Skeletonized wave-equation Q_s tomography using surface-waves

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SUMMARY

We present a skeletonized inversion method that inverts surface-wave data for the Q_s quality factor. Similar to the inversion of dispersion curves for the S-wave velocity model, the complicated surface-wave arrivals are skeletonized as simpler data, namely the amplitude spectra of the windowed Rayleigh-wave arrivals. The optimal Q_s model is then found that minimizes the difference in the peak frequencies of the predicted and observed Rayleigh wave arrivals using a gradient-based wave-equation optimization method. Solutions to the viscoelastic wave-equation are used to compute the predicted Rayleigh-wave arrivals and the misfit gradient at every iteration. This procedure, denoted as wave-equation Q_s tomography (WQ_s), does not require the assumption of a layered model and tends to have fast and robust convergence compared to Q full waveform inversion (Q-FWI). Numerical examples with synthetic and field data demonstrate that the WQ_s method can accurately invert for a smoothed approximation to the subsurface Q_s distribution as long as the V_s model is known with sufficient accuracy.

INTRODUCTION

Surface waves play an important role in the characterization of the near surface for earthquake, engineering and environmental studies. Inverting and imaging surface waves can be an effective means for characterizing the subsurface at different scales (Dasgupta and Clark, 1998; Xia et al., 1999; Lin et al., 2008; Li and Hanafy, 2016), and can be important for site response and seismic hazard studies. Since the surface waves are sensitive to the near-surface elastic properties, estimation of the surface-wave velocity V_s and quality factor Q_s are of significant interest in exploration and earthquake seismology (Xia, 2014).

The near-surface S-wave velocity model is usually estimated from the dispersion curves of the recorded surface-wave arrivals (Park et al., 1998). However, in a real dissipative media, the propagation of the surface waves is strongly influenced by elastic damping in the near-surface that results in increasing amplitude loss and attenuation of high frequencies with distance travelled. As a result, the dispersion curves are also sensitive to the near-surface Q_s variations. He et al. (2015) showed that the phase velocity of the fundamental mode of the Rayleigh waves increases with decreasing values of Q_s . Thus, inverting dispersion curves for the S-wave velocity without taking into consideration the effect of Q_s can lead to erroneous estimates of the near-surface S-velocity distribution. Groos et al. (2014) showed that the S-wave velocity tomograms ob-

tained from elastic full waveform inversion (FWI) of Rayleigh waves have lower resolution when compared to the tomograms obtained by viscoelastic FWI when the near-surface is strongly anelastic. The increased resolution in S-wave tomograms obtained by taking into account the effect of Q_s can be useful in delineating near-surface faults or local anomalies (Pinson et al., 2008).

In this paper, we present a novel wave-equation Q_s tomography method (WQ_s) that inverts skeletonized surface waves for the quality factor Q_s . The skeletonized data are the frequency shifts of the observed and predicted surface-wave's spectral peaks. The Q_s method is similar to the wave-equation Q_p inversion developed by Dutta (2016) and Dutta and Schuster (2016), except it inverts for Q_s from surface waves rather than Q_p from body-wave arrivals. The Q_s model is then found that minimizes the squared differences between the predicted and observed peak-frequency shifts associated with the Rayleigh wave arrivals. For this method, the isotropic viscoelastic wave equation based on the standard linear solid model (Robertsson et al., 1994) is used to generate the predicted Rayleigh-wave arrivals. The adjoint viscoelastic wave-equation is then used to backpropagate the residual traces that are obtained by weighting the observed Rayleigh-wave arrivals with their corresponding frequency shifts. The gradient for WQ_s can be interpreted as the zero-lag cross-correlation between the forward propagated source wavefield and the backprojected weighted residual wavefield. Numerical examples on synthetic and field data validate the effectiveness of the proposed approach.

THEORY OF WAVE-EQUATION Q_s SKELETONIZED INVERSION

The theory of WQ_s is derived in a manner that is similar to that for wave-equation travelttime inversion (Luo and Schuster, 1991) and surface-wave dispersion inversion (Li and Schuster, 2016; Li et al., 2017b). These steps include: (1) define a frequency-shift misfit function, (2) define a connective function that connects the frequency-shift residual of the Rayleigh-wave arrivals with the particle velocity seismograms, and (3) derive the gradient of the misfit function with respect to Q_s using the isotropic viscoelastic wave equation and the connective function in step (2).

In our analysis, we assume that the wave propagation honors the 2D isotropic viscoelastic equations of motion based on the standard linear solid (SLS) mechanism (Robertsson et al.,

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1994):

$$\begin{aligned}\rho \frac{\partial u}{\partial t} &= \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z}, \\ \rho \frac{\partial w}{\partial t} &= \frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z}, \\ \frac{\partial \sigma_{xx}}{\partial t} &= \pi \frac{\tau_\epsilon^p}{\tau_\sigma} \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) - 2\mu \frac{\tau_\epsilon^s}{\tau_\sigma} \frac{\partial w}{\partial z} + r_{xx} + S_{xx}, \\ \frac{\partial \sigma_{zz}}{\partial t} &= \pi \frac{\tau_\epsilon^p}{\tau_\sigma} \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) - 2\mu \frac{\tau_\epsilon^s}{\tau_\sigma} \frac{\partial u}{\partial x} + r_{zz} + S_{zz}, \\ \frac{\partial \sigma_{xz}}{\partial t} &= \mu \frac{\tau_\epsilon^s}{\tau_\sigma} \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) + r_{xz}, \\ \frac{\partial r_{xx}}{\partial t} &= -\frac{1}{\tau_\sigma} (r_{xx} + \pi \left(\frac{\tau_\epsilon^p}{\tau_\sigma} - 1 \right) \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) - 2\mu \left(\frac{\tau_\epsilon^s}{\tau_\sigma} - 1 \right) \frac{\partial w}{\partial z}), \\ \frac{\partial r_{zz}}{\partial t} &= -\frac{1}{\tau_\sigma} (r_{zz} + \pi \left(\frac{\tau_\epsilon^p}{\tau_\sigma} - 1 \right) \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) - 2\mu \left(\frac{\tau_\epsilon^s}{\tau_\sigma} - 1 \right) \frac{\partial u}{\partial x}), \\ \frac{\partial r_{xz}}{\partial t} &= -\frac{1}{\tau_\sigma} (r_{xz} + \mu \left(\frac{\tau_\epsilon^s}{\tau_\sigma} - 1 \right) \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)).\end{aligned}\quad (1)$$

Here, u and w are the horizontal- and vertical-particle-velocity components, respectively, σ_{ij} denotes the ij -th component of the symmetric stress tensor, r_{ij} is the memory variable, τ_ϵ^p and τ_ϵ^s are the strain relaxation times for the P and SV waves, respectively, and τ_σ is the stress relaxation time for both the P and SV waves. S_{xx} and S_{zz} denote the source wavelets for the special case of an explosive source. The variable $\pi = \lambda + 2\mu$ is related to the Lamé parameters λ and μ whereas the stress and strain relaxation times are related to the quality factors Q_p and Q_s and the reference angular frequency ω as (Carcione et al., 1988):

$$\begin{aligned}\tau_\sigma &= \frac{\sqrt{1 + \frac{1}{Q_p^2}} - \frac{1}{Q_p}}{\omega}, \\ \tau_\epsilon^s &= \frac{1 + \omega Q_s \tau_\sigma}{\omega Q_s - \omega^2 \tau_\sigma}, \\ \tau_\epsilon^p &= \frac{1}{\omega^2 \tau_\sigma}.\end{aligned}\quad (2)$$

Equation 1 is solved for a point source at each shot point by an O(2,4) time-space domain staggered grid finite-difference algorithm. In order to generate surface waves, an explicit free-surface boundary condition is implemented by using the mirroring technique proposed by Levander (1988).

It is possible to approximate a frequency independent seismic quality factor in a limited frequency range. We define the frequency-independent parameter η as:

$$\eta = \frac{\tau_\epsilon^s}{\tau_\sigma} - 1 = \frac{1 + (\sqrt{1 + \frac{1}{Q_p^2}} - \frac{1}{Q_p})^2}{(\sqrt{1 + \frac{1}{Q_p^2}} - \frac{1}{Q_p})[Q_s - (\sqrt{1 + \frac{1}{Q_p^2}} - \frac{1}{Q_p})]}.\quad (3)$$

For the parametrization in WQs, η is used because it is quite sensitive to small changes in Q_s . The relaxation ratio η is inverted at each iteration and the updates in η are then mapped to Q_s using equation 3.

Misfit Function

We denote the Rayleigh-wave arrivals that are extracted from

Figure 1: (a) A common shot gather (CSG) comparing the Rayleigh-wave arrivals with and without Q_s . The blue dashed line shows the window used to extract these arrivals for WQs, (b) comparison between a predicted and an observed surface-wave arrival, and (c) comparison between their amplitude spectra.

the recorded data as $D_{f_g^{obs}}(\mathbf{g}, t; \mathbf{s})^{obs}$ for a vertical-component point source at $\mathbf{s}$ and a vertical-component receiver at $\mathbf{g}$ (the black curve in Figure 1a). The predicted Rayleigh-wave arrivals for the same source-receiver pair are denoted by $D_{f_g^{pred}}(\mathbf{g}, t; \mathbf{s})^{pred}$ (the red curve in Figure 1a). The peak frequency of the observed and predicted spectra are denoted as f_g^{obs} and f_g^{pred} , respectively. Here, $f_g^{pred} = f$ and $f_g^{obs} = f - f_1$, where f is the peak frequency of the event and f_1 is the shift between the peak frequencies of the predicted and the observed traces due to Q_s . A comparison between the windowed observed and predicted Rayleigh-wave arrivals for a given Q_s model is shown in Figure 1b. The amplitude spectra of these arrivals are plotted in Figure 1c, where it is evident that the observed spectrum has a lower peak frequency than the predicted spectrum. The peak frequencies in these spectra are denoted as the skeletonized surface-wave data.

The goal of WQs is to find the attenuation model $Q_s = F(\eta(\mathbf{x}))^{-1}$ so that $f_g^{pred} \approx f_g^{obs}$ for all the traces. In our case, we use the frequency-shift residual $\Delta f = f_g^{pred} - f_g^{obs}$ to form the skeletonized misfit function:

$$\epsilon = \frac{1}{2} \sum_s \sum_g \Delta f(\mathbf{g}, \mathbf{s})^2.\quad (4)$$

The gradient $\gamma(\mathbf{x})$ is given by

$$\gamma(\mathbf{x}) = \frac{\partial \epsilon}{\partial \eta(\mathbf{x})} = \sum_s \sum_g \frac{\partial \Delta f}{\partial \eta(\mathbf{x})} \Delta f(\mathbf{g}, \mathbf{s}).\quad (5)$$

The η model is updated at each iteration using the iterative steepest descent method:

$$\eta^{(k+1)} = \eta^{(k)} - \alpha \sum_s \sum_g \overbrace{\frac{\partial \Delta f}{\partial \eta(\mathbf{x})} \Delta f(\mathbf{g}, \mathbf{s})}^{\gamma(\mathbf{x}) = \text{gradient}},\quad (6)$$

where α is the step-length at the k -th iteration (Nocedal and Wright, 1999). The update for the relaxation ratio η is then mapped to Q_s using equation 3.

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Connective Function

To find an analytic expression for the Fréchet derivative $\frac{\partial \Delta f}{\partial \eta(\mathbf{x})}$ in equation 6, we define the connective function $\Phi(\mathbf{g}, t; \mathbf{s})$ that connects the change in peak frequency of an arrival with the observed and the predicted Rayleigh-wave arrivals in Figure 1b as:

$$\Phi_{f_1}(\mathbf{g}, t; \mathbf{s}) = \int D_f(\mathbf{g}, t; \mathbf{s}) D_{f-f_1}(\mathbf{g}, t; \mathbf{s})^{obs} dt. \quad (7)$$

We seek an optimal Q_s model that minimizes the peak-frequency shift between an observed and a predicted trace. For an accurate background Q_s model, the predicted and the observed arrivals will have the same peak frequency. So, we define $f_1 = \Delta f$ to be the frequency shift associated with the actual background Q_s model. If $\Delta f = 0$, it indicates that the correct background Q_s model has been found and the transmission surface-wave arrivals in the predicted and observed traces have the same peak frequencies. The derivative of Φ_{f_1} with respect to f_1 should be zero at the frequency-shift value $f_1 = \Delta f$, i.e.,

$$\begin{aligned} \Phi_{\Delta f}(\Delta f, \eta(\mathbf{x})) &= \left[\frac{\partial \Phi_{f_1}(\mathbf{g}, t; \mathbf{s})}{\partial f_1} \right]_{f_1=\Delta f} \\ &= \int D_f(\mathbf{g}, t; \mathbf{s}) \dot{D}_{f-\Delta f}(\mathbf{g}, t; \mathbf{s})^{obs} dt = 0, \end{aligned} \quad (8)$$

where $\dot{D}_{f-\Delta f}(\mathbf{g}, t; \mathbf{s})^{obs} = [\partial D_{f-f_1}(\mathbf{g}, t; \mathbf{s})^{obs} / \partial f_1]_{f_1=\Delta f}$. Note that $\Delta f = 0$ if the predicted Q_s model is the actual Q_s model. Equation 8 is the connective function (Luo and Schuster, 1991) that connects the skeletonized data, i.e., the frequency-shift residuals of the Rayleigh-wave arrivals, with the particle-velocity seismograms. Such a connective function is required because there is no wave equation that relates the skeletonized data to a single type of model parameter (Dutta and Schuster, 2016).

Using the implicit function theorem and the connective function in equation 8, the Fréchet derivative with respect to the relaxation ratio $\eta(\mathbf{x})$ can be expressed as

$$\frac{\partial \Delta f}{\partial \eta(\mathbf{x})} = - \frac{\partial \Phi}{\partial \eta(\mathbf{x})} / \frac{\partial \Phi}{\partial \Delta f}, \quad (9)$$

where the numerator on the right-hand side is given by

$$\frac{\partial \Phi_{\Delta f}}{\partial \eta(\mathbf{x})} = \int \frac{\partial D_f(\mathbf{g}, t; \mathbf{s})}{\partial \eta(\mathbf{x})} \dot{D}_{f-\Delta f}(\mathbf{g}, t; \mathbf{s})^{obs} dt, \quad (10)$$

and the denominator by

$$\frac{\partial \Phi_{\Delta f}}{\partial \Delta f} = \int D_f(\mathbf{g}, t; \mathbf{s}) \ddot{D}_{f-\Delta f}(\mathbf{g}, t; \mathbf{s})^{obs} dt = K_2. \quad (11)$$

Using equation 9, the gradient in equation 5 can be written as

$$\gamma(\mathbf{x}) = \frac{\partial \varepsilon}{\partial \eta(\mathbf{x})} = - \sum_s \sum_g \frac{\frac{\partial \Phi}{\partial \eta(\mathbf{x})}}{\frac{\partial \Phi}{\partial \Delta f}} \Delta f(\mathbf{g}, \mathbf{s}). \quad (12)$$

Then, we use the adjoint-state method to derive the Fréchet derivative. Combining equations 10-12, the gradient for WQ_s

Figure 2: (a) True velocity and (b) Q_s models used for generating the observed data, (c) background V_s model used for WQ_s inversion, (d) starting Q_s model, (e) the Q_s tomogram computed by WQ_s , and (f) the Q_s tomogram computed by Q-FWI.

can be written as

$$\begin{aligned} \gamma(\mathbf{x}) &= - \sum_s \sum_g \frac{1}{K_2} \frac{\partial \Phi}{\partial \eta(\mathbf{x})} \Delta f, \\ &= - \sum_s \int (2\mu \frac{\partial w}{\partial z} \hat{\sigma}_{xx} + 2\mu \frac{\partial u}{\partial x} \hat{\sigma}_{zz} - (\mu \frac{\partial u}{\partial z} + \mu \frac{\partial w}{\partial x}) \hat{\sigma}_{xz} \\ &\quad - 2\mu E \frac{\partial w}{\partial z} \hat{r}_{xx} - 2\mu E \frac{\partial u}{\partial x} \hat{r}_{zz} + \mu E (\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}) \hat{r}_{xz}) dt. \end{aligned} \quad (13)$$

Here $(\hat{u}, \hat{w}, \hat{\sigma}_{xx}, \hat{\sigma}_{zz}, \hat{\sigma}_{xz}, \hat{r}_{xx}, \hat{r}_{zz}, \hat{r}_{xz})$ are the adjoint-state variables of $(u, w, \sigma_{xx}, \sigma_{zz}, \sigma_{xz}, r_{xx}, r_{zz}, r_{xz})$, $E = \frac{1}{\tau_\sigma}$ and K_2 is defined in equation 11. It can be shown that the residual trace, $\hat{f}_w$, that is backpropagated at every iteration is given by:

$$\hat{f}_w = \frac{1}{K_2} \dot{D}(\mathbf{g}, t; \mathbf{s})^{obs} \Delta f(\mathbf{g}, \mathbf{s}). \quad (14)$$

NUMERICAL TESTS

We now compare the performance of WQ_s against that of Q-FWI for the near-surface V_s and Q_s models shown in Figure 2. A smooth version of the true S-wave velocity model, shown in Figure 2c, is used as the background velocity model for WQ_s and Q-FWI. The grid spacing and time sampling intervals for the 2D viscoelastic finite-difference algorithm are 1 m and 0.025 ms, respectively, and the center frequency of the source wavelet is 35 Hz. The observed data are generated by 40 shots evenly distributed on the surface and the data are recorded by 70 receivers every 2 m on the surface. The initial Q_s model is a homogeneous half space (Figure 2d).

The Q_s tomograms from the WQ_s and Q-FWI methods after 21 iterations are shown in Figures 2e and 2f, respectively. Figure

WQs Tomography

Figure 3: The peak-frequencies for different source-receiver pairs in (a) the observed data, (b) the predicted data after WQ_s , and (c) their differences.

Figure 4: (a) P-wave tomogram with ray tracing tomography, and (b) S-wave velocity tomogram by wave equation dispersion inversion (Li et al., 2017a).

3 reveals that the predicted peak frequencies from WQ_s are very close to the actual ones for many source-receiver pairs.

The WQ_s method is now tested on a near-surface field data set recorded over the Qademah fault. There are 60 shots and 60 receivers on the surface placed at 10 m intervals. The first-arrival traveltimes are picked in all the CSGs and inverted using ray-based traveltime tomography to obtain the P-wave velocity model shown in Figure 4a. The S-wave tomogram is shown in Figure 4b, which is obtained using the wave-equation surface wave dispersion method (Li et al., 2017b). These velocity models are used as the background velocity models for WQ_s . For WQ_s , the starting Q_s model is taken to be homogeneous with $Q_s = 1000$ and the inverted Q_s tomogram is shown in Figure 5a. There is reasonable geological agreement between the S-wave velocity model in Figure 4b and the Q_s tomogram in Figure 5a. The high attenuation regions in the Q_s tomogram (low Q_s values) correspond to the low S-wave velocity regions. Previous work by Zhang et al. (2015) demonstrated that areas with high V_p/V_s ratios tend to have low Q values (or high attenuation). We calculate the V_p/V_s ratio using the tomograms in Figure 4 and the ratio is shown in Figure 5b. It can be seen from this figure that areas with high V_p/V_s ratio have low Q_s values.

As a final sanity check, the inverted Q_s tomogram is compared

Figure 5: (a) WQ_s tomogram, (b) ratio (V_p/V_s) computed from a) and b), and (c) common offset gather (offset=50 m) profile after data processing.

to a common offset gather (COG) profile (Hanafy et al., 2015). Figure 5c shows a COG profile using the processed data for an offset of 50 m. The black dashed line in this figure shows the location of the fault which is between 250-300 m. The locations of the Q_s anomalies in the WQ_s tomogram and the low-velocity area in the S-wave tomogram are consistent with the location of the fault, as indicated in the COG profile.

CONCLUSIONS

We presented a skeletonized surface-wave wave-equation Q_s inversion method, where a Q_s model is found that minimizes the sum of the squared difference between the peak-frequencies of the observed and the predicted surface-wave arrivals. The gradient for WQ_s is obtained by a zero-lag cross-correlation between the forward propagated viscoelastic source wavefield and the weighted backprojected residuals that are obtained by weighting the observed particle velocity seismograms with the residual frequency shifts. This method does not require a layered-medium assumption used in conventional Q_s estimation techniques and also does not suffer from the limitations of ray-tracing based Q tomography methods or Q-FWI.

EDITED REFERENCES

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REFERENCES

- Carcione, J. M., D. Kosloff, and R. Kosloff, 1988, Wave propagation simulation in a linear viscoelastic medium: *Geophysical Journal International*, **95**, 597–611, <http://dx.doi.org/10.1111/j.1365-246X.1988.tb06706.x>.
- Dasgupta, R. and R. A. Clark, 1998, Estimation of Q from surface seismic reflection data: *Geophysics*, **63**, 2120–2128, <http://dx.doi.org/10.1190/1.1444505>.
- Dutta, G., 2016, Skeletonized wave-equation inversion for Q: 86th Annual International Meeting, SEG, Expanded Abstracts, 3618–3623, <http://dx.doi.org/10.1190/segam2016-13839306>.
- Dutta, G. and G. T. Schuster, 2016, Wave-equation Q tomography: *Geophysics*, **81**, no. 6, R471–R484, <http://dx.doi.org/10.1190/geo2016-0081.1>.
- Groos, L., M. Schäfer, T. Forbriger, and T. Bohlen, 2014, The role of attenuation in 2D full-waveform inversion of shallow-seismic body and Rayleigh waves: *Geophysics*, **79**, no. 6, R247–R261, <http://dx.doi.org/10.1190/geo2013-0462.1>.
- Hanafy, S. M., A. AlTheyab, and G. T. Schuster, 2015, Controlled noise seismology: 85th Annual International Meeting, SEG, Expanded Abstracts, 5102–5106, <http://dx.doi.org/10.1190/segam2015-5906063.1>.
- He, Y., J. Gao, and Z. Chen, 2015, On the comparison of properties of Rayleigh waves in elastic and viscoelastic media: *International Journal of Numerical Analysis and Modeling*, **12**, 254–267.
- Levander, A. R., 1988, Fourth-order finite-difference P-SV seismograms: *Geophysics*, **53**, 1425–1436, <http://dx.doi.org/10.1190/1.1442422>.
- Li, J., G. Dutta, and G. T. Schuster, 2017a, Wave-equation Qs inversion of skeletonized surface waves: *Geophysical Journal International*, **209**, 979–991, <https://doi.org/10.1093/gji/ggx051>.
- Li, J., Z. Feng, and G. Schuster, 2017b, Wave-equation dispersion inversion: *Geophysical Journal International*, **208**, 1567–1578, <https://doi.org/10.1093/gji/ggw465>.
- Li, J. and S. Hanafy, 2016, Skeletonized inversion of surface wave: Active source versus controlled noise comparison: *Interpretation*, **4**, SH11–SH19, <http://dx.doi.org/10.1190/INT-2015-0174.1>.
- Li, J. and G. Schuster, 2016, Skeletonized wave-equation of surface wave dispersion inversion: 86th Annual International Meeting, SEG, Expanded Abstracts, 3630–3635, <http://dx.doi.org/10.1190/segam2016-13770057.1>.
- Lin, F.-C., M. P. Moschetti, and M. H. Ritzwoller, 2008, Surface wave tomography of the Western United States from ambient seismic noise: Rayleigh and Love wave phase velocity maps: *Geophysical Journal International*, **173**, 281–298, <http://dx.doi.org/10.1111/j.1365-246X.2008.03720.x>.
- Luo, Y. and G. T. Schuster, 1991, Wave-equation travelttime inversion: *Geophysics*, **56**, 645–653, <http://dx.doi.org/10.1190/1.1443081>.
- Nocedal, J. and S. Wright, 1999, Numerical optimization: Springer Series in Operations Research, Springer Company 35.
- Park, C. B., R. D. Miller, and J. Xia, 1998, Imaging dispersion curves of surface waves on multi-channel record: 68th Annual International Meeting, SEG, Expanded Abstracts, 1377–1380, <http://dx.doi.org/10.1190/1.1820161>.
- Pinson, L. J., T. J. Henstock, J. K. Dix, and J. M. Bull, 2008, Estimating quality factor and mean grain size of sediments from high-resolution marine seismic data: *Geophysics*, **73**, no. 4, G19–G28, <http://dx.doi.org/10.1190/1.2937171>.

- Robertsson, J. O., J. O. Blanch, and W. W. Symes, 1994, Viscoelastic finite-difference modeling: *Geophysics*, **59**, 1444–1456, <http://dx.doi.org/10.1190/1.1443701>.
- Xia, J., 2014, Estimation of near-surface shear-wave velocities and quality factors using multichannel analysis of surface-wave methods: *Journal of Applied Geophysics*, **103**, 140–151, <http://dx.doi.org/10.1016/j.jappgeo.2014.01.016>.
- Xia, J., R. D. Miller, and C. B. Park, 1999, Estimation of near-surface shear-wave velocity by inversion of Rayleigh waves: *Geophysics*, **64**, 691–700, <http://dx.doi.org/10.1190/1.1444578>.
- Zhang, L., D. Zhu, and X. Zhang, 2015, Seismic attributes method for prediction of unconsolidated sand reservoirs of heavy oil: *The Open Fuels & Energy Science Journal*, **8**, 14–18, <http://dx.doi.org/10.2174/1876973X01508010014>.