Elastic reflection waveform inversion with variable density

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Summary

Elastic full waveform inversion (FWI) provides a better description of the subsurface than those given by the acoustic assumption. However it suffers from a more serious cycle skipping problem compared with the latter. Reflection waveform inversion (RWI) provides a method to build a good background model, which can serve as an initial model for elastic FWI. Therefore, we introduce the concept of RWI for elastic media, and propose elastic RWI with variable density. We apply Born modeling to generate the synthetic reflection data by using optimized perturbations of P- and S-wave velocities and density. The inversion for the perturbations in P- and S-wave velocities and density is similar to elastic least-squares reverse time migration (LSRTM). An incorrect initial model will lead to some misfits at the far offsets of reflections; thus, can be utilized to update the background velocity. We optimize the perturbation and background models in a nested approach. Numerical tests on the Marmousi model demonstrate that our method is able to build reasonably good background models for elastic FWI with absence of low frequencies, and it can deal with the variable density, which is needed in real cases.

Introduction

Elastic waveform inversion aims to estimate elastic model parameters for seismic imaging and interpretation, such as the P- and S-wave velocities and density, by minimizing the misfit between the predicted and observed multicomponent data (Tarantola, 1986; Virieux and Operto, 2009). Although elastic multiparameter inversion can utilize multicomponent data and extract more medium properties compared with acoustic inversion, the nonlinearity of inversion will increase because of the additional degrees of freedom. The parameterization and inversion scheme, which are widely studied, are crucial for the stability of multiparameter inversion (Kohn et al., 2012; Jeong et al., 2012; Operto et al., 2013). Additionally, an initial model, close enough to the true model, is prerequisite for the convergence to the global minimum in elastic FWI (Symes, 2008). Xu et al. (2012) and Wang et al. (2013), based on the previous work (Mora, 1989; Plessix et al., 1995), proposed a reflection waveform inversion method to construct the background model using reflections generated from migration/demigration. The low wavenumber update of the background model requires a true amplitude migration that can be used to predict the reflections in the migration/demigration process. With a least-square optimization to the predicted image, Wu and

Alkhalifah (2015) proposed a new optimization problem where the background model and perturbations are inverted simultaneously. Subsequently, Guo and Alkhalifah (2016) brought the concept of RWI to elastic media and developed elastic reflection waveform inversion that aims to invert for the long wavelength components of P- and S-wave velocities. However, the variable density as a potential reflector, with little influence on the propagation of waves, can not be neglected as a perturbation parameter. The absence of perturbation density can cause amplitude errors in predicted reflections and thus expose our RWI to some artifacts. The inverted density in elastic FWI plays the role of absorbing high-wavenumber components mainly on the layer interfaces (Xu and McMechan, 2014). We invert for the perturbations and background models simultaneously, and the inverted model parameters include the perturbations of P- and S-wave velocities and density and the background P- and S-wave velocities.

Base on the Born approximation, we use the objective function proposed by Alkhalifah and Wu (2014). The gradient for the perturbations of P- and S-wave velocities and density and the background part of P- and S-wave velocities can be derived using the adjoint state method (Plessix, 2006). We then establish elastic reflection waveform inversion scheme with variable density. Finally, numerical tests on the Marmousi model demonstrate that our proposed method has the potential to construct good initial models for elastic FWI in the absence of low frequencies.

Theory

The 2-D elastic wave displacement-stress equation is given as:

$$\begin{cases} \rho \frac{\partial^2 u_x}{\partial t^2} = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} \\ \rho \frac{\partial^2 u_z}{\partial t^2} = \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z} \\ \tau_{xx} = (\lambda + 2\mu) \frac{\partial u_x}{\partial x} + \lambda \frac{\partial u_z}{\partial z} \\ \tau_{zz} = \lambda \frac{\partial u_x}{\partial x} + (\lambda + 2\mu) \frac{\partial u_z}{\partial z} \\ \tau_{xz} = \mu \frac{\partial u_x}{\partial z} + \mu \frac{\partial u_z}{\partial x} \end{cases}, \quad (1)$$

where u_x and u_z are displacement components, τ_{xx} , τ_{xz} and τ_{zz} are stress components, λ and μ are lame parameters, and ρ is the bulk density.

We split the elastic medium parameters into background and perturbation components, which can be written as:

Elastic RWI with variable density

$$\rho_{all} = \rho + \delta\rho \quad \lambda_{all} = \lambda + \delta\lambda \quad \mu_{all} = \mu + \delta\mu \quad (2)$$

Correspondingly, the total wavefield can be defined as the summation of the background and perturbed wavefields:

$$\mathbf{u}_{all} = \mathbf{u} + \delta\mathbf{u}, \quad \boldsymbol{\tau}_{all} = \boldsymbol{\tau} + \delta\boldsymbol{\tau}. \quad (3)$$

We subtract the wave equation for the background wavefield (eq. 1) from that for the total wavefield, and apply the Born approximation. After further simplification, the perturbed wavefields satisfy the following wave equations:

$$\begin{cases} \rho \frac{\partial^2 \delta u_x}{\partial t^2} = \frac{\partial \delta \tau_{xx}}{\partial x} + \frac{\partial \delta \tau_{xz}}{\partial z} - \delta \rho \frac{\partial^2 u_x}{\partial t^2} \\ \rho \frac{\partial^2 \delta u_z}{\partial t^2} = \frac{\partial \delta \tau_{zx}}{\partial x} + \frac{\partial \delta \tau_{zz}}{\partial z} - \delta \rho \frac{\partial^2 u_z}{\partial t^2} \\ \delta \tau_{xx} = (\lambda + 2\mu) \frac{\partial \delta u_x}{\partial x} + \lambda \frac{\partial \delta u_z}{\partial z} + (\delta\lambda + 2\delta\mu) \frac{\partial u_x}{\partial x} + \delta\lambda \frac{\partial u_z}{\partial z} \\ \delta \tau_{zz} = \lambda \frac{\partial \delta u_x}{\partial x} + (\lambda + 2\mu) \frac{\partial \delta u_z}{\partial z} + (\delta\lambda + 2\delta\mu) \frac{\partial u_x}{\partial x} + \delta\lambda \frac{\partial u_z}{\partial z} \\ \delta \tau_{xz} = \mu \frac{\partial \delta u_x}{\partial x} + \mu \frac{\partial \delta u_z}{\partial z} + \delta\mu \frac{\partial u_x}{\partial x} + \delta\mu \frac{\partial u_z}{\partial z} \end{cases} \quad (4)$$

The perturbed wavefield refers to the single scattered wavefield, which is produced when the background wavefield encounters the perturbation of medium parameters. That is to say, eq. (4) expresses the demigration operator.

We employ the objective function (Wu and Alkhalifah, 2015):

$$\min_{\mathbf{m}, \delta\mathbf{m}} O(\mathbf{m}, \delta\mathbf{m}) = \frac{1}{2} \sum_{s,r} \|\mathbf{u} + \delta\mathbf{u} - \mathbf{u}_{obs}\|^2. \quad (5)$$

where $\mathbf{u}_{obs}$ is the observed multicomponent data, $\mathbf{m} = (\lambda, \mu, \rho)$ and $\delta\mathbf{m} = (\delta\lambda, \delta\mu, \delta\rho)$ are background and perturbed model parameters, respectively. The background and perturbed model parameters are inverted simultaneously to minimize the misfit between the modeled and observed data. Specifically, we update the model perturbations to fit the near-offset data well. Thus, when the background models are closer to the true models, the residuals at the far offsets will then be used to further correct the background models.

Given the objective function, we deduce the gradient of the functional with respect to model perturbations using the adjoint state method (Plessix, 2006):

$$\begin{aligned} \nabla O_{\delta\lambda} &= -\sum_s \int_0^T \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} \right) \left(\frac{\partial u_x^\dagger}{\partial x} + \frac{\partial u_z^\dagger}{\partial z} \right) dt \\ \nabla O_{\delta\mu} &= -\sum_s \int_0^T \left\{ 2 \left(\frac{\partial u_x}{\partial x} \frac{\partial u_x^\dagger}{\partial x} + \frac{\partial u_z}{\partial z} \frac{\partial u_z^\dagger}{\partial z} \right) + \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} \right) \left(\frac{\partial u_x^\dagger}{\partial x} + \frac{\partial u_z^\dagger}{\partial z} \right) \right\} dt \\ \nabla O_{\delta\rho} &= \sum_s \int_0^T \frac{\partial u_x}{\partial t} \frac{\partial u_x^\dagger}{\partial t} + \frac{\partial u_z}{\partial t} \frac{\partial u_z^\dagger}{\partial t} dt \end{aligned} \quad (6)$$

and the gradient for the background model:

$$\begin{aligned} \nabla_x O &= -\sum_s \int_0^T \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} \right) \left(\frac{\partial u_x^\dagger}{\partial x} + \frac{\partial u_z^\dagger}{\partial z} \right) dt + \int_0^T \left(\frac{\partial \delta u_x}{\partial x} + \frac{\partial \delta u_z}{\partial z} \right) \left(\frac{\partial u_x^\dagger}{\partial x} + \frac{\partial u_z^\dagger}{\partial z} \right) dt \\ &\quad + \int_0^T \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} \right) \left(\frac{\partial \delta u_x^\dagger}{\partial x} + \frac{\partial \delta u_z^\dagger}{\partial z} \right) dt \\ \nabla_\mu O &= -\sum_s \int_0^T \left\{ 2 \left(\frac{\partial u_x}{\partial x} \frac{\partial u_x^\dagger}{\partial x} + \frac{\partial u_z}{\partial z} \frac{\partial u_z^\dagger}{\partial z} \right) + \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} \right) \left(\frac{\partial u_x^\dagger}{\partial x} + \frac{\partial u_z^\dagger}{\partial z} \right) \right\} dt \\ &\quad + \int_0^T \left\{ 2 \left(\frac{\partial \delta u_x}{\partial x} \frac{\partial u_x^\dagger}{\partial x} + \frac{\partial \delta u_z}{\partial z} \frac{\partial u_z^\dagger}{\partial z} \right) + \left(\frac{\partial \delta u_x}{\partial x} + \frac{\partial \delta u_z}{\partial z} \right) \left(\frac{\partial u_x^\dagger}{\partial x} + \frac{\partial u_z^\dagger}{\partial z} \right) \right\} dt \\ &\quad + \int_0^T \left\{ 2 \left(\frac{\partial u_x}{\partial x} \frac{\partial \delta u_x^\dagger}{\partial x} + \frac{\partial u_z}{\partial z} \frac{\partial \delta u_z^\dagger}{\partial z} \right) + \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} \right) \left(\frac{\partial \delta u_x^\dagger}{\partial x} + \frac{\partial \delta u_z^\dagger}{\partial z} \right) \right\} dt \\ \nabla_\rho O &= \sum_s \int_0^T \frac{\partial u_x}{\partial t} \frac{\partial u_x^\dagger}{\partial t} + \frac{\partial u_z}{\partial t} \frac{\partial u_z^\dagger}{\partial t} dt + \int_0^T \frac{\partial \delta u_x}{\partial t} \frac{\partial u_x^\dagger}{\partial t} + \frac{\partial \delta u_z}{\partial t} \frac{\partial u_z^\dagger}{\partial t} dt \\ &\quad + \int_0^T \frac{\partial u_x}{\partial t} \frac{\partial \delta u_x^\dagger}{\partial t} + \frac{\partial u_z}{\partial t} \frac{\partial \delta u_z^\dagger}{\partial t} dt \end{aligned} \quad (7)$$

where $\mathbf{u}^\dagger$ is the wavefield propagating backward in time generated from the residual data at the receiver positions, $\delta\mathbf{u}^\dagger$ is the perturbed wavefield propagating backward generated from the wavefield $\mathbf{u}^\dagger$ at the model perturbations.

The trade-off between different parameters is inevitable in the multiparameter inversion. We should select a proper parameterization to suppress the crosstalk, especially for the important parameters that we care about. Here, we use the velocity parameterization, where the investigation of other parameterizations will be left to future study. We can obtain the gradients with respect to P- and S-wave velocities and density by using the chain rule, given by:

$$\begin{aligned} \nabla O_{v_p} &= 2\rho v_p \nabla O_\lambda \\ \nabla O_{v_s} &= -4\rho v_s \nabla O_\lambda + 2\rho v_s \nabla O_\mu \\ \nabla O_\rho &= (v_p^2 - 2v_s^2) \nabla O_\lambda + v_s^2 \nabla O_\mu + \nabla O_\rho \end{aligned} \quad (8)$$

Considering that the stable update of the background model needs an accurate perturbation image, the perturbations of P- and S-wave velocities and density are simultaneously optimized in the elastic least-squares migration to produce the reflections. The background parts of P- and S-wave velocities are updated iteratively in the elastic RWI.

Examples

We test our method on the elastic Marmousi model. We add a constant-velocity layer (we keep it fixed through the inversion) above the original model to reduce the source near field effect. The true P-wave velocity model is shown in Figure 1. The true S-wave velocity model is built with a poisson ratio of 0.25. The true density model is constructed based on the Gardner formula (Gardner et al., 1974). The explosive source, a Ricker wavelet with peak frequency of 5 Hz after filtering out frequencies below 2.5 Hz, is used in synthesizing the wavefields. We ignite 47 evenly distributed shots on the surface. The initial P-wave velocity is shown in Figure 2. Similarly, we build the initial S-wave velocity and density. Given the initial models, we first update the model perturbations (δv_p , δv_s , $\delta\rho$)

Elastic RWI with variable density

simultaneously using the near-offset residuals. Actually, the optimization of model perturbations is equivalent to elastic least-squares reverse time migration, however with an inaccurate velocity. The optimized model perturbations (i.e., δv_p , δv_s and $\delta \rho$) are shown in Figure 3. The inverted background P- and S-wave velocities (i.e., v_p and v_s) are shown in Figure 4a and 4b, and the optimal δv_p , δv_s and $\delta \rho$ for the inverted background models are shown in Figure 5a, 5b and 5c, respectively. Most of the reflections and scattered waves are focused and located at the accurate positions. The quality of image is considerably improved compared to the first iteration. Adding the model perturbations to the background models, we can obtain the more exact models, which can be used as initial model for the subsequent conventional elastic FWI, and the results are shown in Figure 6. For comparison, we apply conventional elastic FWI starting from the initial models shown in Figure 2, and obtain the inverted results shown in Figure 7. In addition, we also implement elastic RWI in the case that the density perturbation is not considered, that is to say, $\delta \rho = 0$. The inverted background P- and S-wave velocities shown in Figure 8 contain more high-wavenumber artifacts, compared to the inverted background models using our approach (Figure 4). The comparisons of P- and S-wave velocity profiles at location 3 km are shown in Figure 9. As we can see from these figures, our ERWI approach can deal with the variable-density case, and provide a good starting model for conventional EFWI to avoid cycle skipping even in the absence of the low frequencies below 2.5 Hz.

Figure 1: The true P-wave velocity.

Figure 2: The initial P-wave velocity.

Figure 3: The inverted model perturbations δv_p (a), δv_s (b) and $\delta \rho$ (c) for the initial models.

Figure 4: The inverted background models v_p (a) and v_s (b).

Figure 5: The inverted model perturbations δv_p (a), δv_s (b) and $\delta \rho$ (c) for the background models shown in Figure 4.

Elastic RWI with variable density

Figure 6: The elastic FWI results starting from the background models (Figure 4) with the inverted perturbations (Figure 5) added to them: (a) Inverted v_p and (b) inverted v_s .

Figure 7: The elastic FWI results starting from the initial models shown in Figure 2. Inverted v_p (a) and inverted v_s (b).

Figure 8: The inverted background models v_p (a) and v_s (b) in the case that the density perturbation is not inverted.

Figure 9: The comparison of v_p (a) and v_s (b) profiles at the distance of 3 km. (Red: true velocity, cyan: initial velocity, green: the inverted background velocity, blue: the elastic FWI result starting from the inverted models, dash black: conventional EFWI starting from the initial models shown in Figure 2).

Conclusions

We proposed an elastic RWI method with variable density. The background and perturbation components of elastic medium parameters are updated in a nested way. Considering that the density influences the amplitudes of reflections, the perturbation components of density and velocities are optimized simultaneously to better fit the near-offset reflections. The inclusion of density helps mitigate high wavenumber artifacts from our RWI results. Numerical tests demonstrate that our approach can better recover the background P- and S-wave velocities, which can accurately describe the kinematic information. Using the background models, the elastic LSRTM can be implemented effectively. In addition, high-resolution models are obtained when the model perturbations are added to the background models, and enable elastic FWI to converge to a more accurate result with less iterations.

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EDITED REFERENCES

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