3D elastic full waveform inversion for OBC data using the P-wave excitation amplitude

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Summary

We suggest a fast and efficient 3D elastic full waveform inversion (FWI) algorithm based on the excitation amplitude (maximum energy arrival) of the P-wave in the source wavefield. It evaluates the gradient direction significantly faster than its conventional counterpart. In addition, it removes the long-wavelength artifacts from the gradient, which are often originated from SS correlation process. From these advantages, the excitation approach offers faster convergence not only for the S wave velocity, but also for the entire process of multi-parameter inversion, compared to the conventional FWI. The feasibility of the proposed method is demonstrated through the synthetic Marmousi and a real OBC data from North Sea.

Introduction

Multi-parameter elastic FWI is in the forefront of our efforts to recover sub-surface information from measured seismic data (Operto et al., 2013; Alkhalifah et al., 2016). Although promising, it suffers from the endemic problems of a large computational cost and for Null space in its quest in recovering the unknowns despite accommodating better physics into the inversion framework (Tarantola, 1986; Brossier et al., 2009; Prioux et al., 2013; Vigh et al., 2014). However, as usual, trade-off exist between the computational advantages and the accuracy of Physics. To ameliorate the former, numerous solutions exist, including boundary saving (Raknes and Weibull, 2016) schemes, time-frequency domain approaches (Sirgue et al., 2008) and wavefield compression techniques (Boehm et al., 2016). Alternately, Kalita and Alkhalifah (2016) reduce the computational overhead by approximating the source wavefield with the excitation amplitude (the most energetic arrival) and its arrival time (excitation time) in acoustic FWI. In this abstract, we employ the excitation approach (ExA) under the elastic FWI realm to mitigate both the computational and convergence issues of large-scale 3D problems. We show the versatility of elastic excitation method on Marmousi II model and 3D real data. In addition to the computational advantages, ExA removes long-wavelength artifacts from SS correlations, thus, results in a cleaner gradient than its conventional counterpart.

P-wave ExA in elastic FWI

The global correlation norm (Choi and Alkhalifah, 2012) that measures the mismatch between observed (d_o) and modeled (d_m) data is defined as

$$E = -\sum_r \left\{ \hat{d}_m(\mathbf{p}) \cdot \hat{d}_o \right\} \quad (1)$$

where $\hat{d}_m(\mathbf{p}) = d_m(\mathbf{p}) / \|d_m(\mathbf{p})\|$ and $\hat{d}_o = d_o / \|d_o\|$.

The gradient of this objective function with respect to a model parameter $\mathbf{p}$ is given by

$$\frac{\partial E}{\partial \mathbf{p}}(\mathbf{x}) = \int \left[\frac{\partial c_{ijkl}}{\partial \mathbf{p}}(\mathbf{x}) \frac{\partial u_i}{\partial x_k}(\mathbf{x}, t) \frac{\partial u_i^a}{\partial x_j}(\mathbf{x}, t_{\max} - t) \right] dt, \quad (2)$$

where c_{ijkl} represents the elastic constants as a function of space coordinates $\mathbf{x}$; u and u^a denote the source and adjoint wavefields, respectively. The source wavefield, u , is a solution of elastic wave equation,

$$\rho(\mathbf{x}) \frac{\partial^2 u_i}{\partial t^2}(\mathbf{x}, t) - \frac{\partial}{\partial x_j} \left[c_{ijkl}(\mathbf{x}) \frac{\partial u_l}{\partial x_k}(\mathbf{x}, t) \right] = f_i(\mathbf{x}_s, t), \quad (3)$$

where ρ and f indicate the density and seismic source at a position, $\mathbf{x}_s$. The adjoint wavefield, u^a , can be computed by backpropagating the following adjoint source,

$$f^a(\mathbf{x}_r, t) = \frac{1}{\|d_m(\mathbf{p})\|} \left\{ \hat{d}_m(\mathbf{p}) \left(\hat{d}_m(\mathbf{p}) \cdot \hat{d}_o \right) - \hat{d}_o \right\}. \quad (4)$$

Eq. (2) requires both the source and adjoint wavefields to be present for the dot product operation, which is computationally cumbersome especially in large 3D problems. However, ExA mitigates those requirements using the following steps (Kalita and Alkhalifah, 2016):

- (I) Computation of the excitation time and amplitude,
- (II) modification in the adjoint source by a temporal cross-correlation process of source function,
- (III) evaluation of gradient direction only at the excitation time.

Therefore, ExA does not require storing entire wavefields. Neither it requires additional wavefield extrapolation, often used in boundary saving schemes (Rakness and Weibull, 2016). As a result, ExA completes the gradient evaluation accessing a limited memory block and investing shorter time than its conventional counterpart. Unlike acoustic media, displacement fields include both P- and S-waves together in the elastic case. Therefore, it is hard to segregate the excitation time of P-wave in elastic media. As suggested by Nguyen and McMechan (2015) in the context of reverse time migration, we compute excitation time of the divergence of wavefields (S_v), which includes only P-wave motions. Next, we store the original displacement wavefield at this excitation time. Mathematically,

$$t_{ex} = t_{ex}(\mathbf{x}) = \arg \max_{t \in [1, nt]} \left\{ S_v(\mathbf{x}, t) \right\} \quad (5)$$

represents the excitation time of P waves. Using ExA in eq. (2), the gradient can be reduced as follows:

$$\frac{\partial E}{\partial \mathbf{p}}(\mathbf{x}) = \frac{\partial c_{ijkl}}{\partial \mathbf{p}}(\mathbf{x}) \frac{\partial u_i}{\partial x_k}(\mathbf{x}, t_{ex}(\mathbf{x})) \frac{\partial \tilde{u}_i^a}{\partial x_j}(\mathbf{x}, t_{\max} - t_{ex}(\mathbf{x})), \quad (6)$$

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where $\tilde{u}^a$ is the modified adjoint source in step (II).

Automatic mode separation using P-wave ExA

In addition to enhanced computational speed, ExA offers the possibility of an automatic mode separation in elastic FWI. As eq. (2) shows, the conventional gradient direction incurs all possible correlations between P- and S-waves. However, using the excitation approximation with P-waves, the gradient evaluation process automatically filters out the SP and SS contributions, which mainly includes artifacts in marine surveys.

Figure 1 shows the P- and S-wave velocities of the true Marmousi II model. The original Marmousi model has a soft sea bottom, which induces slow S-wave velocity. However, to avoid numerical dispersion, S-wave velocities are modified in this example. We use a homogeneous density model (1g/cm^3) and update only seismic velocities. For initial models, we use smoothed P- and S-wave velocities. As Kalita and Alkhalifah (2016) demonstrated, the gradient directions for P-wave velocity from the boundary saving method and ExA show quite similar features. This is because, in elastic FWI, the P-wave velocity perturbation still generates only PP waves (Tarantola, 1986). However, the gradient direction for the S-wave velocity using conventional methods includes some long wavelength artifacts (yellow arrows in Figure 2a). These long wavelength artifacts are products of SS correlations (Figure 2c). As discussed earlier, the gradient direction using ExA contains neither SP nor SS correlations, thus, resembles the mode-separated gradient (Wang and Cheng, 2017) as shown in Figure 2b. Moreover, those long-wavelength artifacts aggravate the convergence of S-wave velocity, so does the overall convergence rate of multi-parameter FWI (Figures 3a and 3b). Thus, ExA improves computational speed and reduces number of required iterations involved in completing the FWI.

Figure 1: The true (a) P- and (b) S-wave velocities.

In anisotropic media, mode separation requires additional work in solving the Christoffel equation (Cheng and Kang, 2014), which increases the cost in large 3D elastic FWI. However, in marine survey (P-wave source), excitation

times for P-waves can be computed using the divergence operator in anisotropic media, in which the divergence operator results in much stronger P-wave than S-wave energy unless anisotropy is extremely strong.

Figure 2: The gradient directions for the S-wave velocity at 1st iteration from (a) conventional method, (b) mode-separated PP + PS, (c) mode-separated SP + SS waves and (d) P-wave ExA.

Figure 3: The inverted P- (a and c) and S-wave (b and d) velocities using the conventional method (a and b) and ExA (c and d).

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Real data application: 2C 3D OBC data

We further apply the P-wave ExA to 3D real OBC data from the North Sea (Szydluk et al., 2007). We use a smoothed initial model (Figure 4) derived from the tomography result. The direct waves are muted by the data-owner so we opt to use the global correlation norm over the least squares. FWI is conducted for data with frequencies ranging from 2.75 to 10 Hz. We randomly choose 200 sources out of the total of 3400 at each iteration to avoid artifacts (Díaz and Guitton, 2011).

Figure 4: The initial P-wave velocity model.

Figure 5: The schematic diagram illustrating the FWI strategy.

Figure 6: (a) Inverted P-wave velocity after conventional acoustic FWI of the vertical component, (b) initial S-wave velocity for elastic FWI and inverted S-wave velocities from inline component using (c) conventional method and (d) P-wave ExA.

Figure 7: The gradient directions for the S-wave velocity from (a) P-wave ExA, (b) the conventional method and (c) mode-separated SP+SS waves.

Figure 5 shows the FWI strategy for 2-component Volve data set. As Sears et al. (2010) suggested, we separately invert for v_p from vertical component and then v_s from

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inline component, which helps to reduce the trade-off (Oh and Alkhalifah, 2016). Since PS waves in the vertical component are also muted by the data-owner, conventional acoustic FWI is good enough for the vertical component (Figures 6a, 8a and 9a). For elastic FWI in the 2nd stage, we build up an initial S-wave velocity from inverted P-wave velocity with a fixed Poisson’s ratio (Figure 6b). The density is assumed to satisfy Gardner’s formula (Gardner et al., 1974) and is, thus, not updated. As Figure 9a (blue arrows) shows, it seems that the Volve area has significant anisotropy. For this reason, we use smoothed background anisotropic parameters extracted from tomography without any update. For the elastic FWI applied in the 2nd stage, we compare the performance of using the P-wave ExA in the FWI with using the conventional approach. Figure 7 displays gradient directions of the S-wave velocity. As we expected, conventional gradients (Figure 7b) have long-wavelength artifacts induced by SS correlation (Figure 7c) whereas ExA (Figure 7a) produces clean gradient directions. Although long wavelength features are not properly recovered due to lack of long wavelength components in PP and PS radiation patterns (Tarantola, 1986), ExA captures low S-wave velocity layers (Figure 6d) providing a better convergence rate and computational speed (Figure 8b) compared to the conventional FWI. The vertical profiles (Figure 8) also support that the proposed approach produces both P- and S-wave velocities with less trade-offs, particularly at 2.4 km in depth. The seismogram (Figure 9) demonstrates that PP waves are well inverted after an acoustic FWI of the vertical component and PS waves are partially inverted after elastic FWI for the horizontal component (red arrows).

Conclusions

To improve the computational speed and convergence rate of multi-parameter FWI in large 3D elastic anisotropic media, we proposed an elastic FWI with P-wave excitation amplitude of the source wavefields. The excitation method invests shorter computational time in gradient evaluation than its conventional counterpart. In addition, it is mainly immune from artifacts resulting from unwanted SS correlation, which is inevitable in conventional methods. Although multipathing is theoretically a limitation of this method, it does not impede the quality of the inversion process, as justified through the synthetic and real data examples.

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Figure 8: (a) Vertical profiles extracted from the well, which starts from the red dot on the top and ends at the blue dot at the bottom in Figure 4 and (b) computational time of boundary saving method and ExA.

Figure 9: Comparisons of (a) vertical and (b) inline components of observed and modeled data from inverted model after acoustic FWI and after elastic FWI using P-wave excitation amplitude. The source (black dot) and receiver (black dashed line) locations are marked in Figure 4. We also display modeled data from initial models for comparison. Notice that the acoustic modeling is used for (a) and elastic modeling is used for (b).

EDITED REFERENCES

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