Estimation of fracture parameters using elastic full-waveform inversion

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SUMMARY

Current methodologies to characterize fractures at the reservoir scale have serious limitations in spatial resolution and suffer from uncertainties in the inverted parameters. Here, we propose to estimate the spatial distribution and physical properties of fractures using full-waveform inversion (FWI) of multicomponent surface seismic data. An effective orthorhombic medium with five clusters of vertical fractures distributed in a checkboard fashion is used to test the algorithm. A shape regularization term is added to the objective function to improve the estimation of the fracture azimuth, which is otherwise poorly constrained. The cracks are assumed to be penny-shaped to reduce the nonuniqueness in the inverted fracture weaknesses and achieve a faster convergence. To better understand the inversion results, we analyze the radiation patterns induced by the perturbations in the fracture weaknesses and orientation. Due to the high-resolution potential of elastic FWI, the developed algorithm can recover the spatial fracture distribution and identify localized “sweet spots” of intense fracturing. However, the fracture azimuth can be resolved only using long-offset data.

INTRODUCTION

Estimation of location, orientation, and physical properties of fractures at the reservoir scale from surface data is of significant interest in reservoir characterization and Oil and Gas production. The fracture compliances (weaknesses) are indicators of the density and the infill of fractures. The spatial distribution and preferential direction of fracture networks control the flow and transportation of fluids in the subsurface (Tsvankin and Grechka, 2011). In theory, these properties can be estimated from the parameters of the fracture-induced azimuthally anisotropic effective medium, which often has orthorhombic symmetry. Bakulin et al. (2000a,b,c) provide a comprehensive analysis of the anisotropic seismic signatures associated with vertical fractures. They show how to estimate the physical properties (such as the normal and tangential compliances) and azimuth of the fractures from multicomponent data. However, existing methods of fracture detection are mostly based on reflection traveltimes (moveout analysis) and/or amplitudes (AVO analysis). Because these methods utilize different parts of the recorded information, they often produce contradictory results (Pérez et al., 1999; Tsvankin and Grechka, 2011). Here we show that inverting waveforms can improve estimates of both the spatial distribution and physical properties of fractures.

In general, full-waveform inversion (FWI) aims to employ entire waveforms of diving and/or reflected waves to reconstruct

the subsurface velocity field. Masmoudi et al. (2016) analyzed the feasibility of high-resolution fracture characterization for orthorhombic media using a waveform-inversion method based on an acoustic approximation. Zhang et al. (2017) applied elastic FWI to invert for all three fracture weaknesses. Oh and Alkhalifah (2017) proposed an efficient inversion workflow for elastic orthorhombic media with unknown azimuths of the vertical symmetry planes.

To simulate wave propagation in a realistic fractured medium (e.g., fractured shale), here we consider an effective elastic orthorhombic medium formed by parallel vertical fractures embedded in a transversely isotropic background with a vertical symmetry axis (VTI). It is one of the most common physical models for fractured reservoirs (Bakulin et al., 2000b). The effective compliance tensor is obtained from the linear-slip theory (Schoenberg, 1980), and the elastic wave equation is solved by a staggered finite-difference method. In the linear-slip theory, fractures are described by three dimensionless parameters—one normal (Δ_N) and two tangential (Δ_V and Δ_H) weaknesses (normalized compliances). These parameters typically are much smaller than unity, which allows us to linearize the modeling equations. An iterative local optimization method is applied to update the model.

After describing the proposed method, we test it on an effective orthorhombic medium with five clusters of vertical fractures. One cluster is assumed to be a sweet spot with relatively large tangential weaknesses. Because seismic signatures are almost insensitive to the fracture shape, we assume the simplified model of penny-shaped cracks ($\Delta_V = \Delta_H$) with different strike directions (azimuths) distributed in a check-board fashion. In the current implementation, we invert only for the fracture weaknesses and orientations using a known background VTI model.

THEORY

We have devised a multiparameter elastic FWI algorithm for orthorhombic media to estimate the fracture weaknesses and orientation. The objective function contains a standard data misfit term and a shape regularization term (Gallardo and Meju, 2003):

$$J(\mathbf{m}) = \|\mathbf{W}_d(\mathbf{d}^{pre} - \mathbf{d}^{obs})\|^2 + \alpha \|\nabla \mathbf{m}_{\Delta_T} \times \nabla \mathbf{m}_{\phi}\|^2, \quad (1)$$

where $\mathbf{d}^{pre}$ and $\mathbf{d}^{obs}$ are the vectors of the multicomponent predicted and observed data, $\mathbf{W}_d$ is a weighting operator ($\mathbf{W}_d = \sigma_d \mathbf{I}$), and σ_d is the standard deviation of the predicted data. The shape regularization term imposes a consistent spatial behavior on the inverted horizontal weakness and azimuth (“ ∇ ” denotes the spatial gradient and “ $\times$ ” denotes the cross product). The parameter α controls the level of similarity between

these two parameters. Our numerical examples and radiation pattern analysis (see below) show that Δ_T is the best resolved parameter. Hence, Δ_T is used in the regularization term to improve the accuracy of the inverted ϕ , which otherwise may be insufficient.

The elastic wave equation for anisotropic media can be written as:

$$\begin{pmatrix} \rho \mathbf{I}_3 & 0 \\ 0 & \mathbf{c}^{-1} \end{pmatrix} \frac{\partial \Psi}{\partial t} - \begin{pmatrix} 0 & E^T \\ E & 0 \end{pmatrix} \Psi - \mathbf{s} = 0, \quad (2)$$

where $\mathbf{I}_3$ is a 3×3 identity matrix, $\mathbf{c}$ is the invertible stiffness matrix, $\Psi = (\mathbf{v}, \sigma)^T$ is the wavefield vector that includes the particle velocity and stress fields, E is the differential matrix, and “ T ” denotes a transpose.

According to the linear-slip theory (Schoenberg, 1980), the effective compliance tensor of a fractured medium can be obtained by adding the fracture compliances $\mathbf{s}_f$ to that of the VTI background ($\mathbf{s}_b$):

$$\mathbf{c} = (\mathbf{s}_b + \mathbf{s}_f)^{-1} = [(\mathbf{I} + \mathbf{s}_f \mathbf{s}_b^{-1}) \mathbf{s}_b]^{-1} = \mathbf{c}_b [\mathbf{I} + \mathbf{s}_f \mathbf{c}_b]^{-1}. \quad (3)$$

Equation 3 can be expanded in a Taylor series if all elements of $\mathbf{s}_f \mathbf{c}_b$ are much smaller than unity, which is the case if the weaknesses are sufficiently small. Then equation 3 can be approximated by retaining only linear terms:

$$\mathbf{c} \approx \mathbf{c}_b - \mathbf{c}_b \mathbf{s}_f \mathbf{c}_b = \mathbf{c}_b - \delta \mathbf{c}. \quad (4)$$

We assume a set of parallel fractures with the normal in the x_I -direction embedded in a VTI medium. The fracture compliance matrix linearized in the weaknesses is given by (Bakulin et al., 2000b):

$$\mathbf{s}_f^{lin} = \begin{bmatrix} \frac{\Delta_N}{c_{11b}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\Delta_T}{c_{55b}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\Delta_T}{c_{66b}} \end{bmatrix}. \quad (5)$$

For more general cases of multiple fracture sets with normals not perpendicular to the x_I -direction, the effective stiffness tensor can be obtained using the Bond transformation (Bakulin et al., 2000c):

$$\mathbf{s}_f^{rot} = \mathbf{N} \mathbf{s}_f^{lin} \mathbf{N}^T, \quad (6)$$

where the matrix $\mathbf{N}$ has the following form for a clockwise rotation around the vertical direction (z):

$$\mathbf{N} = \begin{bmatrix} \cos^2 \phi & \sin^2 \phi & 0 & 0 & 0 & \frac{\sin 2\phi}{2} \\ \sin^2 \phi & \cos^2 \phi & 0 & 0 & 0 & -\frac{\sin 2\phi}{2} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos \phi & -\sin \phi & 0 \\ 0 & 0 & 0 & \sin \phi & \cos \phi & 0 \\ -\sin 2\phi & \sin 2\phi & 0 & 0 & 0 & \cos 2\phi \end{bmatrix}. \quad (7)$$

The stiffness matrix for the rotated orthorhombic model in equation 2 is then calculated from equation 4.

The gradient of the misfit (objective) function in equation 1 can be calculated from the adjoint-state method:

$$\mathbf{g} = \left(\frac{\partial \mathbf{d}^{pre}}{\partial \mathbf{m}} \right)^T \mathbf{W}_d^T \mathbf{W}_d (\mathbf{d}^{pre} - \mathbf{d}^{obs}) - 2\alpha \delta_{\mathbf{m}, \mathbf{m}_{\Delta_T}} \nabla \cdot [\nabla \mathbf{m}_\phi \times (\nabla \mathbf{m}_{\Delta_T} \times \nabla \mathbf{m}_\phi) - 2\alpha \delta_{\mathbf{m}, \mathbf{m}_\phi} \nabla \cdot [\nabla \mathbf{m}_{\Delta_T} \times (\nabla \mathbf{m}_\phi \times \nabla \mathbf{m}_{\Delta_T})], \quad (8)$$

where $\delta_{\mathbf{m}, \mathbf{m}_{\Delta_T}}$ and $\delta_{\mathbf{m}, \mathbf{m}_\phi}$ are the Dirac delta functions.

The model is updated iteratively using gradient-based optimization:

$$\mathbf{m} = \mathbf{m}_0 - \gamma \mathbf{g}; \quad (9)$$

γ is the step length calculated by a line-search method.

NUMERICAL EXAMPLE

The model includes five clusters of vertical fractures (Table 1) embedded in the 3D SEAM VTI model (Fehler and Keliher, 2011) used as the background medium. The staggered finite-difference method is implemented to solve the wave equation 2 (Virieux, 1986). There are 24 inlines and crosslines evenly covering the surface, each with 24 explosive sources and 48 receivers placed at constant increments. The recorded data are three-component (3C) particle velocities. One fracture set, which has a relatively large tangential compliance (due to the higher crack density), plays the role of a potential sweet spot to be detected from surface data. To illustrate the fracture distribution, the actual and initial models of the stiffness C_{11} are shown in Figure 1. The shaded blue blocks mark the fractured areas. The assumed sweet spot, which has a relatively large crack density, is located in the center of the model (Figure 2). The initial weaknesses Δ_N and Δ_T and the initial azimuth ϕ are set to zero throughout the model.

Figure 1: Actual (a) and initial (b) stiffness C_{11} . The shaded blocks (blue) mark the five fractured areas.

Three peak frequencies, 3.5 Hz, 5 Hz and 7.5 Hz, are used in a multistage inversion. We applied the L-BFGS method with a proper preconditioning to the gradients during the iterative model updating. The inverted weaknesses Δ_N and Δ_T and the azimuth ϕ are shown in Figure 3. The assumed sweet spot can be reliably identified through its large tangential weakness. However, the azimuth ϕ is not sufficiently resolved at depth mainly because it mostly influences the relatively far-offset

Table 1: Embedded fracture sets.

Fracture set	A (sweet spot)	B	C	D	E
Location of center (z,x,y)	(0.35,0.5,0.5)	(0.53,0.35,0.35)	(0.25,0.35,0.65)	(0.53,0.75,0.35)	(0.25,0.75,0.65)
Δ_N	0.1	0.1	0.1	0.1	0.1
Δ_T	0.15	0.08	0.08	0.08	0.08
ϕ (deg)	10	5	-5	5	-5

Figure 2: Actual weaknesses Δ_N (a) Δ_T (b) and the azimuth ϕ (c).

data, which were missing in our acquisition. Vertical parameter profiles across each fracture set are displayed in Figure 4. As expected, fractures located near the edge of the model are inverted with a lower resolution compared to those close to the center. A larger offset-to-depth ratio is needed to better resolve the fracture azimuth. An important quality control factor is the change of the normalized misfit with iterations plotted in Figure 5 for stage 1 (3.5 Hz). It takes only about 10 iterations for the inversion to converge.

Figure 3: Inverted weaknesses Δ_N (a) and Δ_T (b) and azimuth ϕ (c). The area of large Δ_T indicates a possible “sweet spot”.

RADIATION PATTERN ANALYSIS

To better understand the inversion results, Figure 6 shows the radiation (reflection) patterns of the fracture weaknesses and orientation for the PP-wave that dominates the wavefield (Oh and Alkhalifah, 2016). The radiation patterns depend on the background medium parameters; here we analyze only the initial model (VTI background with an orthorhombic perturbation). This simplified model is adequate in the weak-anisotropy approximation. The patterns exhibit more sensitivity to the parameters Δ_N and ϕ at large opening angles; therefore, these parameters need large offset-to-depth ratios to be resolved. The weakness Δ_T generates more energy at medium-to-large opening angles so that it can be inverted with a reasonable resolution from conventional-spread data. The pattern for Δ_N has a larger magnitude which should help in the recovery of the nor-

Figure 4: Vertical profiles of Δ_N (a), Δ_T (b), and ϕ (c). From top to the bottom are fracture sets A, B, C, D and E respectively. Blue: actual model; Cyan: initial model; Pink: inverted model.

mal weakness. Estimation of the azimuth ϕ depends on the accuracy of the resolved weaknesses. Because the S-wavelength is much smaller than that of the P-wave, the converted waves PS1 and PS2 could potentially provide higher-resolution FWI results. In terms of azimuthal coverage, Δ_N can be recovered from narrow-azimuth data (the fracture planes are nearly perpendicular to the inline direction), but Δ_T and ϕ need a wider range of azimuths. There are some overlaps in the radiation patterns which indicate a potential for crosstalk between the corresponding parameters. However, the trade-offs are mitigated in the employed parameterization because Δ_N is generally separated from Δ_T and ϕ . Indeed, Δ_T and ϕ are both

Figure 5: Change of the normalized objective function with iterations. The inversion is terminated when the function flattens out.

sensitive to large opening angles but at different azimuths, and Δ_T has additional constraints from medium opening angles.

Figure 6: Azimuthally varying radiation patterns of PP-waves for the fracture weaknesses Δ_N (a), Δ_T (b) and azimuth ϕ (c). The radial direction indicates the opening angle and the azimuth is shown around the plot.

CONCLUSIONS

We proposed a new approach to estimate the spatial distribution, orientation, and physical properties of fractures by waveform inversion of multicomponent surface seismic data. The high-resolution potential of elastic full-waveform inversion allows the developed algorithm to reconstruct the spatial fracture distribution and identify localized “sweet spots” of intense fracturing. However, reliable estimation of the fracture orientation requires large offset-to-depth ratios and well-resolved weaknesses. Both radiation pattern analysis and numerical results indicate that the inversion for all fracture parameters is better posed for long-offset data. The radiation patterns also reveal that whereas the normal weakness can be recovered from narrow-azimuth data, the inversion for the tangential weakness and orientation requires a wider azimuthal coverage. The algorithm can potentially produce high-resolution results using converted modes recorded on conventional spreads because PS-waves have smaller opening angles. The azimuthal angle ϕ proved to be the least constrained parameter, so a shape-regularization term is added to the objective function to improve the accuracy in ϕ . Note that fracture azimuth typically remains consistent laterally and can often be estimated by AVO analysis or moveout inversion. The proposed method has the same limitations as more conventional FWI algorithms: it requires a good background model and sufficient subsurface illumination.

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EDITED REFERENCES

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