressure-interfered and their cumulative production bends down when plotted vs. the square root of time, because of the exponential decline rate of production that becomes detectable at $t/\tau > 0.6$.

The frequency distributions of interference times, τ , are shown in **Fig. 12** for the three classes of GOR values. Note that all three distributions have quite pronounced peaks around five years (60 months). It appears, therefore, that the most likely ratios pressure diffusivities to the square of interfracture distance are about the same regardless of well location and design. These ratios should be used when planning for infill wells.

Note that on average our physics-based modeling predictions are in agreement with a statistical study of the Eagle Ford wells by EIA, but there are significant differences, see **Table 2**:

- We select only wells that produce oil with API gravity less than 40 deg and cumulative GOR less than 2500 scf/stb, identifying three GOR classes with the median GORs of 200, 500, and 1300 scf/stb.
- As far as we can tell, EIA selects all wells, including gas condensate and gas wells.
- We select wells with at least 6 months of production in contrast to the EIA's criterion of at least three months on production.
- Almost all oil wells in the Eagle Ford have aggregated production records. We use our own, mass-conserving but painful procedure for the record disaggregation. We don't know what EIA did.
- We use a physics-based model of production and shut off wells when their production is less than 10 boe/day, while EIA uses hyperbolic decline curves and extrapolates them to 30 years. In this case, hyperbolic decline extrapolations always overestimate well EURs.
- Table 2 shows that our EURs for the black-oil wells in the six counties in which EIA reported their results bear little correlation with the EIA's EURs; three are lower and three are higher.
- Nevertheless, to two significant digits, both sets of EURs have identical averages. This agreement could be a fortuitous coincidence, because one would expect the EIA estimate to be higher.

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Table 2: Our EURs for the Eagle Ford shale black oil wells compare well with EIA's 2014 Annual Energy Outlook by D. Van Wagener

County	SPE 187226 Predictions			EIA's Energy Outlook 2014*	
	EUR (bbls)	Number of Wells	Average EUR (bbls/well)	Number of Wells	Average EUR (bbls/well)
Atascosa	67,219,000	377	178,000	-	-
Bastrop	176,000	1	176,000	-	-
Brazos	6,428,000	47	137,000	-	-
Burleson	6,412,000	49	131,000	-	-
Caldwell	116,000	5	23,000	-	-
Dewitt	8,914,000	42	212,000	453	334,000
Dimmit	3,915,000	45	87,000	820	137,000
Fayette	8,482,000	53	160,000	-	-
Frio	29,871,000	203	147,000	-	-
Gonzales	54,211,000	329	165,000	486	198,000
Karnes	105,634,000	446	237,000	975	226,000
Lasalle	67,089,000	329	204,000	755	153,000
Lavaca	3,794,000	28	135,000	-	-
Lee	3,040,000	40	76,000	-	-
Leon	389,000	6	65,000	-	-
Live Oak	15,332,000	96	160,000	-	-
Madison	378,000	4	94,000	-	-
Maverick	24,000	2	12,000	10	25,000
Mcmullen	22,990,000	172	134,000	455	127,000
Milam	19,000	1	19,000	-	-
Robertson	1,335,000	13	103,000	-	-
Titus	1,401,000	27	52,000	-	-
Wilson	11,911,000	99	120,000	-	-
Wood	2,112,000	25	84,000	-	-
Zavala	16,006,000	172	93,000	-	-
Total/Mean	437,198,000	2,611	167,000	3,954	171,000*

* The study by EIA (2014) covered 32 counties in Texas with 5,384 wells, whose mean EUR was 168,000 bbl. However, data from just seven counties have been reported.

Figures

Figure 1: Increases of production of crude oil & lease condensate worldwide and in the U.S. relative to Jan 2005. In October 2004, global production stopped growing for six years and then increased exactly as oil production in the U.S. This means that all new petroleum projects in the world cancelled production declines, even with the highest E&P investment ever. Source: U.S. DOE EIA, accessed 01/18/2017.

Figure 2: A horizontal well with 10 – 20 hydrofracture “stages,” spaced uniformly along its entire length. There is one equivalent hydrofracture per stage. The common hydrofracture height is H and the tip-to-tip length of each fracture is $2L$. The distance between between the hydrofractures is $2d$. The exterior hydrofractures at the left- and right-hand end of the well drain the reservoir volume beyond the volume spanned by these hydrofractures and the length of the lateral. The hydrofractures are perpendicular to the wellbore. The flow of oil and gas (red arrows) is horizontal and perpendicular to the fracture planes. Once inside the hydrofracture, the fluids flows more or less radially into the well perforations connected to each hydrofracture. The permeability of a propped hydrofracture is infinite in comparison with the effective permeability of the rock matrix and natural fractures feeding the fluids into this hydrofracture.

Figure 3: Rock temperature as a function of depth. Data source: *Gong (2013)*.

Figure 4: The black oil wells in Eagle Ford align nicely with the dip, becoming more condensate-like downdip to the S-SE. There are 2,611 wells with three GOR classes (1,123 leases) that qualify for further analysis.

Figure 5: Three classes of cumulative GORs were identified for further analysis. The horizontal lines are medians of each class. From bottom to top, these median GORs are approximately 300, 500, and 1300 scf/stb, respectively. Note that wells with fewer than six months on production have been rejected.

Figure 6: The cumulative mass of stock tank oil plus stock tank gas allocated from 1,123 selected leases (equivalent to 2,611 wells). Note that some of the wells with the highest GOR are outliers in terms of oil and gas production. The median well produces 7.8 kt of total mass for $\text{GOR} \leq 300$ scf/stb. The other two classes of wells produce 2.09 and 3.97 times more mass, respectively. Therefore, gas production is an important component of total mass produced, ranging from 23% of the mass of oil for $\text{GOR} \leq 300$ scf/stb to 150% for $\text{GOR} > 900$ scf/stb.

Figure 7: Assuming that average gas saturation increases linearly with a decrease of average reservoir pressure below the adjusted bubble, P'_b , point (red curve), the resulting gas compressibility (blue curve) dominates the overall system compressibility. The total compressibility of oil, water and pore space is shown as the black line.

Figure 8: Integration of the total compressibility under the assumption that average gas saturation never builds above the critical value, S_{gc} . Above bubble point, RF is proportional to the liquid and pore space compressibility multiplied by the pressure drop $\Delta p = p_i - \langle p \rangle$. Below bubble point, gas compressibility kicks in under the assumption that average gas saturation $\langle S_g \rangle \in [0, S_{gc}]$ increases linearly with the decreasing reservoir pressure, and gas compressibility is $c_g \approx 1/p$.

Figure 9: The ratio of hydraulic diffusivity at a reservoir pressure p to that at the initial reservoir pressure $p_i > p_b$. The maximum average saturation of gas bubbles trapped in the pore space is assumed to be 5, 10, and 15%.

Figure 10: 429 allocated wells in 268 leases are still in early time flow, with cumulative production vs. square root of time plotting as straight lines, one for each GOR class. Note that the x -scale in this plot is the square root of time on production, but the nonlinearly distributed labels are the corresponding times.

Figure 11: 2,182 allocated wells in 855 leases have already pressure-interfered and their cumulative production bends down when plotted vs. square root of time.

Figure 12: Frequency distributions of interference times in wells with inter-hydrofracture pressure interference. Note that the tail values of τ above 120 months are increasingly uncertain.

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Appendix A: Overall Mass and Momentum Balance for Single-Phase Flow in a Slightly Compressible Fluid/Rock System

The fluids consist of oil that flows, as well as water and gas that don't. The fluids form three immiscible phases. The oil is surrounded with immovable water that lines pore corners, and nooks and crannies in the pore walls. At a low enough local pressure, stationary gas bubbles evolve in the oil. The fluids are compressible and rock's solid skeleton is slightly compressible. We set out to derive an approximate equation governing the single-phase flow of oil above and somewhat below the oil's bubble pressure. The result is a nonlinear pressure diffusion equation with an appropriately defined hydraulic diffusivity.

Figure A1: At the reference state, we imagine a colored rock grain sample, in blue, filled with colored fluid, in red. First, we remove the red fluid into a beaker and fill the pore space with ordinary fluid. Second, we change the stress on the solid and the pore pressure, and “measure” the new pore volume, V_ϕ . Third, we measure the new red fluid volume under the new pore pressure, V_{fluid} . In general, the new pore volume and fluid volume will not be equal to each other, and the fluid will have to flow in/out of the blue rock volume. Image source: *Jin et al. (2004)*.

Volumetric Fluid Content. The density of the fluid mass m_{fluid} occupying the volume V_{fluid} in a beaker is

$$\rho_{\text{fluid}} = \frac{m_{\text{fluid}}}{V_{\text{fluid}}} \quad (\text{A1})$$

The bulk density of the fluid inside a porous rock is

$$\rho_b = \frac{m_{\text{fluid}}}{V} = \left(\frac{V_{\text{fluid}}}{V} \right) \left(\frac{m}{V_{\text{fluid}}} \right) = \phi \rho_{\text{fluid}} \quad (\text{A2})$$

At an arbitrary reference or initial condition, the fluid mass in the pore space is $m_{\text{fluid},0}$, and the initial fluid volume, $V_{\text{fluid},0}$, is equal to the initial pore space volume, $V_{\phi 0}$. The initial bulk volume of the rock is V_0 .

We apply stress on the solid skeleton and change the fluid pressure at the same time, see **Fig. A1**. Both the fluid volume, V_f , and the pore space volume, V_ϕ , will change. In the end, the new pore space volume will be filled entirely with the fluid at the new fluid pressure and the new fluid density. But, initially, we can think about these two volume changes independently, and then introduce flow into or from the rock pore space to arrive at the final state. At the new state we can write formally

$$m_{\text{fluid}} = m_{\text{fluid},0} \frac{V_\phi}{V_{\text{fluid}}} \quad (\text{A3})$$

After Biot, we will now introduce the increment of fluid mass per unit initial bulk volume V_0 , normalized by the initial fluid density $\rho_{\text{fluid},0} = m_{\text{fluid},0}/V_{\text{fluid},0}$:

$$\delta\zeta := \frac{\delta m_{\text{fluid}}/V_0}{m_{\text{fluid},0}/V_{\text{fluid},0}} = \frac{\delta\rho_{bf}}{\rho_{\text{fluid},0}} = \left(\frac{V_{\text{fluid},0}}{V_0}\right) \delta\left(\frac{V_\phi}{V_{\text{fluid}}}\right) = \frac{V_{\text{fluid},0}}{V_0} \frac{\delta V_\phi V_{\text{fluid},0} - \delta V_{\text{fluid}} V_{\phi 0}}{V_{\text{fluid},0}^2} \quad (\text{A4})$$

But, by assumption, $V_{\text{fluid},0} = V_{\phi 0}$, and the initial fluid volume and the pore space volume cancel out.

Therefore, the increment of fluid content, i.e., the difference between the fractional change of pore volume and the fractional change in fluid volume is

$$\delta\zeta = \frac{1}{V_0} (\delta V_\phi - \delta V_{\text{fluid}}) \quad (\text{A5})$$

Let's keep the confining pressure, p_c , constant and only change the fluid pressure, p , see **Fig. A2**. If the fluid is *incompressible*, i.e., when $\delta V_{\text{fluid}} = 0$, the increment of fluid content is

$$\delta\zeta = \frac{\delta V_\phi}{V_0} = \frac{V_{\phi 0}}{V_0} \left(\frac{1}{V_{\phi 0}} \frac{\delta V_\phi}{\delta p} \right)_{p_c} \delta p = \phi_0 c_\phi \delta p \approx \phi c_\phi \delta p \quad (\text{A6})$$

Note that the approximate equality above underscores that the bulk volume and the pore space volume change simultaneously, and the porosity depends on both.

If, conversely, the fluid-filled pore space is *incompressible*

$$\delta\zeta = \frac{V_{\text{fluid},0}}{V_0} \left(-\frac{1}{V_{\text{fluid},0}} \frac{\delta V_{\text{fluid}}}{\delta p} \right) \delta p = \phi_0 c_{\text{fluid}} \delta p \approx \phi c_{\text{fluid}} \delta p \quad (\text{A7})$$

One can impose a constant load on the piston and the side jacket (a hydrostatic stress, $S_1 = S_2 = S_3 = S$, or the confining pressure $p_c = S$) in the apparatus shown in Figure A2, and change (increase or decrease) the fluid pressure independently through the hollow piston. In this case, the bulk isothermal compressibility at constant stress and with simultaneous fluid flow will be:

$$c_{p_c} = \left(\frac{\delta\zeta}{\delta p} \right)_{p_c, T} \approx \left(\frac{\partial\zeta}{\partial p} \right)_{p_c, T} \quad (\text{A8})$$

where the parameters held constant are the confining pressure, p_c , and the temperature, T . From Eqs. (A8), (A6), and (A7), it follows immediately that when both the fluid and the pore space are somewhat compressible

$$c_{p_c} \approx \phi (c_{\text{fluid}} + c_\phi) \quad (\text{A9})$$

Suppose now that the blue fluid in the pore space is actually made up of three immiscible fluids: Flowing oil with the density ρ_o and saturation S_o , practically immovable water with the density of ρ_w and saturation S_w , and bubbles of gas with the density ρ_g and saturation S_g . The thermodynamic reservoir pressure, p , is the oil pressure, since gas is disconnected. The volume-fraction weighted bulk density of the pore fluid that consists of three immiscible phases is:

$$\rho_{\text{fluid},b} = \phi (S_o \rho_o + S_w \rho_w + S_g \rho_g), \quad \frac{\text{kg}}{\text{m}^3 \text{ bulk}} \quad (\text{A10})$$

The overall mass balance of the bulk fluid in the pore space is

$$\frac{\partial \rho_{\text{fluid},b}}{\partial t} = -\nabla \cdot (\rho_o u_o + \rho_w u_w + \rho_g u_g) = -\nabla \cdot \rho_o u_o, \quad \frac{\text{kg}}{\text{m}^3 \text{ bulk-s}} \quad (\text{A11})$$

because neither water nor gas flow. Here u_o is the superficial (Darcy) velocity of oil.

Figure A2: The jacketed triaxial rock compressibility measurement. The vertical stress, S_1 , is applied to a hollow piston. The tube in the piston is used to regulate the pore pressure, p . The lateral stresses, $S_2 = S_3$, are applied to the copper-jacketed specimen by injecting oil through the side tube. The confining pressure is defined as $p_c = \frac{1}{3}(S_1 + S_2 + S_3)$.

Using Eqs. (A4), (A8) – (A10), we obtain:

$$\begin{aligned} \frac{\partial \rho_{\text{fluid},b}}{\partial t} &= \rho_{\text{fluid},0} \left(\frac{1}{\rho_{\text{fluid},0}} \frac{\partial \rho_{\text{fluid},b}}{\partial p} \right)_{p_c, T} \frac{\partial p}{\partial t} = \rho_{\text{fluid},0} c_{p_c} \frac{\partial p}{\partial t} \approx \rho_{\text{fluid},0} \phi (c_{\text{fluid}} + c_\phi) \frac{\partial p}{\partial t} \\ &= \rho_{\text{fluid},0} \phi \underbrace{(S_o c_o + S_w c_w + S_g c_g + c_\phi)}_{=c_{\text{fluid}}} \frac{\partial p}{\partial t} = \rho_{\text{fluid},0} \phi c_t \frac{\partial p}{\partial t} \end{aligned} \quad (\text{A12})$$

Now we insert into the right hand side of Eq. (A11) Darcy's law for horizontal flow of oil, and make an argument that the quantity

$$\nabla \left(\frac{\rho_o}{\mu_o} \right) \cdot \nabla p \quad (\text{A13})$$

is small in the second order. Here k is the rock permeability and μ_o the oil viscosity. The result is:

$$\frac{\partial p}{\partial t} = \left(\frac{k}{\phi c_t \mu_o} \right) \left(\frac{\rho_o}{\rho_{\text{fluid},0}} \right) \nabla^2 p, \quad (\text{A14})$$

We define the hydraulic diffusivity of pressure for the slightly compressible horizontal flow of oil in the presence of immovable water and some gas as

$$\alpha \equiv \left(\frac{k}{\phi c_t \mu_o} \right) \left(\frac{\rho_o}{\rho_{\text{fluid},0}} \right) \quad (\text{A15})$$

If there is enough gas as compressible bubbles, α becomes a function of reservoir pressure.

Table A1: Typical values of key fluid parameters.

Parameter	SI units ^a	Field units ^a	SI units ^b	Field units ^b
Oil compressibility, c_o	$2.13 \times 10^{-9} \text{ Pa}^{-1}$	15 microsips (10^{-6} psi^{-1})		
Water compressibility, c_w	$4.27 \times 10^{-10} \text{ Pa}^{-1}$	3 microsips (10^{-6} psi^{-1})		
Oil viscosity, $\mu_{o,i}$			$5.8 \times 10^{-5} \text{ Pa s}$	0.58 cp
Oil FVF ^c $B_{o,i}$	$1.65^d \text{ m}^3/\text{sm}^3$	1.65^d rbbl/stb	$1.34 \text{ m}^3/\text{sm}^3$	1.34 rbbl/stb
Oil density/gravity	958 kg/sm^3	41 ⁰ API	958 kg/sm^3	41 ⁰ API
Gas gravity, γ_g			0.905	0.905
GOR ^e	$178 \text{ sm}^3/\text{m}^3$	1000 scf/stb	$89 \text{ sm}^3/\text{m}^3$	500 scf/stb
Saturation pressure, p_b	30 MPa	4300 ^f psi	14.4 MPa	2053 psi

^a Data source: *Gong* (2013)^b Data source: *Orangi et al.* (2011)^c Formation volume factor^d From *Orangi et al.* (2011) for GOR=1000 oil^e Solution gas-oil ratio^f $p_b \sim 3250$ psi reported for a GOR=1000 scf/stb oil in Figure 2 in *Orangi et al.* (2011)**Table A2: Typical values of key reservoir and well parameters.**

Parameter	SI units ^a	Field units ^a	SI units ^b	Field units ^b
Rock porosity, ϕ	0.086	0.086		
Rock permeability, k	$0.1 \times 10^{-18} \text{ m}^2$	0.1 microdarcy		
Rock compressibility, c_ϕ	$1.42 \times 10^{-10} \text{ Pa}^{-1}$	1 microsip (10^{-6} psi^{-1})		
Initial oil saturation, S_{oi}	0.67	0.67		
Irreducible water saturation, S_{we}	0.33	0.33		
Initial pressure, p_i	36.7 MPa	5600 psi	44.3 MPa	6300 psi
Reservoir temperature, T	383 K	230 ⁰ F	387 K	237 ⁰ F
Fracture pressure ^c , p_f	10.5 MPa	1500 psi		
Horizontal well length, L_w	1500 m	5000 ft		
# of fracture stages, N	12	12		
Fracture height, H	30 m	100 ft		
Tip-to-tip fracture length, $2L$	400 m	700 ft		

^a Data source: *Gong* (2013)^b Data source: *Orangi et al.* (2011)^c The uniform fluid pressure in the hydrofractures

Appendix B: Decomposition of Lumped Production of Eagle Ford Oil Wells

Texas Oil Tax Regulation and its Consequences. Evaluation of oil wells in the Eagle Ford is a challenge caused the rules of production reporting. The Texas Railroad Commission tax regulations for oil entities (Chapter 202) permit operators to report production from a variable number of wells as one lease production record. The operators may gather production from adjacent wells into well headers, connect these headers to a crude oil transmission pipeline, and report the total production from all lease wells as a single production record (bbl oil per month and GOR) based on the crude oil volumes delivered to the lease oil separation tanks and the total lease gas. It is therefore important to have a fast and consistent method of disaggregating lease production data back to individual allocated wells. In contrast, gas wells have individual wellhead production meters and are not aggregated.

Example of Extreme Lumping of Eagle Ford Lease Production. Fig. B1 shows 71 oil wells that are connected to the same pipeline and reported as one lease production record assigned to a well located at the border between the Dimmit and La Salle counties. The actual production data have been carefully tabulated and analyzed. We have found that these 71 wells

Figure B1: Map of a lease in the Eagle Ford with 71 wells connected to a single pipeline (after: RRC Texas, accessed on April 3, 2017). The production data for all wells are lumped together and assigned to well number 50.

span the bottom-hole area of 10,500 acres.

In the DrillingInfo software, this particular lease example is reported as one unallocated well with the API number 42-283-32XXX-00 and the production data detailed in **Table B1**. The well count varies in each month and mostly increases until the maximum well count of 65 is reached. This well count, reported by DrillingInfo, is inaccurate because we counted 71 wells that belong to the same lease.

The cumulative mass produced by all wells, along with the corresponding number of the wells, are plotted vs. time on production in **Fig. B2**. During the first four months, there was only one well contributing to the production. Later on,

more wells were drilled, resulting in the cumulative total oil mass curving up parabolically, until the maximum well count reached 65 wells. Naturally, as the reservoir pressure declines, the rate of oil production of each well must decrease with time, resulting in the cumulative oil production that curves down. Consequently, the cumulative oil mass curve shown in Fig. B2 cannot be matched with any physics-based model³ and must be back-allocated to individual wells.

Figure B2: The cumulative mass produced from an Eagle Ford lease in Fig. B1. The blue and black lines are the unallocated production and number of wells that contribute each month. The allocated production of an equivalent average well is shown as the green line.

Well Allocation Method. Oil production from unallocated total production records must be allocated to individual wells so that we can use our predictive model for shale oil production. We consider the total mass produced as the sum of oil and gas masses:

$$\text{mass produced} = \text{monthly Oil (bbl)} \times 5.615 \frac{\text{cuft}}{\text{bbl}} \times (0.3048)^3 \frac{\text{m}^3}{\text{cuft}} \times \left(\rho_o + \rho_g \times \text{GOR} \times \frac{1}{5.615} \frac{\text{bbl}}{\text{cuft}} \right) \quad (\text{B1})$$

where mass produced is in kg, GOR is gas-oil ratio in scf/STB and oil and gas density in kg/m³ are:

$$\rho_o = \frac{141.5}{131.5 + \text{API}} \times \rho_{\text{water}} \quad \text{and} \quad \rho_g = \text{SG}_{\text{gas}} \times \rho_{\text{air}}$$

Once we have the monthly mass production, the simplest way to perform the allocation is to divide the total mass produced in a given month by the well corresponding count:

$$\text{allocated mass produced} = \frac{\text{unallocated mass produced}}{\text{number of wells}} \quad (\text{B2})$$

The allocated cumulative mass produced in each month can then be written as follows:

$$\bar{N}_i = \sum_{i=1}^n \left(\frac{m_i}{x_i} \right) \quad (\text{B3})$$

Here $\bar{N}_i$ is the allocated cumulative mass produced at the month- i , m_i is the unallocated monthly mass produced at month- i (lease production), x_i is the number of wells in month- i , and i counts months on production ($i = 1, 2, \dots, n$). All steps in our well allocation procedure are detailed in **Table B2**. Before the allocation, cumulative production has the shape of a convex-down parabola that violates the natural depletion mechanisms for a physical well. Our allocation method results in a convex-up cumulative mass production curve that follows the reduction of reservoir pressure during production, see the green curve in Fig. B2.

³The hyperbolic decline model will yield rates that never go to zero and arbitrarily large cumulative production.

Reconstruction of Lease Production. Once we have allocated lease production, we are able to predict the EUR for each well. However, in order to conserve mass, the predicted production for each allocated well must be mapped back to lease production. This back-mapping procedure is not difficult as long as we record the number of wells at each time step. Following Eq. (B2), we can quickly obtain the unallocated mass production of each month as follows:

$$\text{lease production} = \text{allocated mass produced} \times \text{number of wells} \quad (\text{B4})$$

In practice, we may only have the cumulative production curve for the average allocated well depicted in Fig. B2. We can calculate the unallocated lease production in each month by differencing cumulative production of the allocated well at each time step. The cumulative lease production can be written as:

$$N_i = \sum_{i=1}^n x_i (\bar{N}_i - \bar{N}_{i-1}) \quad (\text{B5})$$

Where N_i is the cumulative lease mass produced at the month- i , $\bar{N}_i$ is the allocated cumulative mass produced in month- i (we take $\bar{N}_0 = 0$ ktons), x_i is the number of wells in month- i , and $(i = 1, 2, \dots, n)$, where n is the number of months on production in the lease record.

Table B1: Monthly production data of Lease 153XX or API 42-283-323XX-00

Production Date	Oil (bbl)	Gas (Mscf)	Water (bbl)	Well Count
12/1/2010	11,755	12,350	7,423	1
1/1/2011	9,427	5,249	5,953	1
2/1/2011	11,903	5,711	7,517	1
3/1/2011	32,693	19,562	20,645	1
4/1/2011	5,527	1,557	3,490	2
5/1/2011	15,934	7,546	10,062	4
6/1/2011	40,382	32,629	25,500	6
7/1/2011	40,715	33,813	25,711	6
8/1/2011	39,333	42,469	24,838	7
9/1/2011	33,462	13,968	21,131	8
10/1/2011	55,802	21,863	35,238	8
11/1/2011	57,737	21,748	36,460	8
12/1/2011	98,339	66,987	62,099	10
1/1/2012	83,726	56,841	52,871	10
2/1/2012	86,653	79,465	54,720	10
3/1/2012	94,809	119,370	59,870	12
4/1/2012	106,934	144,435	67,527	15
5/1/2012	81,498	123,674	51,464	17
6/1/2012	102,460	115,400	64,702	17
7/1/2012	166,885	243,417	105,385	17
8/1/2012	177,752	277,829	112,247	19
9/1/2012	157,201	290,617	99,269	19
10/1/2012	157,442	293,739	99,422	19
11/1/2012	155,811	263,451	98,392	21

12/1/2012	188,747	276,631	119,190	23
1/1/2013	185,089	286,820	115,903	23
2/1/2013	187,400	361,561	117,350	23
3/1/2013	183,155	428,418	114,692	23
4/1/2013	155,920	383,981	97,637	25
5/1/2013	203,920	430,697	127,695	27
6/1/2013	168,164	431,030	105,305	31
7/1/2013	237,870	528,778	148,955	34
8/1/2013	204,620	515,141	128,133	34
9/1/2013	213,397	579,924	133,629	32
10/1/2013	201,047	612,705	125,896	32
11/1/2013	178,213	592,611	111,597	34
12/1/2013	242,573	690,535	151,900	37
1/1/2014	219,223	655,856	122,326	37
2/1/2014	191,462	553,561	106,835	37
3/1/2014	213,225	604,804	118,979	40
4/1/2014	286,031	651,832	159,604	43
5/1/2014	267,200	690,462	149,097	43
6/1/2014	250,062	612,366	139,534	47
7/1/2014	255,191	578,674	142,396	50
8/1/2014	330,378	736,667	184,350	50
9/1/2014	267,676	788,338	149,362	50
10/1/2014	250,374	686,188	139,708	53
11/1/2014	327,482	733,613	182,734	56
12/1/2014	277,747	707,103	154,982	59
1/1/2015	306,664	697,782	217,629	59
2/1/2015	245,764	634,524	174,410	59
3/1/2015	221,023	564,284	156,852	61
4/1/2015	243,057	528,138	172,489	65
5/1/2015	288,341	534,509	204,625	65
6/1/2015	246,013	525,615	174,587	65
7/1/2015	215,486	266,194	152,923	65
8/1/2015	208,437	569,353	147,920	65
9/1/2015	185,976	515,584	131,981	65
10/1/2015	187,137	468,540	132,805	65
11/1/2015	165,843	450,327	117,693	65
12/1/2015	157,109	456,515	111,495	65
1/1/2016	134,315	391,612	34,622	65
2/1/2016	132,082	347,745	34,046	65
3/1/2016	138,433	354,005	35,683	65
4/1/2016	123,178	332,495	31,751	65
5/1/2016	122,456	342,188	31,565	65
6/1/2016	123,606	409,034	31,862	65

7/1/2016	118,240	430,513	30,478	65
8/1/2016	114,365	444,446	29,479	65
9/1/2016	106,273	421,164	27,394	65
10/1/2016	107,956	416,936	27,827	65
11/1/2016	99,859	386,615	25,740	65

Table B2: Allocating steps summarized in a table

Time (Months)	Mass Produced (ktons)	Unallocated Cumulative Mass	Well Count	Mass/Wcnt ktons	Allocated Cumulative Mass
1.00	4.94	4.94	1	4.94	4.94
2.03	3.83	8.77	1	3.83	8.77
3.07	4.84	13.61	1	4.84	13.61
4.00	14.72	28.33	1	14.72	28.33
5.03	2.25	30.57	2	1.13	29.46
6.03	6.69	37.27	4	1.67	31.13
7.07	16.42	53.68	6	2.74	33.86
8.07	17.10	70.79	6	2.85	36.71
9.10	15.99	86.78	7	2.28	39.00
10.13	13.60	100.38	8	1.70	40.70
11.13	23.44	123.83	8	2.93	43.63
12.17	23.47	147.30	8	2.93	46.56
13.17	41.31	188.61	10	4.13	50.69
14.20	34.04	222.65	10	3.40	54.10
15.23	35.23	257.88	10	3.52	57.62
16.20	41.20	299.08	12	3.43	61.05
17.23	43.47	342.56	15	2.90	63.95
18.23	34.24	376.80	17	2.01	65.97
19.27	41.66	418.45	17	2.45	68.42
20.27	70.11	488.56	17	4.12	72.54
21.30	72.27	560.83	19	3.80	76.34
22.33	63.91	624.74	19	3.36	79.71
23.33	66.14	690.88	19	3.48	83.19
24.37	63.35	754.23	21	3.02	86.21
25.37	79.29	833.52	23	3.45	89.65
26.40	75.25	908.77	23	3.27	92.92
27.43	76.19	984.96	23	3.31	96.24
28.37	82.44	1067.40	23	3.58	99.82
29.40	63.39	1130.79	25	2.54	102.36
30.40	85.67	1216.46	27	3.17	105.53
31.43	68.37	1284.82	31	2.21	107.74
32.43	99.93	1384.76	34	2.94	110.67
33.47	83.19	1467.94	34	2.45	113.12

34.50	86.76	1554.70	32	2.71	115.83
35.50	84.46	1639.16	32	2.64	118.47
36.53	72.45	1711.62	34	2.13	120.60
37.53	101.91	1813.52	37	2.75	123.36
38.57	89.13	1902.65	37	2.41	125.77
39.60	77.84	1980.49	37	2.10	127.87
40.53	95.98	2076.46	40	2.40	130.27
41.57	116.29	2192.75	43	2.70	132.97
42.57	112.25	2305.00	43	2.61	135.58
43.60	101.66	2406.67	47	2.16	137.75
44.60	107.21	2513.88	50	2.14	139.89
45.63	134.32	2648.19	50	2.69	142.58
46.67	108.82	2757.02	50	2.18	144.75
47.67	105.18	2862.20	53	1.98	146.74
48.70	133.14	2995.34	56	2.38	149.12
49.70	116.68	3112.02	59	1.98	151.09
50.73	124.68	3236.70	59	2.11	153.21
51.77	99.92	3336.62	59	1.69	154.90
52.70	99.49	3436.10	61	1.63	156.53
53.73	98.82	3534.92	65	1.52	158.05
54.73	121.13	3656.05	65	1.86	159.92
55.77	100.02	3756.07	65	1.54	161.45
56.77	90.53	3846.60	65	1.39	162.85
57.80	84.74	3931.34	65	1.30	164.15
58.83	75.61	4006.95	65	1.16	165.31
59.83	78.62	4085.56	65	1.21	166.52
60.87	67.42	4152.99	65	1.04	167.56
61.87	66.00	4218.99	65	1.02	168.58
62.90	54.61	4273.60	65	0.84	169.42
63.93	53.70	4327.30	65	0.83	170.24
64.90	60.16	4387.46	65	0.93	171.17
65.93	50.08	4437.54	65	0.77	171.94
66.93	51.44	4488.98	65	0.79	172.73
67.97	50.25	4539.23	65	0.77	173.50
68.97	49.67	4588.91	65	0.76	174.27
70.00	46.50	4635.40	65	0.72	174.98
71.03	43.21	4678.61	65	0.66	175.65
72.03	45.35	4723.96	65	0.70	176.35
73.07	40.60	4764.56	65	0.62	176.97