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Q-Least Squares Reverse Time Migration with Viscoacoustic Deblurring Filters

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Q-LSRTM with Viscoacoustic Deblurring Filter

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ABSTRACT

Viscoacoustic least-squares reverse time migration, also denoted as Q-LSRTM, linearly inverts for the subsurface reflectivity model from lossy data. Compared to the conventional migration methods, it can compensate for the amplitude loss in the migrated images due to strong subsurface attenuation and can produce reflectors that are accurately positioned in depth. However, the adjoint Q propagators used for backward propagating the residual data are also attenuative. Thus, the inverted images from Q-LSRTM are often observed to have lower resolution when compared to the benchmark acoustic LSRTM images from acoustic data. To increase the resolution and accelerate the convergence of Q-LSRTM, we propose using viscoacoustic deblurring filters as a preconditioner for Q-LSRTM. These filters can be estimated by matching a simulated migration image to its reference reflectivity model.

Numerical tests on synthetic and field data demonstrate that Q-LSRTM combined with viscoacoustic deblurring filters can produce images with higher resolution and more balanced amplitudes than images from acoustic reverse time migration (RTM), acoustic LSRTM and Q-LSRTM when there is strong attenuation in the background medium. The proposed preconditioning method is also shown to improve the convergence rate of Q-LSRTM by more than 30 percent in some cases and significantly compensate for the lossy artifacts in RTM images.

INTRODUCTION

Migration deconvolution (MD) is used to deblur migration images corrupted by artifacts due to coarse source and receiver sampling (Hu and Schuster, 1998), limited aperture width, strong velocity contrasts, and uneven subsurface illumination (Hu et al., 2001). The MD filter assumes a homogeneous layered medium around the image point and a sufficiently wide recording aperture. In this case, the migration Green’s function (Schuster and Hu, 2000), sometimes denoted as a point spread function (PSF) (Jansson and Richardson, 1997), can be efficiently computed by invoking a layered-medium assumption (under this assumption, the migration response can be horizontally invariant) localized around the trial image point. Therefore, the migration filter can be approximated as a lateral convolution between the reflectivity model and the migration Green’s function. Thus, the MD operation can be used as an approximation to the inverse of the Hessian to mitigate migration artifacts.

Hu and Schuster (1998) and Hu et al. (2001) designed a migration deconvolution operator in the space-wavenumber domain to suppress the migration artifacts of poststack migration. Yu et al. (2006) extended the application of MD from poststack to prestack depth migration. Instead of using MD in the space-wavenumber domain, Guitton (2004) approximated the inverse of the Hessian in the space domain with a bank of matching filters. These filters are similar to deblurring filters and have been used as a preconditioner for conventional least-squares migration (LSM) (Aoki and Schuster, 2009). For multisource LSM, Dai and Schuster (2009) and Dai et al. (2011) used deblurring filters to reduce the crosstalk noise and accelerate the convergence of multisource LSM.

The previous work on MD assumed a lossless background medium. However, strong subsurface attenuation can significantly distort the amplitudes and phases of seismic waves

(Aki and Richards, 1980). In this case, conventional acoustic reverse time migration (RTM) and least-squares reverse time migration (LSRTM) cannot completely correct for the attenuation loss.

To account for attenuation, Blanch and Symes (1994) and Blanch and Symes (1995) used the adjoint state method and proposed a linearized inversion method to recover the short wavelength model components in a viscoacoustic medium in both the $\tau - p$ domain and space-time domains. Ribodetti et al. (1995) developed an analytical inversion in the frequency domain for viscoacoustic media using the Born approximation and asymptotic solutions for Green functions. Dai and West (1994), Yu et al. (2002), Wang (2007) and Valenciano et al. (2011) used the one-way wave-equation migration in the frequency domain for attenuation compensation. For reverse time migration, Zhang et al. (2010), Suh et al. (2012), Fletcher et al. (2012), Zhu et al. (2014) and Zhu and Harris (2015) proposed different viscoacoustic wave equations with separate controls over phase and amplitude to compensate for the attenuation loss. Viscoacoustic least-squares reverse time migration, also denoted as Q-LSRTM, has been shown to compensate for the attenuation loss and produce images with more balanced amplitudes and accurately positioned reflectors than standard migration techniques (Dutta and Schuster, 2014; Dai et al., 2015; Sun et al., 2016). However, the inverted images from Q-LSRTM sometimes have lower resolution when compared to the original acoustic LSRTM images. This is because the adjoint Q propagators used for backpropagating the data residual are also attenuative. Hence, a large number of least-squares iterations are required to get the desired uplift in the image quality, which makes the Q-LSRTM technique computationally expensive when compared to standard RTM.

To mitigate these problems, we propose using viscoacoustic deblurring filters as a pre-

conditioner for Q-LSRTM. A reference reflectivity model is first constructed using a uniform distribution of point scatterers while the background velocity and Q models are kept the same. The viscoacoustic data generated from these background models are then migrated by viscoacoustic reverse time migration (Q-RTM) to obtain a reference migration image. The viscoacoustic deblurring filters are then estimated for different parts of the migration image using local matching filters to transform the simulated migration image into its reference reflectivity model. These filters are then used as a preconditioner during the Q-LSRTM iterations. The estimation of these local filters can be done in parallel using the Message Passing Interface (MPI) language, which makes the preconditioning process very efficient. Numerical results with these deblurring filters show a much faster convergence rate for Q-LSRTM and a much improved image for Q-RTM. These benefits only require the extra computational cost of constructing the deblurring filters, which is no more than one migration of the data.

This paper is divided into four sections. After the introduction, the second section presents the theory of Q-LSRTM with viscoacoustic deblurring filters. Numerical tests on synthetic and field data are then used to demonstrate the advantages of the proposed preconditioning method. The conclusions are in the last section.

THEORY

In an acoustic medium, the forward modeling of the data can be represented mathematically as

$$\mathbf{d} = \mathbf{L}\mathbf{m}_0, \tag{1}$$

where $\mathbf{L}$ represents a linear modeling operator, $\mathbf{d}$ is the data vector, and $\mathbf{m}_0$ is a vector that represents the subsurface reflectivity model. The migration image $\mathbf{m}_{mig}$ is computed by applying the migration operator $\mathbf{L}^\dagger$ to the observed data to give

$$\mathbf{m}_{mig} = \mathbf{L}^\dagger \mathbf{d} = \overbrace{\mathbf{L}^\dagger \mathbf{L}}^{\text{blurring operator}} \mathbf{m}_0, \quad (2)$$

where $\mathbf{L}^\dagger$ denotes the adjoint of the forward modeling operator $\mathbf{L}$. Since the adjoint operator $\mathbf{L}^\dagger$ in equation 2 is not the inverse of the forward modeling operator, the computed migration image $\mathbf{m}_{mig}$ is a blurred version of the true reflectivity model $\mathbf{m}_0$. Here, the blurring operator is defined as the $\mathbf{L}^\dagger \mathbf{L}$ operator in equation 2. The blurring operator blurs the true reflectivity model $\mathbf{m}_0$ to give the migration image $\mathbf{m}_{mig}$, which often suffers from artifacts because of poor sampling of the source and receiver intervals and uneven illumination. The blurring can be mostly corrected by applying the inverse of $\mathbf{L}^\dagger \mathbf{L}$ to the migration image as

$$\mathbf{m}_0 = \overbrace{(\mathbf{L}^\dagger \mathbf{L})^{-1}}^{\text{deblurring operator}} \mathbf{m}_{mig}. \quad (3)$$

However, computing the direct inverse of $\mathbf{L}^\dagger \mathbf{L}$ is computationally prohibitive for practical seismic imaging problems. One possible solution is iterative least-squares migration (Lailly, 1983; Nemeth et al., 1999), which typically requires an order-of-magnitude more computations than standard migration. To accelerate the convergence, deblurring operators can be designed to approximate the inverse of the Hessian operator $(\mathbf{L}^\dagger \mathbf{L})^{-1}$, which can then be used as a preconditioning operator (Hu et al., 2001; Guitton, 2004; Yu et al., 2006; Aoki and Schuster, 2009; Dai and Schuster, 2009).

Guitton (2004) and Aoki and Schuster (2009) proposed localized deblurring filters which

deblur the migration image in the space domain. The deblurring filters are estimated using a reference model and its migration image. Following Aoki and Schuster (2009), the reference reflectivity model is constructed using a uniform distribution of point scatters as shown in Figure 1a. The reference data $\mathbf{d}_{ref}$ are generated from these reference reflectivity and background velocity models, which are then migrated to get a reference migration image $\mathbf{m}_{mig-ref}$ as

$$\mathbf{m}_{mig-ref} = \mathbf{L}^\dagger \mathbf{L} \mathbf{m}_{ref} = \mathbf{L}^\dagger \mathbf{d}_{ref}. \quad (4)$$

As shown in Figure 1, $\mathbf{m}_{ref}$ and $\mathbf{m}_{mig-ref}$ are then divided into subdomains. A subdomain or a local window is chosen such that a point scatterer is at the center of the window and the size of the window is large enough to cover the main part of the migration butterfly (Hu et al., 2001). Within the local window, the migration Green's function can be assumed to be approximately invariant with respect to the scatterer's location. The deblurring filter for each local window is then estimated by locally matching the reference migration image with the true reference reflectivity model as (Aoki and Schuster, 2009):

$$m_{ref}(x, y, z)_i = \int_{V_0} F(x - x_0, y - y_0, z - z_0)_i m_{ref-mig}(x_0, y_0, z_0)_i dV_0, \quad (5)$$

which can also be written in the matrix-vector notation:

$$[\mathbf{m}_{ref}]_i = [\mathbf{F}]_i \otimes [\mathbf{m}_{ref-mig}]_i, \quad (6)$$

where $\otimes$ denotes spatial convolution, and i indicates the i th local window. Here, $[\mathbf{F}]_i$, $[\mathbf{m}_{ref}]_i$ and $[\mathbf{m}_{ref-mig}]_i$ denote the deblurring filter, the reference reflectivity model and

the reference migration image within the i th local window, respectively. Similar to the localized migration Green's function, each deblurring filter is also shift invariant within its local window.

Since the convolution operation is commutative, we can write

$$[\mathbf{F}]_i \otimes [\mathbf{m}_{ref_mig}]_i = [\mathbf{m}_{ref_mig}]_i \otimes [\mathbf{F}]_i, \quad (7)$$

so that, for the 2D case, the reference migration image $[\mathbf{m}_{mig_ref}]_i$ in the local window (shown as the black dashed square in Figure 1) can be reformulated as a convolution matrix $\mathbf{M}_{ref_mig}$ of size $M \times N$. Also, the corresponding deblurring filter (shown as the red dash square in Figure 1) and the reflectivity model can be reformulated as a vector $\mathbf{f}_i$ of size $N \times 1$ and a vector of $\mathbf{m}_i$ of size $M \times 1$, respectively. Here, $N = fx \times fz$ and $M = wx \times wz$, where fx and fz are the filter lengths in the horizontal and vertical directions, respectively, and wx and wz are the window lengths in the horizontal and vertical directions, respectively. Therefore we can rewrite equation 5 as

$$[\mathbf{M}_{ref_mig}]_i [\mathbf{f}]_i = [\mathbf{m}]_i. \quad (8)$$

Here, equation 8 represent an overdetermined problem which has $M \times 1$ equations and $N \times 1$ unknowns (usually, N is smaller than M), and can be easily solved by a least-squares method. Multiplying $[\mathbf{M}_{ref_mig}]_i^\dagger$ on both sides of equation 8 gives the normal equations

$$[\mathbf{M}_{ref_mig}]_i^\dagger [\mathbf{M}_{ref_mig}]_i [\mathbf{f}]_i = [\mathbf{M}_{ref_mig}]_i^\dagger [\mathbf{m}]_i. \quad (9)$$

The unknown deblurring filters can be computed by solving these normal equations using

LU decomposition. In practice the dimension of the matrix is only related to the filter length and window size, so the calculation of the inverse matrix is computational inexpensive. In practice, the window is no more than 1.5 times of wavelengths in width and height.

Deblurring filters in a viscoacoustic medium

Previous MD research concentrated on lossless media. However, strong subsurface attenuation can significantly distort the amplitudes and phases of seismic waves (Aki and Richards, 1980). To mitigate this problem, Q-LSRTM (Dutta and Schuster, 2014) was developed to generate migration images with more balanced amplitudes and accurately positioned reflectors than standard migration techniques. However, the inverted images from Q-LSRTM sometimes tend to have lower resolution when compared to the benchmark acoustic LSRTM images using lossless data because the adjoint Q propagators used for backward propagating the data residual are also attenuative.

This loss of resolution can be explained by analyzing the migration Green's function for a viscoacoustic medium. For a homogeneous medium with velocity v_0 and a monochromatic point source at $\mathbf{x}_s = (x_s, z_s)$ with angular frequency ω , the acoustic Green's function $G(\mathbf{x}, \mathbf{x}_s)$ is given by (Schuster, 2017)

$$G(\mathbf{x}, \mathbf{x}_s) = \frac{\exp\{i\omega \frac{|\mathbf{x}_s - \mathbf{x}|}{v_0}\}}{|\mathbf{x}_s - \mathbf{x}|}. \quad (10)$$

If the medium is lossy, the viscoacoustic Green's function can be derived by replacing the acoustic phase velocity v_0 with the complex phase velocity given by (Aki and Richards, 1980):

$$v(\omega) = v_0[1 + \frac{1}{\pi Q} \ln \frac{\omega}{\omega_0}](1 - \frac{i}{2Q}), \quad (11)$$

where Q is the quality factor used to quantify the attenuation in the subsurface and ω_0 is the reference frequency. After substituting equation 11 into equation 10, we get the viscoacoustic Green's function as

$$G(\mathbf{x}, \mathbf{x}_s) = \frac{1}{|\mathbf{x}_s - \mathbf{x}|} \overbrace{\exp\{\frac{i\omega|\mathbf{x}_s - \mathbf{x}|}{v_0\xi}\}}^{\text{phase distortion}} \overbrace{\exp\{-\frac{\omega|\mathbf{x}_s - \mathbf{x}|}{2Qv_0\xi}\}}^{\text{amplitude/frequency attenuation}}, \quad (12)$$

where $\xi = [1 + \frac{1}{\pi Q}(\ln(\frac{\omega}{\omega_0}))][1 + \frac{1}{4Q^2}]$. The first exponential term is the phase distortion term and the second exponential term represents the amplitude/high-frequency loss term.

Therefore, the acoustic migration Green's function are given by

$$\Gamma_{acou} = \int_{\omega} d\omega \sum_s \sum_r \frac{\exp\{i\frac{\omega}{v_0}(\frac{rr}{\xi} - rr_0)\} \exp\{-\frac{\omega}{2Qv_0\xi}(rr)\}}{|\mathbf{x}_s - \mathbf{x}| \cdot |\mathbf{x}_r - \mathbf{x}| \cdot |\mathbf{x}_s - \mathbf{x}_0| \cdot |\mathbf{x}_r - \mathbf{x}_0|}, \quad (13)$$

and the viscoacoustic migration Green's function can be approximated by

$$\Gamma_Q = \int_{\omega} d\omega \sum_s \sum_r \frac{\exp\{i\frac{\omega}{v_0\xi}(rr - rr_0)\} \exp\{-\frac{\omega}{2Qv_0\xi}(rr + rr_0)\}}{|\mathbf{x}_s - \mathbf{x}| \cdot |\mathbf{x}_r - \mathbf{x}| \cdot |\mathbf{x}_s - \mathbf{x}_0| \cdot |\mathbf{x}_r - \mathbf{x}_0|}, \quad (14)$$

where $rr = |\mathbf{x}_s - \mathbf{x}| + |\mathbf{x}_r - \mathbf{x}|$ and $rr_0 = |\mathbf{x}_s - \mathbf{x}_0| + |\mathbf{x}_r - \mathbf{x}_0|$. Equation 13 shows that if acoustic migration (acoustic adjoint operator) is used to migrate viscoacoustic data, the reflection energy will be focused at the wrong subsurface location $\mathbf{x}$ instead of $\mathbf{x}_0$. If viscoacoustic migration (viscoacoustic adjoint operator, i.e. Q-RTM) is used, the reflector will be imaged at the right subsurface location $\mathbf{x}_0$. However, the backward propagated receiver wavefield in Q-RTM is further attenuated because of the $\exp\{-\frac{\omega}{2Qv_0\xi}(rr_0)\}$ term

on the RHS of equation 14. Thus, the inverted images from Q-LSRTM will have lower resolution when compared to the images computed by acoustic LSRTM on acoustic data.

To increase the resolution of the Q-LSRTM images and accelerate the convergence of the least-squares iterations, we propose the use of local viscoacoustic deblurring filters to approximate the inverse of the Hessian in a viscoacoustic medium. Assuming the standard linear solid (SLS) model (Christensen, 1982; Carcione et al., 1988; Blanch et al., 1995) and the Born approximation, the observed data $d_Q(\mathbf{x}_r, t; \mathbf{x}_s)$, excited by a point source at $\mathbf{x}_s$ and recorded by a receiver at $\mathbf{x}_r$ in a viscoacoustic medium can be represented as

$$d_Q(\mathbf{x}_r, t; \mathbf{x}_s) = \int_{V_0} -\delta K(\mathbf{x}_0) \left\{ (\tau(\mathbf{x}_0) + 1) [g_P(\mathbf{x}_r, t; \mathbf{x}_0, 0) * \nabla \cdot \mathbf{v}(\mathbf{x}_0, t; \mathbf{x}_s)] \right. \quad (15) \\ \left. + \frac{\tau(\mathbf{x}_0)}{\tau_\sigma(\mathbf{x}_0)} [g_{r_p}(\mathbf{x}_r, t; \mathbf{x}_0, 0) * \nabla \cdot \mathbf{v}(\mathbf{x}_0, t; \mathbf{x}_s)] \right\} dV_0,$$

where K represents the bulk modulus, $\mathbf{v}$ is the particle velocity vector, $g_P(\mathbf{x}_r, t; \mathbf{x}_0, 0)$ and $g_{r_p}(\mathbf{x}_r, t; \mathbf{x}_0, 0)$ are the pressure and memory variable Green's functions, respectively. Here, τ_σ and τ_ϵ represent the stress and strain relaxation times, respectively, and $\tau = \frac{\tau_\epsilon}{\tau_\sigma} - 1$, which is also related to the quality factor Q . Using a matrix-vector notation, equation 15 can also be written as

$$\mathbf{d}_Q = \mathbf{L}_Q \mathbf{m}_0, \quad (16)$$

where $\mathbf{L}_Q$ is a linear viscoacoustic modeling operator, $\mathbf{d}_Q$ represent the lossy data traces, and $\mathbf{m}_0$ is the reflectivity of the medium. The perturbation in the image, δm , is associate with the perturbation in the bulk modulus, δK , which in turn can be obtained by zero-lag

cross-correlation of the adjoint fields with the background wavefields

$$\delta m \approx \delta K = \int_0^T (1 + \tau)(\nabla \cdot \mathbf{v})q + \frac{\tau}{\tau_\sigma}(\nabla \cdot \mathbf{v})s dt, \quad (17)$$

where, q and s are the adjoint-state variables of pressure P and memory variable r_p , respectively (Blanch and Symes, 1994; Dutta and Schuster, 2014, 2016). Equation 17 can also be represented by the matrix-vector notation

$$\mathbf{m}_{mig} = \mathbf{L}_Q^\dagger \mathbf{d}_Q, \quad (18)$$

where the detailed derivation of equations 15 to 18 are shown in the appendix B.

Similar to the acoustic case, the viscoacoustic deblurring filter is estimated by defining a uniform distribution of point scatterers as the reference reflectivity model. The viscoacoustic synthetic data are generated using this reference reflectivity model and the background velocity and Q models. The data are then migrated by Q-RTM to obtain a reference Q-RTM image (detailed derivations are shown in the appendix C). Viscoacoustic deblurring filters for different subdomains of the Q-RTM image are then estimated using matching filters as described in the previous section. The viscoacoustic deblurring filter for the i th local window are estimated in the space domain by solving the system of equations given by

$$[\mathbf{F}_Q]_i \otimes [\mathbf{L}_Q^\dagger \mathbf{L}_Q \mathbf{m}_{ref}]_i \approx [\mathbf{m}_{ref}]_i, \quad (19)$$

where $[\mathbf{F}_Q]_i$ and $[\mathbf{m}_{ref}]_i$ represent the viscoacoustic deblurring filter and reference reflectivity model in the i th local window, respectively. As the reference migration image is generated from the same source-receiver configuration as the original field experiment and by using the same velocity and Q models, the application of these deblurring filters to the migration image is an acceptable approximation to the true inverse Hessian operator $[\mathbf{L}_Q^\dagger \mathbf{L}_Q]^{-1}$.

Q-LSRTM using viscoacoustic deblurring filters

The misfit function for Q-LSRTM is given by (Dutta and Schuster, 2014)

$$\epsilon = \frac{1}{2} \|\mathbf{L}_Q \mathbf{m}^{(k)} - \mathbf{d}_Q^{obs}\|^2, \quad (20)$$

where $\mathbf{d}_Q^{obs}$ denotes the observed data that have suffered from attenuation, $\mathbf{m}^k$ represents the migration image at the k^{th} iteration and $\mathbf{L}_Q$ is the linearized viscoacoustic forward modeling operator. The Gauss-Newton gradient for this misfit function is given by

$$(\mathbf{L}_Q^\dagger \mathbf{L}_Q) \Delta \mathbf{m}^{(k)} = \mathbf{L}_Q^\dagger [\mathbf{L}_Q \mathbf{m}^{(k)} - \mathbf{d}_Q^{obs}] = \mathbf{g}^{(k)}. \quad (21)$$

It can be seen from this equation that the update at each iteration $\mathbf{g}^{(k)}$ is a blurred version of the desired update $\Delta \mathbf{m}^{(k)}$ caused by the viscoacoustic migration Green's function $\mathbf{L}_Q^\dagger \mathbf{L}_Q$. Thus, a preconditioner for the gradient in equation 21 can be written as the deblurring approximation $[\mathbf{L}_Q^\dagger \mathbf{L}_Q]^{-1} \approx \mathbf{F}_Q$, so that the preconditioned gradient in equation 21 can be used in the iterative update equation

$$\mathbf{m}^{(k+1)} = \mathbf{m}^{(k)} - \alpha \mathbf{F}_Q \otimes (\mathbf{L}_Q^\dagger (\mathbf{L}_Q \mathbf{m}^{(k)} - \mathbf{d}_Q^{obs})), \quad (22)$$

where α is the step length.

NUMERICAL RESULTS

The effectiveness of Q-LSRTM with viscoacoustic deblurring filters is now demonstrated with synthetic data generated from the Marmousi II model and the 3D SEG/EAGE Overthrust model. Also, we apply this method to the benchmark viscoacoustic data released by Schlumberger for the BP2004Q model and field data recorded in a crosswell experiment in Friendswood, Texas. The data are migrated using acoustic RTM, acoustic LSRTM, Q-RTM, Q-LSRTM and Q-LSRTM with viscoacoustic deblurring filters. The migration results in the same figure are compared at the same colorbar scale.

Marmousi II model

The preconditioned Q-LSRTM method is first tested on the Marmousi II model. Figures 2a and 2b shows the true velocity and Q models, respectively, used for generating the observed data. The migration velocity and Q models are shown in Figures 2c and 2d, respectively, which are smoothed by a 5×5 window. The Q model is chosen such that the attenuation layers overlie the deeper anticlines. A time-domain viscoacoustic finite-difference modeling algorithm is used with one standard linear solid model for both data simulation and migration. A Ricker wavelet with a peak frequency of 15 Hz is used as the source wavelet. A fixed-spread acquisition geometry is employed where there are 150 sources evenly distributed on the surface at an interval of 50 m. The data are recorded by

800 receivers for each shot uniformly distributed every 10 m on the surface.

Conventional acoustic RTM and LSRTM images obtained from the viscoacoustic data are shown in Figures 3a and 3b, respectively. Both these images fail to recover the amplitudes of the reflectors at the deeper parts. The Q-LSRTM image, shown in Figure 3d, shows improvement in the deeper layers when compared to the acoustic migration results. However, the Q-LSRTM image has lower resolution for the reflectors below the Q anomaly when compared to the benchmark acoustic LSRTM image, shown in Figure 3f, that has been obtained from acoustic data generated using the same velocity model in Figure 2a. As discussed in the previous section, this low resolution problem with Q-LSRTM is due to the attenuation properties of the adjoint operator $\mathbf{L}_Q^\dagger$. However, the preconditioned Q-LSRTM image in Figure 3e computed with the viscoacoustic deblurring filters has better resolution when compared to the Q-LSRTM image in Figure 3d. The amplitudes in the preconditioned Q-LSRTM image are also better balanced when compared to the acoustic RTM and LSRTM images in Figures 3a and 3b, respectively.

The magnified views of these images are compared in Figures 4 and 5, where the black arrows point to the areas in which noticeable improvements in resolution can be seen. Figure 4e shows the wavenumber spectrum of a vertical slice at $x = 3.11$ km. The wavenumber spectrum clearly shows the improvement in resolution with preconditioned Q-LSRTM, which is also confirmed in Figure 5e which shows the wavenumber spectrum of a vertical slice at $x = 5.13$ km. It is evident from these plots that the high-wavenumber details in the image are successfully recovered in the preconditioned Q-LSRTM image and these images have a similar resolution as the benchmark image obtained from applying acoustic LSRTM to lossless acoustic data.

The data residual as a function of iteration number for LSRTM, Q-LSRTM and preconditioned Q-LSRTM is plotted in Figure 6. This plot shows that the convergence rate for preconditioned Q-LSRTM is much faster than Q-LSRTM especially at the early iterations. The residuals in the 3rd and 9th iterations of preconditioned Q-LSRTM are almost equal to the residuals at the 6th and 20th iterations of Q-LSRTM, respectively. Thus, a speedup of around 50% can be seen with the proposed preconditioning method. The convergence rate for preconditioned Q-LSRTM is better than that of standard Q-LSRTM because the viscoacoustic deblurring filters are an effective approximation of the inverse Hessian operator.

Sensitivity of deblurred images with respect to errors in the Q model

To check the sensitivity of the preconditioned Q-LSRTM method to errors in the migration Q model, numerical tests are carried out for different background Q models. Errors are introduced in the background Q model by varying the Q value at the anomalies. In Figure 7, as the errors in the background Q model increase, the amplitudes of the reflectors below the Q anomalies become weaker with increasing errors in the Q model. On the contrary, the PSFs, which are the migration response of the point scatterers, in the same areas become stronger as the Q value increases and thus, they are not able to take into account the effect of Q during deblurring. Therefore, a fairly accurate estimation of the background Q model is required to see noticeable improvements in the image quality with the proposed preconditioned Q-LSRTM method.

BP viscoacoustic benchmark data

The preconditioned Q-LSRTM method is now tested on the B2004Q data generated by Schlumberger (Billette and Brandsberg-Dahl, 2005; Cavalca et al., 2013). The observed data are generated using a Ricker wavelet with a peak frequency of 19 Hz, where the original data consist of 1348 shot gathers and each shot is recorded by 2401 receivers. The shot spacing is 50 m while the receivers are uniformly distributed on both sides of a shot at a spacing of 12.5 m. The sources and receivers are placed at a depth of 12.5 m.

For our numerical tests, we only focus on the salt region, which has a large attenuation zone ($Q=50$) near the left of salt flank. The velocity and Q models used for migration are shown in Figures 8a and 8b, respectively, where only 236 shots are included in this region. The true reflectivity model in Figure 9f is used as the groundtruth for the acoustic and Q-LSRTM images. In the acoustic LSRTM image in Figure 9b, it is difficult to delineate the reflectors near the salt flank. The Q-LSRTM image in Figure 9d has better balanced amplitudes than the acoustic LSRTM image. However, the preconditioned Q-LSRTM image in Figure 9e has reflectors with better balanced amplitudes and better resolution than the standard Q-LSRTM and acoustic LSRTM images. Magnified views of these images in Figure 10a-10d, also illustrate the same. Figure 10e shows the wavenumber spectrum of a vertical slice at $x = 17.05$ km, and the brown curve represents the spectrum of the preconditioned Q-LSRTM image that clearly shows an improvement in resolution when compared with Q-LSRTM indicated by the blue line.

The zoomed views especially in the red box also reveal some artifacts parallel to the salt boundary that become amplified in the preconditioned Q-LSRTM image. These artifacts can also be seen in the acoustic RTM and LSRTM images and can be reduced by non-local

means filter (Huang et al., 2014). The residual as a function of iteration number for LSRTM, Q-LSRTM and preconditioned Q-LSRTM is plotted in Figure 11. The convergence rate for preconditioned Q-LSRTM is much faster than Q-LSRTM and acoustic LSRTM, especially for the first few iterations.

3D SEG/EAGE Overthrust Model

To further illustrate the effectiveness of viscoacoustic deblurring filters, we apply our method to data generated from the 3D SEG/EAGE Overthrust model. Similar to the 2D examples, we use 3D viscoacoustic deblurring filters as a preconditioner for the 3D Q-LSRTM. The number of grid points in the original model is $187 \times 801 \times 801$. For our numerical tests, we only use part of the model with a size of $110 \times 240 \times 240$ grid points. Figures 12a and 12b shows the true velocity and Q models, respectively, used for generating the observed data. A Ricker wavelet with a peak frequency of 15 Hz is used as the source wavelet. A fixed spread acquisition geometry is used where there are 4225 sources evenly distributed on the surface at an interval of 40 m in the inline and crossline directions. The data are recorded by 130 receiver lines uniformly distributed at an interval of 20 m on the surface. Each receiver line has 130 receivers with a spacing of 20 m between each receiver.

Figure 13 compares the images from 3D Q-LSRTM and 3D preconditioned Q-LSRTM. The 3D Q-LSRTM and preconditioned Q-LSRTM images are obtained after 6 and 2 iterations, respectively. The preconditioned Q-LSRTM image in Figure 13c has reflectors with more balanced amplitudes and better resolution than the standard Q-LSRTM in Figure 13b. The residual as a function of iteration number for the 3D Q-LSRTM and preconditioned Q-LSRTM images is plotted in Figure 14. The convergence rate for preconditioned

Q-LSRTM is much faster than Q-LSRTM. The residuals in the second iterations of preconditioned Q-LSRTM are even smaller than the residuals for the 6th iteration of Q-LSRTM. This fast convergence further proves that the estimated visco deblurring filter is a good approximation of the true Hessian inverse.

Friendswood crosswell field data

As a final example, we test our preconditioned Q-LSRTM method on the Friendswood crosswell field data set. Two 305-m-deep cased wells separated by 183 m were used as the source and receiver wells. Downhole explosive charges were fired at intervals of 3 m from 305 m to 9 m in the source well, and the receiver well had 96 receivers placed at depths ranging from 293 m to 3 m. The data were recorded with a sampling interval of 0.25 ms for a total recording time of 0.375 s (Chen et al., 1990). During processing, the data are Wiener-filtered to transform the original wavelet to a Ricker wavelet with a 200-Hz peak frequency. For these data, the migration velocity and Q models are shown in Figures 15a and 15b, respectively. The migration velocity model is estimated by early-arrival waveform inversion and the migration Q model is estimated by wave-equation Q tomography (Dutta, 2016; Dutta and Schuster, 2016).

The comparison between the acoustic LSRTM and Q-LSRTM images after 20 iterations are shown in Figures 16a and 16b, respectively. Similar to the synthetic examples, the amplitudes are more balanced in the Q-LSRTM image in Figure 16d than in the acoustic RTM and LSRTM images in Figures 16a and 16b, respectively, while the resolution becomes lower. The preconditioned Q-LSRTM image is shown in Figure 16e. When compared to the standard Q-LSRTM image, the preconditioned LSRTM image has better resolution at

depths of 10-80 m. Magnified views of these areas are shown in Figure 17 that further validate the improvement in resolution with preconditioning. The black arrows in these figures depict the areas that have become more pronounced in the preconditioned Q-LSRTM image.

As a sanity check, reflectivity slices from the preconditioned Q-LSRTM image are compared to the well log data taken at a distance of 12 m from the source well. The comparison between the synthetic seismogram computed from the well-log profile and the standard and the preconditioned Q-LSRTM reflectivity profiles is shown in Figures 18a and 18b, respectively. It is evident that the well log agrees better with the preconditioned Q-LSRTM image than with the Q-LSRTM image. The black arrows in this figure highlight some of these agreements.

CONCLUSIONS AND RECOMMENDATIONS

A preconditioned Q-LSRTM method is presented that uses viscoacoustic deblurring filters to compensate for the amplitude and resolution losses due to strong subsurface attenuation. Numerical tests on synthetic and field data validate that the proposed preconditioning method mitigates the problem of low resolution associated with standard Q-LSRTM and can produce images with better balanced amplitudes and better resolution than acoustic RTM and LSRTM. The viscoacoustic deblurring filters are estimated from a reference model with evenly distributed point-scatterers and its Q-RTM image is estimated by local matched filters. The proposed preconditioning method is also shown to improve the convergence rate of iterative LSM by more than 30 percent in some cases. Similar to standard Q-LSRTM, a fairly accurate estimation of the background Q model is required to see noticeable improvements in the image quality with the preconditioned Q-LSRTM method. The limitation

of this procedure is that the localized deblurring filter will not reduce artifacts associated with strong migration artifacts far from the scattering point. In this case, the deblurring filters must be computed by using much wider windows, which can significantly increase the computational costs.

ACKNOWLEDGMENTS

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APPENDIX A

ACOUSTIC MODELING AND BLURRING OPERATOR

Under the Born approximation, the observed data $d(\mathbf{x}_r|\mathbf{x}_s, \omega)$, excited by a point source at $\mathbf{x}_s$ and recorded by a receiver at $\mathbf{x}_r$, can be represented as

$$d(\mathbf{x}_r|\mathbf{x}_s, \omega) = \int_{V_0} w(\omega) G(\mathbf{x}_r|\mathbf{x}_0, \omega) G(\mathbf{x}_0|\mathbf{x}_s, \omega) m(\mathbf{x}_0), \quad (\text{A-1})$$

where ω denotes the angular frequency, $w(\omega)$ denotes the spectrum of the second-order time derivative of the source wavelet, $m(\mathbf{x}_0)$ is the reflectivity distribution at the subsurface

location $\mathbf{x}_0$ and V_0 is the 3D integration volume. $G(\mathbf{x}'|\mathbf{x}, \omega)$ denotes the background Green's function for a source at $\mathbf{x}$ and a receiver at $\mathbf{x}'$. Using a matrix-vector notation, equation A-1 can also be written as

$$\mathbf{d} = \mathbf{L}\mathbf{m}_0, \quad (\text{A-2})$$

where $\mathbf{L}$ represents a linear modeling operator, $\mathbf{d}$ is the data vector, and $\mathbf{m}_0$ is a vector that represents the subsurface reflectivity model. The migration image $\mathbf{m}_{mig}$ is computed by applying the migration operator $\mathbf{L}^\dagger$ to the observed data to give

$$\mathbf{m}_{mig} = \mathbf{L}^\dagger \mathbf{d} = \overbrace{\mathbf{L}^\dagger \mathbf{L}}^{\text{blurring operator}} \mathbf{m}_0, \quad (\text{A-3})$$

where $\mathbf{L}^\dagger$ denotes the adjoint of the forward modeling operator $\mathbf{L}$. Using Green's function notation and equation A-1, equation 3 can also be expressed as

$$\begin{aligned} m_{mig}(\mathbf{x}) &= \int_{V_0} \int_{-\infty}^{\infty} d\omega \sum_s \sum_r |w(\omega)|^2 G^*(\mathbf{x}_r|\mathbf{x}, \omega) G^*(\mathbf{x}|\mathbf{x}_s, \omega) G(\mathbf{x}_r|\mathbf{x}_0, \omega) G(\mathbf{x}_0|\mathbf{x}_s, \omega) m(\mathbf{x}_0) dV_0, \\ &= \int_{V_0} \Gamma(\mathbf{x}|\mathbf{x}_0) m(\mathbf{x}_0) dV_0, \end{aligned} \quad (\text{A-4})$$

where the migration Green's function (Schuster and Hu, 2000) is defined as

$$\Gamma(\mathbf{x}|\mathbf{x}_0) = \int_{-\infty}^{\infty} d\omega \sum_s \sum_r w(\omega) w(\omega)^* G^*(\mathbf{x}_r|\mathbf{x}, \omega) G^*(\mathbf{x}|\mathbf{x}_s, \omega) G(\mathbf{x}_r|\mathbf{x}_0, \omega) G(\mathbf{x}_0|\mathbf{x}_s, \omega). \quad (\text{A-5})$$

Here, $*$ indicates the complex conjugate and $\Gamma(\mathbf{x}|\mathbf{x}_0)$ denotes the migration Green's function response at $\mathbf{x} = (x, y, z)$ for a point scatterer at $\mathbf{x}_0 = (x_0, y_0, z_0)$.

APPENDIX B

ADJOINT EQUATIONS AND GRADIENT FOR VISCOACOUSTIC LEAST-SQUARES MIGRATION

Based on the SLS model, the equations of motion for a 2D viscoacoustic medium can be written as (Carcione et al., 1988; Blanch and Symes, 1994).

$$\frac{\partial P}{\partial t} + K(\tau + 1)(\nabla \cdot \mathbf{v}) + r_p = S(\mathbf{x}_s, t), \quad (\text{B-1})$$

$$\frac{\partial \mathbf{v}}{\partial t} + \frac{1}{\rho} \nabla P = 0,$$

$$\frac{\partial r_p}{\partial t} + \frac{1}{\tau_\sigma} (r_p + \tau K(\nabla \cdot \mathbf{v})) = 0,$$

where ρ represents the density, P represents the pressure wavefield, K represent the bulk modulus, r represent the memory variables and $\mathbf{v}$ represents the particle velocity vector. Here, τ is related to the quality factor Q , and the reference angular frequency ω , which can be expressed as

$$\tau = \frac{\tau_\epsilon}{\tau_\sigma} - 1 = \frac{2}{Q} \left(\frac{1}{Q} + \sqrt{1 + \frac{1}{Q^2}} \right), \quad (\text{B-2})$$

where τ_σ and τ_ϵ represents the stress and strain relaxation times, respectively. If we perturb K by an amount δK , the perturbed wavefields can be written as

$$\begin{aligned} \frac{\partial \delta P}{\partial t} + K(\tau + 1)(\nabla \cdot \delta \mathbf{v}) + \delta r_p &= -\delta K(\tau + 1)(\nabla \cdot \mathbf{v}), \\ \frac{\partial \delta \mathbf{v}}{\partial t} + \frac{1}{\rho} \nabla \delta P &= 0, \\ \frac{\partial \delta r_p}{\partial t} + \frac{1}{\tau_\sigma}(\delta r_p + \tau K(\nabla \cdot \delta \mathbf{v})) &= -\frac{\tau}{\tau_\sigma} \delta K(\nabla \cdot \mathbf{v}). \end{aligned} \quad (\text{B-3})$$

Using the Green's function $g_P(\mathbf{x}_r, t; \mathbf{x}_0, 0)$ and $g_{r_p}(\mathbf{x}_r, t; \mathbf{x}_0, 0)$, equation B-3 can also be expressed as

$$\begin{aligned} \delta P(\mathbf{x}_r, t; \mathbf{x}_s) &= \int_{V_0} -\delta K(\mathbf{x}_0) \left((\tau(\mathbf{x}_0) + 1)(g_P(\mathbf{x}_r, t; \mathbf{x}_0, 0) * \nabla \cdot \mathbf{v}(\mathbf{x}_0, t; \mathbf{x}_s)) \right. \\ &\quad \left. + \frac{\tau(\mathbf{x}_0)}{\tau_\sigma(\mathbf{x}_0)}(g_{r_p}(\mathbf{x}_r, t; \mathbf{x}_0, 0) * \nabla \cdot \mathbf{v}(\mathbf{x}_0, t; \mathbf{x}_s)) \right) dV_0, \end{aligned} \quad (\text{B-4})$$

where K represent the bulk modulus, $\mathbf{v}$ represents the particle velocity vector, τ_σ represents the stress relaxation time, τ is related to the quality factor Q , $g_P(\mathbf{x}_r, t; \mathbf{x}_0, 0)$ and $g_{r_p}(\mathbf{x}_r, t; \mathbf{x}_0, 0)$ are the pressure and memory variable Green's functions, respectively. In the context of Q-LSRTM, the solution to equations B-4 is equivalent to the matrix-vector operation

$$\mathbf{d}_Q = \mathbf{L}_Q \mathbf{m}_0, \quad (\text{B-5})$$

where $\mathbf{d}_Q$ denotes the Born-modeled data with attenuation, $\mathbf{L}_Q$ is a linear viscoacoustic modeling operator and $\mathbf{m}_0$ is related to the reflectivity of the medium. The adjoint equations

for equation B-3 can be derived using the adjoint-state method (Lailly, 1983; Blanch and Symes, 1994; Blanch et al., 1995) and is given by

$$\begin{aligned}\frac{\partial q}{\partial t} + \nabla \cdot \left(\frac{1}{\rho} \mathbf{u} \right) &= -\Delta d(\mathbf{x}_g, t; \mathbf{x}_s), \\ \frac{\partial \mathbf{u}}{\partial t} + \left[\nabla (K(1 + \tau)q) + \nabla \left(\frac{1}{\tau_\sigma} K \tau s \right) \right] &= 0, \\ \frac{\partial s}{\partial t} - \frac{s}{\tau_\sigma} - q &= 0.\end{aligned}\tag{B-6}$$

Here $(q, \mathbf{u}, s)$ are the adjoint-state variables of the state variables $(P, \mathbf{v}, r_p)$ and Δd represents the data residual for the predicted and the observed pressure data at every iteration.

The perturbation in the image, δm , is related to the perturbation in the bulk modulus, δK , which in turn can be obtained by zero-lag cross-correlation of the adjoint fields with the background wavefields from equation B-3 as

$$\delta m \approx \delta K = \int_0^T (1 + \tau)(\nabla \cdot \mathbf{v})q + \frac{\tau}{\tau_\sigma}(\nabla \cdot \mathbf{v})s \, dt,\tag{B-7}$$

Using Green's function notation, equation B-7 can also be expressed as

$$\begin{aligned}\delta m \approx \delta K &= \sum_r \sum_s \int_0^T dt \left((\tau(\mathbf{x}) + 1) \nabla \cdot \mathbf{v}(\mathbf{x}, t; \mathbf{x}_s) (g_P(\mathbf{x}_r, -t; \mathbf{x}, 0) * (\Delta P(\mathbf{x}_r, t; \mathbf{x}_s))) \right. \\ &\quad \left. + \frac{\tau(\mathbf{x})}{\tau_\sigma(\mathbf{x})} \nabla \cdot \mathbf{v}(\mathbf{x}, t; \mathbf{x}_s) (g_{r_p}(\mathbf{x}_r, -t; \mathbf{x}, 0) * \Delta P(\mathbf{x}_r, t; \mathbf{x}_s)) \right), \\ &= \sum_s \int dt \left((\tau(\mathbf{x}) + 1) (\nabla \cdot \mathbf{v}(\mathbf{x}, t; \mathbf{x}_s) q(\mathbf{x}, t; \mathbf{x}_s)) + \left(\frac{\tau(\mathbf{x})}{\tau_\sigma(\mathbf{x})} \nabla \cdot \mathbf{v}(\mathbf{x}, t; \mathbf{x}_s) s(\mathbf{x}, t; \mathbf{x}_s) \right) \right).\end{aligned}\tag{B-8}$$

Here, q and s are the adjoint-state variables of the state variable P and r_p , respectively. Equation B-8 is the complete migration Green's function in a viscoacoustic medium, and can also be represented by the matrix-vector notation

$$\mathbf{m}_{mig} = \mathbf{L}_Q^\dagger \mathbf{d}_Q. \quad (\text{B-9})$$

APPENDIX C

DERIVATION OF THE VISCOACOUSTIC DEBLURRING FILTER

Same as acoustic deblurring filter, viscoacoustic deblurring filters are also estimated using a reference model and it's Q migration image. The reference model is constructed using a uniform distribution of point scatterers. The reference data $\mathbf{d}_{ref}^Q$ are generate from the reference reflectivity, background velocity model and Q models, which are then migrated to get a reference Q migration image $\mathbf{m}_{mig-ref}^Q$ as

$$\mathbf{m}_{mig-ref}^Q = \mathbf{L}_Q^\dagger \mathbf{L}_Q \mathbf{m}_{ref} = \mathbf{L}_Q^\dagger \mathbf{d}_{ref}^Q. \quad (\text{C-1})$$

Viscoacoustic deblurring filters for different subdomains of the Q migration image are then estimated by locally matching the reference Q migration image with the reference reflectivity model as

$$m_{ref}(x, y, z)_i = \int_{v_0} F_Q(x - x_0, y - y_0, z - z_0)_i m_{ref-mig}^Q(x_0, y_0, z_0)_i dV_0, \quad (\text{C-2})$$

which can be written in the matrix-vector notation:

$$[\mathbf{m}_{ref}]_i = [\mathbf{F}_Q]_i \otimes [\mathbf{m}_{ref_mig}^Q]_i. \tag{C-3}$$

Here, $[\mathbf{F}_Q]_i$, $[\mathbf{m}_{ref}]_i$ and $[\mathbf{m}_{ref_mig}^Q]_i$ denote the viscoacoustic deblurring filter, the reference reflectivity model and the reference Q migration image within the i th local window, respectively.

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Figure 1. Illustration of local matching filters that transform the migration image to its reference model.

245x173mm (300 x 300 DPI)

Figure 2. The Marmousi model: (a) true velocity model, (b) true Q model, (c) migration velocity model, and (d) migration Q model.

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171x179mm (300 x 300 DPI)

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196x234mm (300 x 300 DPI)

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83x51mm (300 x 300 DPI)

Figure 7. Sensitivity of preconditioned Q-LSRTM to errors in the migration Q model. The left panel shows the preconditioned Q-LSRTM images while the right panel shows the corresponding point spread functions.

226x313mm (300 x 300 DPI)

Figure 8. BP2004Q model: (a) true velocity and (b) true Q models.

86x30mm (300 x 300 DPI)

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214x260mm (300 x 300 DPI)

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296x355mm (300 x 300 DPI)

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82x54mm (300 x 300 DPI)

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145x127mm (300 x 300 DPI)

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246x410mm (300 x 300 DPI)

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218x369mm (300 x 300 DPI)

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101x62mm (300 x 300 DPI)

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246x313mm (300 x 300 DPI)