Acoustic VTI wavefield tomography of P-wave surface and VSP data

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SUMMARY

Transversely isotropic (TI) models have become standard in depth imaging and are often used in waveform inversion. Here, we develop a robust wave-equation-based tomographic algorithm for building acoustic VTI (transversely isotropic with a vertical symmetry axis) velocity models from P-wave surface reflection and vertical seismic profiling (VSP) data. Wavefield extrapolation is performed with an integral operator to avoid generating shear-wave artifacts. Focusing energy in extended images produced by reverse-time migration (RTM) makes it possible to update the zero-dip NMO velocity V_{nmo} and the anellipticity parameter η . To constrain the anisotropy coefficient δ and improve the accuracy in V_{nmo} and η , we employ borehole information by introducing an additional objective-function term designed to fit VSP data. Image-guided smoothing is applied to both data- and image-domain gradients to steer the inversion towards geologically plausible solutions. Testing on the VTI Marmousi model shows that the joint inversion of surface and VSP data helps estimate all three relevant medium parameters.

INTRODUCTION

Migration velocity analysis (MVA) algorithms update the background velocity model by enforcing the consistency of the migrated image for different illumination angles (Al-Yahya, 1989). Compared to conventional ray-based techniques, wave-equation (wavefield) tomography provides more reliable results for complex geologic structures. Angle-dependent illumination in wavefield tomography is typically evaluated using extended images (Sava and Vasconcelos, 2011), and different energy-focusing criteria are employed to update the velocity model (Symes and Carazzone, 1991; Zhang and Shan, 2013).

Transversely isotropic models are often built in the pseudoacoustic approximation (Alkhalifah, 1998), which helps reduce the computational cost. Integral pseudoacoustic operators (Du et al., 2014) allow one to model pure P-modes without the shear-wave artifacts.

It is well known that robust estimation of the Thomsen parameters V_{P0} (symmetry-direction velocity), ϵ , and δ just from P-wave reflections is seldom feasible (Tsvankin, 2012; Wang and Tsvankin, 2013). Weibull and Arntsen (2014) attempt to simultaneously estimate V_{P0} , ϵ , and δ in the image domain using the elastic wave equation. However, they apply a purely isotropic wave-mode separation technique and exclude shear-wave information. E. Li et al. (2016) mitigate tradeoffs between the model parameters by using structure-guided steering filters and prior rock-physics information. Their results, however, show that realistic errors in the covariance matrix re-

sult in insufficient updates in the anisotropy coefficients. Here, we combine P-wave surface and VSP data to simultaneously estimate all three relevant VTI parameters using the adjoint-state gradients derived by V. Li et al. (2017) for the image- and data-domain objective functions.

METHODOLOGY

P-mode wavefield extrapolation in VTI media

P-wave kinematics in VTI media is controlled by the vertical velocity V_{P0} and Thomsen parameters ϵ and δ (Tsvankin and Thomsen, 1994). Alternatively, one could parameterize the VTI model with the zero-dip normal-moveout (NMO) velocity $[V_{\text{nmo}} = V_{P0}\sqrt{1+2\delta}]$, the anellipticity parameter η , and either V_{P0} or δ . The parameters V_{nmo} and η are primarily responsible for P-wave reflection moveout (Alkhalifah and Tsvankin, 1995; Tsvankin, 2012) and can be potentially constrained by surface P-wave data.

Integral wave-equation solutions use the P-wave dispersion relation to obtain the phase shift for extrapolating (time-stepping) the wavefield (Du et al., 2014). In the pseudoacoustic approximation, the 2D P-wave dispersion relation for VTI media can be written as (Alkhalifah, 1998):

$$\omega^2 = \frac{1}{2} \left[(1+2\epsilon)V_{P0}^2 k_x^2 + V_{P0}^2 k_z^2 \right] \times \left\{ 1 + \sqrt{1 - \frac{8(\epsilon-\delta)k_x^2 k_z^2}{[(1+2\epsilon)k_x^2 + k_z^2]^2}} \right\}, \quad (1)$$

where k_x and k_z are the horizontal and vertical wavenumbers, respectively. Assuming that the term containing $\epsilon - \delta$ under the radical is small, the first-order Padé expansion in that term yields (Schleicher and Costa, 2015):

$$\omega^2 = (1+2\epsilon)V_{P0}^2 k_x^2 + V_{P0}^2 k_z^2 - 2(\epsilon-\delta)V_{P0}^2 \frac{k_x^2 k_z^2}{k_x^2 + k_z^2} \times \left[1 - 2\epsilon \frac{k_x^2}{k_x^2 + k_z^2} + 2(\epsilon-\delta) \frac{k_x^2 k_z^2}{(k_x^2 + k_z^2)^2} \right]. \quad (2)$$

Here, the Padé coefficients α and β in equation 17 of Schleicher and Costa (2015) are set to 1/2 and 1/4 respectively.

Objective function

Our algorithm iteratively updates the background model by focusing energy in the extended image domain. To reduce the ambiguity in parameter estimation, the objective function includes a term that measures the similarity between the observed and predicted VSP data:

$$J = J_{\text{IM}} + \alpha J_{\text{VSP}}, \quad (3)$$

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where J_{IM} is the term that enforces the consistency in extended images, J_{VSP} is designed to fit VSP data, and α is a model-dependent weighting factor chosen by numerical testing. Both terms of the objective function are described in more detail below.

Extended images contain information about angle-dependent illumination needed for wave-equation MVA. The general form of the imaging condition can be written as (Sava and Vasconcelos, 2011):

$$I(\mathbf{x}, \lambda, \tau) = \sum_{e,t} W_s(\mathbf{x} - \lambda, t - \tau) W_r(\mathbf{x} + \lambda, t + \tau), \quad (4)$$

where $I(\mathbf{x}, \lambda, \tau)$ is the extended image, W_s and W_r are the source and receiver wavefields, respectively, λ is the space lag, τ is the time lag, and e indicates summation over experiments (shots). Model updating can be based on penalizing energy at nonzero lags or maximizing it at zero-lag. In this paper, we use the “partial” stack-power objective function designed to focus energy at small horizontal lags λ_x (Zhang and Shan, 2013):

$$J_{IM} = -\frac{1}{2} \|H(\lambda_x) I(x, z, \lambda_x)\|_{\ell_2}^2, \quad (5)$$

where H is a Gaussian operator centered at zero-lag. Partial stack power has proved to be more suitable for typical TI models than differential semblance optimization (V. Li et al., 2017).

The conventional data-difference misfit function is suboptimal for fitting VSP data because the initial model is likely to be inaccurate and cause cycle-skipping. Here, we follow Wu and Alkhalifah (2017) and employ the “selective correlation” objective function that operates with time lags to overcome cycle-skipping problems. The envelope-based objective function helps boost low frequencies in the adjoint source and stabilize the inversion of VSP data.

Inversion algorithm

The gradients for both terms of the objective function in equation 3 are obtained with the adjoint-state method. It is convenient to invert for the parameters V_{nmo}^2 , $2V_{nmo}^2\eta$, and V_{p0}^2 , which form a linear parameter set for the dispersion relation employed in the integral extrapolator. The gradients for these parameters can be obtained from the ones described in V. Li et al. (2017) by applying the chain rule. Because this parameterization is linear, the gradients are computed just from wavefield correlation and do not involve any model-parameter terms. The inverted parameters are then recomputed into V_{nmo} , η , and δ .

Because P-wave reflections alone cannot constrain the vertical velocity, the image-domain part of the algorithm is used to update only the parameters V_{nmo}^2 , $2V_{nmo}^2\eta$, whereas the parameter V_{p0}^2 is updated by matching VSP records. If walkaway VSP data are available, the algorithm can be modified to invert J_{VSP} for all three parameters. Similar to the methodology of Wang and Tsvankin (2013), we apply image-guided smoothing (Hale, 2009) to the VSP-data gradients to propagate the updates away from the borehole locations. This approach is close to that described by Guitton et al. (2012) for isotropic FWI. The gradients are used in the L-BFGS (Nocedal and Wright, 2006) model-updating algorithm.

SYNTHETIC EXAMPLE

The algorithm is tested on the constant-density VTI Marmousi model in Figure 1 (Guitton and Alkhalifah, 2016). The parameters V_{nmo} , η , and δ are estimated simultaneously by combining P-wave reflections with short-offset VSP data. The V_{nmo} -field of the initial model is substantially distorted, and the initial anisotropy coefficients η and δ are set to zero. The balancing factor α is set to two, which assigns a larger weight to the VSP objective-function term. Applying image-guided smoothing to the VSP-data gradient helps extrapolate the resulting updates along the structure away from the borehole (Figure 2).

After several iterations of the joint tomography, the parameters V_{nmo} , η , and δ are updated in the correct direction (Figure 3). Figure 4 shows that the model refinement significantly improves event focusing and continuity in the RTM image, especially in the fault area (between the lateral coordinates 3.5 and 6 km). Because image-guided smoothing does not propagate the VSP-data gradients through the fault area, the depth scale of the section is correctly reconstructed only to the left of the faults (i.e., where the borehole is located). Further improvements can be achieved by mitigating illumination and aperture-truncation artifacts in the extended images.

CONCLUSIONS

We developed a wave-equation-based algorithm for joint inversion of P-wave surface and VSP data in VTI media. Signatures of reflection events in the extended domain are employed to update the key moveout parameters V_{nmo} and η . Including borehole information is crucial for estimating the anisotropy coefficient δ and reconstructing the depth scale of the section. Image-guided smoothing of the inversion gradients computed from VSP data steers the inversion results towards geologically plausible solutions.

The algorithm is applied to the joint surface and VSP data set from the constant-density VTI Marmousi model. Although the initial model was isotropic with a strongly distorted V_{nmo} -field, the inversion provided substantially improved estimates of both V_{nmo} and η . Including VSP data helped recover the depth scale of the section around the borehole location. The updates in η are impeded by the tradeoff between this parameter and V_{nmo} , which can be mitigated by either including long-offset walkaway VSP data or employing multistage inversion strategies. The obtained VTI model considerably improves the quality of the RTM image, especially in the fault area.

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Figure 1: VTI Marmousi model: (a) V_{nmo} , (b) η , and (c) δ . The red and green lines mark the sources and receivers (respectively) for the VSP data set. (d) Initial model for V_{nmo} obtained by applying strong smoothing to the actual V_{nmo} -field on plot (a). The initial coefficients η and δ are set to zero.

Figure 2: Gradient of the VSP term of the objective function (J_{VSP}) with respect to V_{P0}^2 shown (a) before and (b) after applying image-guided smoothing.

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Figure 3: Updated model parameters after several iterations of joint tomography: (a) V_{nmo} , (b) η , and (c) δ .

Figure 4: Conventional RTM image computed with the (a) initial and (b) inverted VTI models.

EDITED REFERENCES

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