Least-squares reverse time migration with Radon preconditioning

Gaurav Dutta, King Abdullah University of Science and Technology, Matteo Giboli, Cyril Agut* and Paul Williamson, TOTAL SA.

Summary

We present a least-squares reverse time migration (LSRTM) method using Radon preconditioning to regularize noisy or severely undersampled data. A high resolution local radon transform is used as a change of basis for the reflectivity and sparseness constraints are applied to the inverted reflectivity in the transform domain. This reflects the prior that for each location of the subsurface the number of geological dips is limited. The forward and the adjoint mapping of the reflectivity to the local Radon domain and back are done through 3D Fourier-based discrete Radon transform operators. The sparseness is enforced by applying weights to the Radon domain components which either vary with the amplitudes of the local dips or are thresholded at given quantiles. Numerical tests on synthetic and field data validate the effectiveness of the proposed approach in producing images with improved SNR and reduced aliasing artifacts when compared with standard RTM or LSRTM.

Introduction

Least-squares migration (LSM) or linearized inversion (Lailly, 1984) has been shown to produce images with balanced amplitudes, better resolution and fewer artifacts than standard migration (Schuster, 1993; Nemeth et al., 1999; Duquet et al., 2000). Besides a migration of the data residual, every iteration of LSM involves Born modeling to estimate the step-length and to update the residual. The potential of LSM, especially LSRTM, to produce images of superior quality than any other conventional migration technique has been well studied in recent years.

The problems associated with LSRTM can be broadly grouped into two major categories: 1) inadequate physics taken into account by the modeling and the adjoint equations, and 2) the computational cost. For reducing the computational cost, phase-encoded migration (Morton and Ober, 1998; Romero et al., 2000) was proposed that was later extended to multisource LSRTM by Dai et al. (2010) and several other authors. A similar approach was proposed by Herrmann and Li (2012) as they used a combination of randomized dimensionality-reduction and divide-and-conquer-techniques to decimate the LSM problem as a series of smaller sub-problems where each sub-problem involved iterating on a small randomized subset of the data. However such approaches typically introduce crosstalk noise which is only slowly attenuated by additional iterations. Herrmann and Li therefore combined their approach with compressive sensing and curvelet-domain

sparse recovery (Candes et al., 2006a and 2006b) to mitigate this noise.

The other major problem which may afflict all migrations, including LSM, is the accuracy of the physics represented by the modeling and the adjoint equations. This may result in both inaccurate amplitudes and/or artefacts in the recovered structures which may also be treated in LSM by some form of regularization or preconditioning, respecting prior information to promote a more accurate representation of the subsurface model and introduce desired qualities.

For example, Wang and Sacchi (2007) use a cost function for one-way wave-equation based LSM with regularization constraints for smoothness along offset-domain common image gathers (CIGs) and reflectivity sparseness in depth. Cabrales-Vargas and Marfurt (2013) also formulated a regularized least-squares Kirchhoff migration problem using a penalty function that controls the amount of roughness in common reflection point gathers (CRPGs).

Another approach is to use a change of basis for the reflectivity using linear sparse transforms or some other form of model reparametrization (Harlan, 1995; Fomel and Guitton, 2006) into a basis where one believes a priori that the reflectivity is sparse. Constraints can then be imposed on the image in the new domain to enforce the sparseness. Suitable transforms should be 1) perfectly invertible, 2) efficient to compute, 3) be compatible with the conjugate gradient method, and 4) minimally redundant (Kingsbury, 2001). Miller et al. (2005) used the Dual Tree Complex Wavelet Transform as a basis for the reflectivity and demonstrated that this reduces noise and migration artifacts while preserving the discontinuities better than standard LSM for very sparse data. Herrmann et al. (2009) and Herrmann and Li (2012) used curvelets and Dutta (2015) used seislets as basis functions for the reflectivity; both showed that with sparsity promoting imaging techniques, it is possible to recover high-quality images from undersampled or noisy data.

In this paper, we promote sparseness in the reflectivity in the local Radon domain by applying a directional Radon-based preconditioning approach in our LSRTM so as to satisfy the prior that the number of dips at each position in the image is limited. The aim is to reduce artefacts in the image when the recorded data are noisy or poorly or irregularly sampled, and the background velocity model is inaccurate. The forward mapping to the local Radon domain and its adjoint is done through 3D discrete Radon transform operators in the Fourier domain. The preconditioning is implemented using model reparametrization and re-weighting where the weights are chosen based on amplitudes of the events in the dip

LSRTM with Radon-based preconditioning

domain. Numerical tests on synthetic and 3D marine data show that Radon-based preconditioning can produce images with fewer migration artifacts and better SNR than standard RTM or LSRTM in a few iterations.

Methodology

For a regularized L2-norm inverse problem, we seek to find a solution that best explains the observed data and is consistent with the prior knowledge that is available before any observations are made. This can be represented by the minimization problem over the image-space $\mathbf{M}$:

$$\min_{\mathbf{m} \in \mathbf{M}} \left[\phi(\mathbf{m}) = \frac{1}{2} \|\mathbf{d} - \mathbf{L}\mathbf{m}\|_2^2 + \lambda R(\mathbf{m}) \right] \quad (1)$$

where $\mathbf{d}$ represents the observed data, $\mathbf{m}$ the reflectivity model, $\mathbf{L}$ a linear modeling operator, $R(\mathbf{m})$ is a discrete regularizer/prior that imposes constraints on the solution $\mathbf{m}$. Finally, λ is a parameter controlling the strength of the regularization term, and thereby the tradeoff between data fit and the prior information encapsulated in the regularization.

In the limit where $\lambda=0$, equation 1 is equivalent to solving an unconstrained optimization where the emphasis is purely on minimizing the misfit between the predicted and the observed data. For incomplete, undersampled or noisy data, emphasizing only the data misfit can lead to images that are degraded in quality with iterations because of over-fitting the noise. In the case of LSM, errors in the migration velocity model can also lead to defocusing of images with iterations (Dutta et al., 2014).

However, the optimal estimation of λ is not trivial, and it may be more appropriate to reformulate the constrained optimization problem (1) as

$$\min_{\mathbf{m} \in \mathbf{M}} \|\mathbf{W}\mathbf{R}\mathbf{m}\|_2 \text{ subject to } \|\mathbf{d} - \mathbf{L}\mathbf{m}\|_2 < \varepsilon \quad (2)$$

where $\mathbf{R}\mathbf{m}$ is a linear transform of the model vector, $\mathbf{m}$, into a domain in which the prior suggests that the L2-norm should be minimized, ε is the tolerance/noise level for the data misfit and $\mathbf{W}$ is a weighting matrix. In our case, we take $\mathbf{R}$ to be the local Radon transform of the image computed using a 3D Fourier-based discrete Radon transform algorithm (Averbuch et al., 2003) and the weights are chosen to be a function of the amplitudes of the local dip events. If $\mathbf{W}$ and $\mathbf{R}$ are invertible we can apply a transform to the model, effectively reparameterizing it, and rewrite equation 2 as

$$\min_{\mathbf{p} \in \mathbf{W}\mathbf{R}\mathbf{m}} \|\mathbf{p}\|_2 \text{ subject to } \|\mathbf{d} - \mathbf{L}\mathbf{R}^{-1}\mathbf{W}^{-1}\mathbf{p}\|_2 < \varepsilon \quad (3)$$

with

$$\mathbf{p} = \mathbf{W}\mathbf{R}\mathbf{m}, \quad \mathbf{m} = \mathbf{R}^{-1}\mathbf{W}^{-1}\mathbf{p}. \quad (4)$$

The normal equations corresponding to the data misfit function in equations 3 are given by

$$(\mathbf{W}^{-T}\mathbf{R}^{-T}\mathbf{L}^T\mathbf{L}\mathbf{R}^{-1}\mathbf{W}^{-1})\mathbf{p} = \mathbf{W}^{-T}\mathbf{R}^{-T}\mathbf{L}^T\mathbf{d} \quad (5)$$

which in practice can be solved using any conjugate-gradient (CG) algorithm (Nocedal and Wright, 1999). With this formalism, the number of internal iterations of the CG controls the model norm and thereby plays the role of regularizer. Even though the system of equations is partially solved at every external iteration because the CG algorithm is stopped before the solution is complete, the total matrix operator resulting from the preconditioning is diagonalized enough so that one can avoid using a damping factor for the smaller eigenvalues. Such a preconditioning approach is useful because it provides a way of incorporating the prior information (the reflectivity should be sparse in the local Radon domain) directly into the forward operator.

Synthetic Data Example

We first demonstrate the effect of Radon-based preconditioning LSRTM on the 3D SEG/EAGE salt model. There are only 45 shots recorded on a 5x9 source grid with a 960m spacing between shots both inline and crossline. Each shot is recorded by a 201x201 receiver grid with 50m spacing between the receivers in each direction. The observed data are simulated using the velocity model of which a 2D slice is shown in Figure 1 for a recording time of 8 seconds with a 4ms sampling rate. For migration, we smooth the true slowness using a triangle smoothing filter with a smoothing radius of 50m in each of the 3 dimensions.

Figure 1: A 2D slice of the 3D SEG/EAGE salt model.

LSRTM with Radon-based preconditioning

Figures 2(a)-2(d) compare 2D slices of the images from standard RTM, standard LSRTM, preconditioned RTM and preconditioned LSRTM, respectively. The RTM image in Figure 2(a) suffers from very strong backscattering noise because of the presence of the salt body. These slices show strong imaging artefacts due to the footprint of the sparse acquisition geometry that dominate the rather weak reflector amplitudes. The LSRTM image in Figure 2(b) shows some improvements over the standard RTM image. The reflector amplitudes are better balanced and the acquisition footprints are mitigated. However, the aliasing noise is still prominent and is severe below the salt body. The preconditioned LSRTM image, shown in Figure 2(d), is free from aliasing noise and the subsalt images are much cleaner when compared to the standard RTM and LSRTM images. The preconditioned images also have a much better SNR than the standard RTM and LSRTM images.

Figure 2: Comparison between images from (a) standard isotropic RTM, (b) standard isotropic LSRTM after 5 iterations, (c) preconditioned isotropic RTM and (d) preconditioned isotropic LSRTM after 5 iterations.

3D Marine Data Example

We applied our preconditioned LSRTM to a subset of a real 3D marine dataset. The acquisition is broadband with a shot interval of 18.75m and a source centre separation of 25m. The receiver spread has 12 cables with an active length of 6km. There are 480 receiver groups with a group interval of 12.5m. The sampling interval is 2ms with a total recording time of 8s. Before starting the PSDM workflow, standard time processing steps were carried out, with particular effort to remove free-surface multiples.

The velocity model was obtained using ray-based tomography and full waveform inversion on the full dataset. For the LSM study carried out here, 10200 shots were used and a bandpass filter of 3-5-25-30 Hz was

LSRTM with Radon-based preconditioning

applied to the traces to accelerate the turnaround of the test. The standard TTI RTM and TTI LSRTM images are shown in Figures 3(a) and 3(b), respectively. It is evident from these images that standard LSRTM improves the image quality (in terms of improved resolution and balanced amplitudes) in the shallow parts. However, the image quality in the depth range of 3-5km, is poor in both the standard RTM and the standard LSRTM images (figures 3a and b). We suspect that this may be due to poor crossline sampling and perhaps some inaccuracy in the background model.

The preconditioned TTI RTM and TTI LSRTM images are shown in Figures 3(c) and 3 (d), respectively. It is evident that the structures of interest are imaged with an improved SNR and that their interpretation will be facilitated.

Figure 3: Comparison between magnified views of the images from (a) standard RTM, (b) standard LSRTM after 5 iterations, (c) preconditioned RTM and (d) preconditioned LSRTM after 5 iterations.

Conclusions

We have presented a Radon-based preconditioning approach for LSRTM which aims to mitigate the problems of low SNR data and aliasing. The preconditioning works by implicitly imposing sparseness constraints on the reflectivity in the local Radon domain. The requisite forward and adjoint mappings of the reflectivity to and from the local Radon domain are efficiently implemented using 3D Fourier-based Radon operators. Tests on synthetic and real data show that our preconditioning approach can discriminate against artefacts in the image resulting from irregular or insufficient acquisition or other sources of noise, and therefore improves the final image at all depths compared with unregularized LSRTM.

EDITED REFERENCES

Note: This reference list is a copyedited version of the reference list submitted by the author. Reference lists for the 2016 SEG Technical Program Expanded Abstracts have been copyedited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

REFERENCES

- Averbuch, A., and Y. Shkolnisky, 2003, 3D fourier based discrete radon transform: Applied and Computational Harmonic Analysis, **15**, no. 1, 33–69. [http://dx.doi.org/10.1016/S1063-5203\(03\)00030-7](http://dx.doi.org/10.1016/S1063-5203(03)00030-7).
- Cabrales-Vargas, A., and K. J. Marfurt, 2013, Amplitude-preserving imaging of aliased data using preconditioned kirchhoff least-squares depth migration: SEG Technical Program Expanded Abstracts, **721**, 3726–3730.
- Candes, E., L. Demanet, D. Donoho, and L. Ying, 2006a, Fast discrete curvelet transforms: Multiscale Modeling & Simulation, **5**, no. 3, 861–899. <http://dx.doi.org/10.1137/05064182X>.
- Candes, E. J., J. K. Romberg, and T. Tao, 2006b, Stable signal recovery from incomplete and inaccurate measurements: Communications on Pure and Applied Mathematics, **59**, no. 8, 1207–1223. <http://dx.doi.org/10.1002/cpa.20124>.
- Dai, W., C. Boonyasiriwat, and G. T. Schuster, 2010, 3D multi-source least-squares reverse time migration: SEG Technical Program Expanded Abstracts, **29**, 3120–3124, <http://dx.doi.org/10.1190/1.3513494>.
- Duquet, B., K. J. Marfurt, and J. A. Dellinger, 2000, Kirchhoff modeling, inversion for reflectivity, and subsurface illumination: Geophysics, **65**, no. 4, 1195–1209. <http://dx.doi.org/10.1190/1.1444812>.
- Dutta, G., 2015, Sparse least-squares reverse time migration using seislets: SEG Technical Program Expanded Abstracts, **810**, 4232–4237, <http://dx.doi.org/10.1190/segam2015-5869595.1>.
- Dutta, G., Y. Huang, W. Dai, X. Wang, and G. T. Schuster, 2014, Making the most out of the least (squares migration): SEG Technical Program Expanded Abstracts, **840**, 4405–4410, <http://dx.doi.org/10.1190/segam2014-1242.1>.
- Fomel, S., and A. Guitton, 2006, Regularizing seismic inverse problems by model reparameterization using plane-wave construction: Geophysics, **71**, no. 5, A43–A47. <http://dx.doi.org/10.1190/1.2335609>.
- Harlan, W. S., 1995, Regularization by model reparameterization: Citeseer.
- Herrmann, F., C. Brown, Y. Erlangga, and P. Moghaddam, 2009, Curvelet-based migration preconditioning and scaling: Geophysics, **74**, no. 4, A41–A46. <http://dx.doi.org/10.1190/1.3124753>.
- Herrmann, F. J., and X. Li, 2012, Efficient least-squares imaging with sparsity promotion and compressive sensing: Geophysical Prospecting, **60**, no. 4, 696–712. <http://dx.doi.org/10.1111/j.1365-2478.2011.01041.x>.
- Kingsbury, N., 2001, Complex wavelets for shift invariant analysis and filtering of signals: Applied and Computational Harmonic Analysis, **10**, no. 3, 234–253. <http://dx.doi.org/10.1006/acha.2000.0343>.
- Lailly, P., 1984, Migration methods: partial but efficient solutions to the seismic inverse problem: Inverse problems of acoustic and elastic waves, **51**, 1387–1403.
- Miller, M. A., N. G. Kingsbury, and R. W. Hobbs, 2005, Seismic image reconstruction using complex wavelets: Electronic Imaging, 27–35.
- Morton, S. A., and C. C. Ober, 1998, Faster shot record depth migrations using phase encoding: SEG Technical Program Expanded Abstracts, **298**, 1131–1134, <http://dx.doi.org/10.1190/1.1820088>.
- Nemeth, T., C. Wu, and G. T. Schuster, 1999, Least-squares migration of incomplete reflection data: Geophysics, **64**, no. 1, 208–221. <http://dx.doi.org/10.1190/1.1444517>.

- Nocedal, J., and S. Wright, 1999, Numerical optimization: Springer Verlag.
<http://dx.doi.org/10.1007/b98874>.
- Romero, L. A., D. C. Ghiglia, C. C. Ober, and S. A. Morton, 2000, Phase encoding of shot records in prestack migration: *Geophysics*, **65**, no. 2, 426–436. <http://dx.doi.org/10.1190/1.1444737>.
- Schuster, G. T., 1993, Least-squares cross-well migration: SEG Technical Program Expanded Abstracts, **12**, 110–113, <http://dx.doi.org/10.1190/1.1822308>.
- Wang, J., and M. D. Sacchi, 2007, High-resolution wave-equation amplitude-variation-with-ray-parameter (AVP) imaging with sparseness constraints: *Geophysics*, **72**, no. 1, S11–S18. <http://dx.doi.org/10.1190/1.2387139>.