Diversity Order Results for MIMO Optical Wireless Communications

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Abstract—An optical wireless multiple-input multiple-output (MIMO) communication system employing intensity-modulation direct-detection (IM/DD) is considered. The maximal diversity order of the channel is characterized by studying the outage probability. Then, spatial repetition coding (RC) is shown to be diversity-optimal as it achieves the channel's maximal diversity order. This diversity order is given by a simple expression which is suitable for any channel statistics of practical interest. The results are specialized to some practical channel statistics, and numerical results are provided to verify the results.

Index Terms—Intensity-modulation direct-detection, optical wireless communications, repetition coding, diversity order.

I. INTRODUCTION

Optical wireless communication (OWC) offers an attractive alternative to current radio frequency (RF) systems for achieving high data rates and contributing to the evolution of wireless networks. This made OWC an attractive research topic for both industry and academia [1].

Different aspects of OWC have been studied in the literature. Of particular interest is the simple and relatively cheap implementation of OWC using IM/DD, whose performance has been studied extensively [2]–[4]. This letter focuses on MIMO OWC, which has been recently studied from error probability perspective [2], and capacity perspective [3], [4]. Since outdoor OWC systems are prone to channel variations due to atmospheric turbulence and pointing errors [5], the optimal diversity order of these systems is an important performance criterion for reliable communications.

Thus, we consider a MIMO IM/DD system, subject to per-aperture optical intensity constraints, which are common and realistic constraints in IM/DD operation. Recently, [4], [6] provided asymptotic capacity expressions and bounds for the MIMO IM/DD channel at high signal-to-noise ratio (SNR). Based on these results, we characterize the channel's outage probability diversity order (OPDO). This is the largest diversity order provided by this channel, i.e., no scheme, no matter how powerful, can achieve a larger diversity order. While the OPDA is an intrinsic *channel property* that can be achieved using information-theoretically optimal codes of

Fig. 1. A MIMO IM/DD system with $N_t = N_r = 2$: $s_i \geq 0$ is the optical intensity of aperture i , $h_{ij} \geq 0$ is the channel gain from aperture j to detector i , and n_i is Gaussian noise.

large block length, it is practical to devise schemes that achieve the optimal diversity order with lower complexity. Here comes the role of the error probability diversity order (EPDO), which is a *scheme property*. We prove analytically that the EPDO of spatial RC coincides with the channels' OPDO, thus proving the diversity-order optimality of spatial RC for the MIMO IM/DD channel. Note that this is in sharp contrast with RF where diversity-optimality requires space-time coding. The result is applicable for any turbulence statistics of practical interest, as long as channels are independent and identically distributed (i.i.d.). The results are specialized to some practical turbulence statistics for comparison.

II. SYSTEM MODEL

Consider an IM/DD OWC system with N_t transmit and N_r receive apertures as shown in Fig. 1. The transmission model in time instant t can be represented as

$$\mathbf{y}(t) = \mathbf{H}(t)\mathbf{s}(t) + \mathbf{n}(t), \quad (1)$$

where $\mathbf{s}(t) \in \mathbb{R}_+^{N_t \times 1}$ is the transmit signal, $\mathbf{H}(t) \in \mathbb{R}_+^{N_r \times N_t}$ is the matrix of the channel coefficients, and $\mathbf{n}(t)$ is an N_r -dimensional vector of independent Gaussian noises with zero mean and variance σ^2 . Noise is i.i.d. through time, and combines thermal noise and background radiation. The signal $\mathbf{s}(t)$ and the channel $\mathbf{H}(t)$ are nonnegative due to the use of IM/DD [5]. The channel model we consider in this paper is that of an IM/DD OWC system. Herein, the intensity of light source (laser, for instance) is modulated to send data to a receiver which detects using a photo-detector. This system is characterized by a nonnegative real transmit signal due to the use of intensity modulation (intensity is nonnegative), and positive channel coefficients due to direct detection (detected intensity is also nonnegative). Clearly, these constraints make the MIMO OWC and the MIMO RF channels fundamentally different. For instance, while Gaussian inputs are capacity-achieving for MIMO RF channels, Gaussian inputs are not even feasible in the MIMO OWC channel due to the non-negativity constraint of the intensity modulation. The positivity of the OWC channel coefficients is another fundamental feature that distinguishes OWC and RF

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TABLE I
DIVERSITY ORDER FOR IM/DD MIMO SYSTEMS.

Channel statistics	Rayleigh	Exponential	Gamma-Gamma(α, β)
Diversity order	$2N_r N_t$	$N_r N_t$	$N_r N_t \min(\alpha, \beta)$

channels whose channel gains may be negative. As we show next, while it is well-known that coherent detection over a Rayleigh-distributed MIMO RF channel provides a maximum diversity gain equal to $N_t N_r$, the maximum diversity gain over a Rayleigh-distributed MIMO OWC channel is equal to $2N_t N_r$. Throughout the paper, we assume the availability of channel state information (CSI) at the receiver, and that $\mathbf{H}(t)$ is quasi-static, i.e., $\mathbf{H}(kB + 1) = \dots = \mathbf{H}(kB + B)$ for $k = 0, 1, \dots$ where B is the block length. This models the very slow fading characteristic of OWC channels [7]. The components of $\mathbf{H}(t)$, denoted $h_{ij}(t)$, are i.i.d. according to a distribution $f_h(x)$, $x \geq 0$, which can be realized if the transmit and receive apertures are properly spaced [5, p. 20]. The system has individual average optical intensity constraints $\mathbb{E}[s_i(t)] \leq \mathcal{E} \forall i \in \{1, \dots, N_t\}$ and $\forall t$. We define SNR as $\gamma = \frac{\mathcal{E}}{\sigma}$.¹ We study the channel's diversity order captured by the OPDO. We further study the EPDO achieved by RC with an M -PAM constellation. Those are defined respectively as

$$d_o = -\lim_{\gamma \rightarrow \infty} \frac{\log \mathbb{P}_o(\gamma, R_0)}{\log \gamma}, \text{ and } d_e = -\lim_{\gamma \rightarrow \infty} \frac{\log \mathbb{P}_e(\gamma, M)}{\log \gamma},$$

where $\mathbb{P}_o(\gamma, R_0)$ and $\mathbb{P}_e(\gamma, M)$ are the channel outage probability at target rate R_0 and the error probability of M -PAM RC as a function of SNR γ , respectively.

III. MAIN RESULTS

Let l_0 be the smallest integer such that $f_h^{(l_0)}(0) \neq 0$, where $f_h^{(k)}(a)$ denotes the k^{th} derivative of $f_h(x)$ at $x = a$. The main results are as follows.

Theorem 1. *The OPDO of the MIMO IM/DD channel is given by $d_o = N_r N_t (l_0 + 1)$*

Theorem 2. *RC achieves the maximum diversity order of the MIMO IM/DD channel, i.e., an EPDO of $d_e = N_r N_t (l_0 + 1)$.*

The proofs are given in Sections IV-A and IV-B, respectively. Since $d_e = d_o$, we conclude that RC is diversity-optimal for the MIMO IM/DD channel.

Applying the results to some practical channel statistics leads to the results given in Table I. The Rayleigh distribution, $f_h(x) = \rho x e^{-\frac{\rho x^2}{2}}$ where $\rho > 0$, models the impact of pointing errors caused by misalignment between the transmitter and receiver due to wind for instance [7]. In this case, $f_h^{(0)}(0) = 0$ and $f_h^{(1)}(0) \neq 0$. Thus, $l_0 = 1$, and $d_o = d_e = 2N_r N_t$. The exponential and Gamma-Gamma distributions model strong and moderate/weak atmospheric turbulence caused by temperature variations along the propagation path, respectively [5, Ch. 11]. The exponential distribution, $f_h(x) = \eta e^{-\eta x}$ with $\eta > 0$, satisfies $f_h^{(0)}(0) \neq 0$ and hence $l_0 = 0$ leading to $d_o = d_e = N_r N_t$. The Gamma-Gamma distribution is given by $f_h(x) =$

¹This is motivated by the high-SNR capacity of the SISO IM/DD channel given by $\log \left(\sqrt{\frac{e}{2\pi}} \frac{\mathcal{E}}{\sigma} \right)$ [8].

(a) Outage probability for $R_0 = 0.1$ nats.

(b) Error probability for $M = 2$.

Fig. 2. Outage and error probability for an IM/DD $N_t \times N_r$ MIMO system as a function of SNR ($\mathcal{E}/\sigma$) under different channel turbulence statistics. The parameters are as follows: $\rho = 0.5$, $\eta = 1$, $\alpha = 5.3$, and $\beta = 2.7$. The markings on the plots show the diversity order.

$\frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} x^{(\alpha+\beta)/2-1} K_{\alpha-\beta}(2\sqrt{\alpha\beta x})$, where α and β are positive parameters, and $K_v(u)$ is the modified Bessel function of the second kind. Using the alternative expression given by $f_h(x) = \sum_{j=0}^{\infty} (a_j(\alpha, \beta)x^{j+\beta-1} + a_j(\beta, \alpha)x^{j+\alpha-1})$ [9], we get $l_0 = \min(\alpha, \beta) - 1$ and $d_o = d_e = N_r N_t \min(\alpha, \beta)$.

The results in this table agree with the literature on electrical-power-constrained MIMO IM/DD systems [9]–[11], and go beyond this point by proving the optimality of these diversity orders. Our result also provides a generic diversity order expression which applies for any analytic $f_h(x)$ on its support. Numerical evaluations are given in Fig. 2. The outage probability is plotted using (11), which is optimal at high SNR (cf. (4)). The worst-case PEP is plotted using (16). The numerical results agree with the derived statements on OPDO and EPDO. The proofs are given next.

IV. DERIVATION OF THE MAIN RESULTS

In what follows, we drop the time index of $\mathbf{H}$, and treat it as a random matrix.

A. Outage-Probability Diversity Order

The outage probability can be defined as:

$$\mathbb{P}_o(\gamma, R_0) = \min_{p_s} \Pr\{I(\mathbf{s}; \mathbf{y}|\mathbf{H}) < R_0\}, \quad (2)$$

where R_0 is the desired target rate, p_s is the distribution of $\mathbf{s}$ satisfying the intensity constraint, and $I(\mathbf{s}; \mathbf{y}|\mathbf{H})$ is the mutual information between $\mathbf{s}$ and $\mathbf{y}$ given $\mathbf{H}$. To derive d_o , we first bound $\mathbb{P}_o(\gamma, R_0)$ as follows

$$\begin{aligned} \Pr\{I_{p_s}(\mathbf{s}; \mathbf{y}|\mathbf{H}) < R_0\} &\geq \mathbb{P}_o(\gamma, R_0) \\ &\geq \Pr\{\max_{p_s|\mathbf{H}} I(\mathbf{s}; \mathbf{y}|\mathbf{H}) < R_0\} \end{aligned} \quad (3)$$

where $I_{p_s}(\mathbf{s}; \mathbf{y}|\mathbf{H})$ is the mutual information when $\mathbf{s}$ follows a distribution p_s independent of $\mathbf{H}$. Next, we focus on the case $N_t \leq N_r$. The other case is relegated to the appendix. Note that $\max_{p_s|\mathbf{H}} I(\mathbf{s}; \mathbf{y}|\mathbf{H})$ is the channel capacity as a function of γ and $\mathbf{H}$. This in turn is upper bounded by the channel capacity under a sum intensity constraint, which is upper bounded by $\frac{1}{2} \log \left| \frac{e\gamma^2}{2\pi} \mathbf{H}^T \mathbf{H} \right| + \epsilon_\gamma$, where $\lim_{\gamma \rightarrow \infty} \frac{\epsilon_\gamma}{\log(\gamma)} = 0$ [6]. Thus,

$$\begin{aligned} \mathbb{P}_o(\gamma, R_0) &\geq \Pr \left\{ \frac{1}{2} \log \det \left| \frac{e\gamma^2}{2\pi} \mathbf{H}^T \mathbf{H} \right| < R_0 - \epsilon_\gamma \right\} \\ &= \Pr \left\{ \sum_{j=1}^{N_t} \log \left(\frac{e\gamma^2}{2\pi} \|\mathbf{h}_j\|_2^2 \right) < 2(R_0 - \epsilon_\gamma) \right\}, \end{aligned} \quad (4)$$

where we used $\log |\mathbf{A}| = \text{tr}(\log(\mathbf{A}))$ (componentwise log) for real $\mathbf{A}$, and where $\mathbf{h}_j$ is j^{th} column of $\mathbf{H}$. Since

$$\Pr \left\{ \sum_{i=1}^N a_i < b \right\} \geq \prod_{i=1}^N \Pr \left\{ a_i < \frac{b}{N} \right\} \quad (5)$$

for mutually independent $a_1, \dots, a_N$, we obtain

$$\begin{aligned} \mathbb{P}_o(\gamma, R_0) &\geq \prod_{j=1}^{N_t} \Pr \left\{ \log \left(\frac{e\gamma^2}{2\pi} \|\mathbf{h}_j\|_2^2 \right) < \frac{2(R_0 - \epsilon_\gamma)}{N_t} \right\} \\ &= \prod_{j=1}^{N_t} \Pr \left\{ \|\mathbf{h}_j\|_2^2 < \frac{2\pi}{\gamma^2} e^{\frac{2(R_0 - \epsilon_\gamma)}{N_t} - 1} \right\} \end{aligned} \quad (6)$$

$$\geq \prod_{j=1}^{N_t} \prod_{i=1}^{N_r} \Pr \left\{ h_{ij} < \frac{k_1}{\gamma} \right\}, \quad (7)$$

using (5) and since $h_{ij} \geq 0$, where $k_1 = \frac{\sqrt{2\pi}}{\sqrt{N_r}} e^{\frac{2(R_0 - \epsilon_\gamma) - N_t}{2N_t}}$. Due to the independence of h_{ij} , this can be written as

$$\mathbb{P}_o(\gamma, R_0) \geq \left[F_h \left(\frac{k_1}{\gamma} \right) \right]^{N_t N_r} \quad (8)$$

where $F_h(x)$ is the CDF of h . Using the Taylor series expansion of $f_h(h)$ at $h = 0$, we obtain

$$\begin{aligned} F_h \left(\frac{k_1}{\gamma} \right) &= \int_0^{\frac{k_1}{\gamma}} f_h(h) dh = \int_0^{\frac{k_1}{\gamma}} \sum_{n=l_0}^{\infty} \frac{f_h^{(n)}(0)}{n!} h^n dh \\ &= c(l_0) \gamma^{-l_0-1} + c(l_0+1) \gamma^{-l_0-2} + \dots \end{aligned} \quad (9)$$

where $f_h^{(n)}(h)$ is the n^{th} derivative of $f_h(h)$, $c(l) = \frac{f_h^{(l)}(0) k_1^{l+1}}{(l+1)!}$ and the second inequality in (9) follows since $f_h(x)$ is analytic on its support. Substituting in (8), taking the logarithm, dividing by $\log(\gamma)$, and taking the limit as $\gamma \rightarrow \infty$ yields $d_o \leq N_r N_t (l_0 + 1)$ if $N_t \leq N_r$.

To derive a lower bound, we fix p_s in (3) to be i.i.d. exponential with mean $\mathcal{E}$. Substituting in the left-hand side of (3), and using the entropy power inequality to lower bound the mutual information as in [4, (30)], yields

$$\mathbb{P}_o(\gamma, R_0) \leq \Pr \left\{ \frac{1}{2} \log \left| \frac{e\gamma^2}{2\pi} \mathbf{H}^T \mathbf{H} \right| < R_0 \right\}. \quad (11)$$

Now we use $\log |\mathbf{A}| = \text{tr}(\log(\mathbf{A}))$, followed by the inequality $\Pr \left\{ \sum_{i=1}^N a_i < b \right\} \leq \prod_{i=1}^N \Pr \{ a_i < b \}$ which holds for mutually independent $a_1, \dots, a_N$, to obtain

$$\mathbb{P}_o(\gamma, R_0) \leq \prod_{j=1}^{N_t} \Pr \left\{ \|\mathbf{h}_j\|_2^2 < \frac{2\pi}{\gamma^2} e^{2R_0-1} \right\} \quad (12)$$

$$\leq \prod_{j=1}^{N_t} \prod_{i=1}^{N_r} \Pr \left\{ h_{ij} < \frac{k_2}{\gamma} \right\}, \quad (13)$$

where $k_2 = \sqrt{2\pi} e^{\frac{2R_0-1}{2}}$. Thus,

$$\mathbb{P}_o(\gamma, R_0) \leq \left[F_h \left(\frac{k_2}{\gamma} \right) \right]^{N_t N_r}. \quad (14)$$

Using (10) leads to the lower bound $d_o \geq N_r N_t (l_0 + 1)$, which coincides with the upper bound, thus proving the OPDO statement in Theorem 1 for $N_t \leq N_r$. The same holds for the case $N_t > N_r$ as shown in the appendix.

B. Error Probability Diversity Order of RC

The error probability of M -PAM RC, $\mathbb{P}_e(\gamma, M)$, can be bounded by using the worst-case PEP, $\mathbb{P}_e^{[ol]}(\gamma, M)$, as [12]:

$$\frac{N_{\min}}{M} \mathbb{P}_e^{[ol]}(\gamma, M) \leq \mathbb{P}_e(\gamma, M) \leq (M-1) \mathbb{P}_e^{[ol]}(\gamma, M), \quad (15)$$

where $N_{\min}$ is the number of symbols that are at the minimum distance from at least one other symbol. From [13], we know that $\mathbb{P}_e^{[ol]}(\gamma, M)$ has following form under individual optical intensity constraint and using RC:

$$\mathbb{P}_e^{[ol]}(\gamma, M) = \mathbb{E}_{\mathbf{H}} \left[Q \left(\frac{\gamma}{(M-1)} \|\mathbf{H} \mathbf{1}_{N_t \times 1}\|_2 \right) \right], \quad (16)$$

where $Q(x)$ is the Q -function, $\mathbf{1}_{N_t \times 1}$ is an all-ones $N_t \times 1$ vector, and $\mathbb{E}_{\mathbf{H}}[\cdot]$ is the expectation with respect to $\mathbf{H}$. We can upper and lower bound this using the following:

$$\sqrt{\frac{e}{2\pi}} \frac{\sqrt{\beta-1}}{\beta} e^{-\frac{\beta x^2}{2}} \leq Q(x) \leq e^{-\frac{x^2}{2}}, \quad \text{for } x > 0, \quad (17)$$

where β is an arbitrary positive real number [14]. Thus,

$$\mathbb{P}_e^{[ol]}(\gamma, M) \leq \mathbb{E}_{\mathbf{H}} \left[e^{-\frac{\gamma^2}{2(M-1)^2} \|\mathbf{H} \mathbf{1}_{N_t \times 1}\|_2^2} \right]. \quad (18)$$

Note that $\|\mathbf{H} \mathbf{1}_{N_t \times 1}\|_2^2 \geq \sum_{i=1}^{N_r} \sum_{j=1}^{N_t} h_{ij}^2$ ($h_{ij} \geq 0$). Using this and the independence of h_{ij} yields

$$\begin{aligned} \mathbb{P}_e^{[ol]}(\gamma, M) &\leq \mathbb{E}_{\mathbf{H}} \left[e^{-s \sum_{i=1}^{N_r} \sum_{j=1}^{N_t} h_{ij}^2} \right] \\ &= \prod_{i=1}^{N_r} \prod_{j=1}^{N_t} \mathbb{E}_{\mathbf{H}} \left[e^{-s h_{ij}^2} \right] = \left[\mathbb{E}_h \left[e^{-s h^2} \right] \right]^{N_r N_t}, \end{aligned} \quad (19)$$

where $s = \frac{\gamma^2}{2(M-1)^2}$. Therefore, $\mathbb{P}_e^{[ol]}(\gamma, M) \leq \left[\int_0^\infty e^{-s h^2} f_h(h) dh \right]^{N_r N_t}$. Using the Taylor series expansion of $f_h(h)$ at $h = 0$, we get

$$\mathbb{P}_e^{[ol]}(\gamma, M) \leq \left[\int_0^\infty e^{-s h^2} \sum_{n=l_0}^{\infty} \frac{f_h^{(n)}(0)}{n!} h^n dh \right]^{N_r N_t} \quad (20)$$

$$= \left[d(l_0) s^{-\frac{l_0+1}{2}} + d(l_0+1) s^{-\frac{l_0+2}{2}} + \dots \right]^{N_r N_t} \quad (21)$$

where $d(l) = \frac{f_h^{(l)}(0)}{l!} \frac{\Gamma(\frac{l+1}{2})}{2}$ and $\Gamma(\cdot)$ is the Gamma function. At high SNR ($\gamma \rightarrow \infty$), the first term dominates in (21). Taking the logarithm of (21), dividing by $\log(\gamma)$, and letting $\gamma \rightarrow \infty$ leads to the EPDO lower bound $d_e \geq N_r N_t (l_0 + 1)$.

Using the lower bound in (17), and the inequality $\|\mathbf{H}_1\|_{N_t \times 1}^2 \leq N_t \sum_{i=1}^{N_r} \sum_{j=1}^{N_t} h_{ij}^2$, and proceeding with similar steps as (19)–(21), we can lower bound (16) as $\mathbb{P}_e^{[o]}(\gamma, M) \geq \left[\int_0^\infty K e^{-\tilde{s} N_t h^2} f(h) dh \right]^{N_r N_t}$, where $\tilde{s} = \frac{\beta \gamma^2}{2(M-1)^2}$ and $K = \sqrt{\frac{e}{2\pi}} \frac{\sqrt{\beta-1}}{\beta}$. Using the Taylor series expansion of $f_h(h)$ leads to $d_e \leq N_r N_t (l_0 + 1)$. Hence, the upper and lower bounds coincide, proving d_e in Theorem 2.

V. CONCLUSION

The outage diversity represents the maximum possible diversity achievable in the system, it represents the physical capability of the channels at high SNR. In this paper, we characterized such a diversity and proved that the error probability diversity of RC is equal to the outage diversity, the maximum possible. Our results hold true irrespective of the optical channel statistics. Specializing our results to most popular turbulence models, we have explicitly derived their diversity orders, thus capturing several independent results.

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APPENDIX

To derive the OPDO for $N_t > N_r$, we consider the capacity upper bound given in [6, (91)] as $C_s(\mathbf{H}, \gamma) \leq \sum_{i=1}^{N_r} \frac{1}{2} \log \left(\frac{e N_t \gamma^2}{2\pi} \|\mathbf{h}_i\|_1^2 \right) + \epsilon_\gamma$. Using this bound, and following similar steps as (4)–(8) leads to $d_o \leq N_r N_t (l_0 + 1)$.

To derive a lower bound by reusing (11), we transform the channel into an effective $N_r \times N_r$ channel as follows. We choose $\mathbf{s} = \mathbf{G}\mathbf{x}$, where $\mathbf{G}$ is an $N_t \times N_r$ matrix and $\mathbf{x}$ is i.i.d. exponential with N_r components, each with mean $\mathcal{E}$. To harness the whole diversity of the channel, we choose $\mathbf{G} = \begin{bmatrix} \mathbf{I}_{N_r-1} & \mathbf{0}_{N_r-1 \times 1} \\ \mathbf{0}_{(N_t-N_r+1) \times (N_r-1)} & \mathbf{1}_{(N_t-N_r+1) \times 1} \end{bmatrix}$, where $\mathbf{I}_a$ is an $a \times a$ identity matrix, and $\mathbf{0}_{a \times b}$ is an $a \times b$ all zeros matrix. Then, transmitting $\mathbf{s}$ is equivalent to transmitting $\mathbf{x}$ over a channel with an effective channel matrix $\mathbf{H}_e = \mathbf{H}\mathbf{G}$. Writing $\mathbf{H} = [\mathbf{H}_1, \mathbf{H}_2]$ where $\mathbf{H}_1$ is the first $N_t - 1$ columns of $\mathbf{H}$, and $\mathbf{H}_2$ the rest, leads to $\mathbf{H}_e = [\mathbf{H}_1, \mathbf{H}_2 \mathbf{1}_{(N_t-N_r+1) \times 1}]$. Then, applying (11) we obtain

$$\mathbb{P}_o(\gamma, R_0) \leq \Pr \left\{ \frac{1}{2} \log \left| \frac{e\gamma^2}{2\pi} \mathbf{H}_e^T \mathbf{H}_e \right| < R_0 \right\}. \quad (22)$$

Using $\log|\mathbf{A}| = \text{tr}(\log(\mathbf{A}))$, we can write $\mathbb{P}_o(\gamma, R_0) \leq \Pr \{A_1 + A_2 < R_0\}$ where

$$A_1 = \frac{1}{2} \sum_{i=1}^{N_r-1} \log \left(\frac{e\gamma^2}{2\pi} \|\mathbf{h}_{1,i}\|_2^2 \right), \quad (23)$$

$$A_2 = \frac{1}{2} \log \left(\frac{e\gamma^2}{2\pi} \|\mathbf{H}_2 \mathbf{1}_{(N_t-N_r+1) \times 1}\|_2^2 \right), \quad (24)$$

and $\mathbf{h}_{1,i}$ is the i^{th} column of $\mathbf{H}_1$. Thus $\mathbb{P}_o(\gamma, R_0) \leq \Pr \{A_1 < R_0\} \Pr \{A_2 < R_0\}$. Using similar steps as (11)–(14) yields $\Pr \{A_1 < R_0\} \leq \left[F_h \left(\frac{k_2}{\gamma} \right) \right]^{N_r(N_r-1)}$ where $k_2 = \sqrt{2\pi} e^{\frac{2R_0-1}{2}}$. Moreover,

$$\begin{aligned} \Pr \{A_2 < R_0\} &= \Pr \left\{ \|\mathbf{H}_2 \mathbf{1}_{(N_t-N_r+1) \times 1}\|_2^2 < \frac{2\pi}{\gamma^2} e^{2R_0-1} \right\} \\ &\leq \prod_{i=1}^{N_r} \Pr \left\{ \|\mathbf{h}_{2,i}\|_1^2 < \frac{2\pi}{\gamma^2} e^{2R_0-1} \right\} \\ &\leq \prod_{i=1}^{N_r} \prod_{j=1}^{N_t-N_r+1} \Pr \left\{ h_{2,ij} < \frac{\sqrt{2\pi}}{\gamma} e^{\frac{2R_0-1}{2}} \right\}, \end{aligned} \quad (25)$$

where $\mathbf{h}_{2,i}$ is the i^{th} row of $\mathbf{H}_2$, and $h_{2,ij}$ is the j^{th} element of $\mathbf{h}_{2,i}$. Thus $\Pr \{A_2 < R_0\} \leq \left[F_h \left(\frac{k_2}{\gamma} \right) \right]^{N_r(N_t-N_r+1)}$. Combining terms yields $\mathbb{P}_o(\gamma, R_0) \leq \left[F_h \left(\frac{k_2}{\gamma} \right) \right]^{N_r N_t}$ which by using (10) leads to $d_o \geq N_r N_t (l_0 + 1)$. The bounds coincide, thus proving Theorem 1 for the case $N_t > N_r$.

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