Full-waveform inversion with reflected waves for 2D VTI media

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SUMMARY

Full-waveform inversion in anisotropic media using reflected waves suffers from the strong non-linearity of the objective function and trade-offs between model parameters. Estimating long-wavelength model components by fixing parameter perturbations, referred to as reflection-waveform inversion (RWI), can mitigate nonlinearity-related inversion issues. Here, we extend RWI to acoustic VTI (transversely isotropic with a vertical symmetry axis) media. To minimize trade-offs between the model parameters, we employ a new hierarchical two-stage approach that operates with the P-wave normal-moveout velocity V_{nmo} and anisotropy coefficients δ and η . First, V_{nmo} is estimated using a fixed perturbation in δ , and then we invert for η by fixing the updated perturbation in V_{nmo} . The proposed 2D algorithm is tested on a horizontally layered VTI model.

INTRODUCTION

Full-waveform inversion (FWI) is a powerful technique capable of building high-resolution velocity models, especially for shallow layers (Virieux and Operto, 2009). Conventional FWI heavily relies on diving waves to update the model. However, existing FWI algorithms often fail to provide a satisfactory background model update for deeper horizons, which are illuminated mostly by reflections.

Conventional FWI uses a least-squares objective function, which has multiple local minima, especially when reflected waves dominate the data. An initial model that does not produce cycle-skipping can be obtained by such optimization methods as migration-based traveltime tomography (Plessix et al., 1995), reflection tomography (Wang and Tsvankin, 2013a,b), or migration velocity analysis (e.g., Biondi and Symes, 2004). Xu et al. (2012) develop an algorithm similar to migration-based traveltime tomography called reflection-waveform inversion (RWI), which is designed to invert reflection data for the long-wavelength components of the velocity model. A migration/demigration approach (Zhou et al., 2012) can be employed to update the background model along the source and receiver wavepath. Wang et al. (2013) implement RWI in the frequency domain and demonstrate that low-frequency data are as essential for RWI as they are for conventional FWI. A spatial- and temporal-correlation-based objective function that can handle phase delays larger than half a period is presented by Chi et al. (2015). Wu and Alkhalifah (2015) propose a new optimization approach that estimates both the background and perturbed model simultaneously.

Here, we extend RWI to anisotropic models and devise a two-stage algorithm to invert reflection data for the parameters of VTI media. We employ parameterization in terms of the P-

wave normal-moveout (NMO) velocity V_{nmo} , the anellipticity parameter η and the Thomsen parameter δ , which yields optimal waveform-inversion results when the data contain high-quality reflections in the conventional offset range (Alkhalifah et al., 2015a).

METHODOLOGY

Two-stage inversion algorithm

The proposed algorithm is designed to estimate just the NMO velocity and parameter η , because δ typically is not well-constrained by reflection data. Stable inversion for δ cannot be performed without additional (e.g., borehole) information (Wang and Tsvankin, 2013a,b). The method operates with P-wave reflection data and includes the following steps:

1. First, we perturb the parameter δ while keeping V_{nmo} and η fixed. This perturbation model is then fixed while the reference NMO velocity is updated. The main goal of this step is to fit the data residuals at the near offsets caused by incorrect initial values of δ and V_{nmo} .
2. Second, the updated NMO velocity is used to generate the perturbation model (image). At this stage, the algorithm inverts only for η to fit the data residuals at the far offsets.

At both stages, we use a correlation-based objective function and compute the gradient from the adjoint-state method. If high-quality reflections are available, parameterization in terms of V_{nmo} , η , and δ helps reduce the trade-offs and provide adequate resolution (Alkhalifah and Plessix, 2014). The velocity V_{nmo} has a purely isotropic (uniform) radiation pattern, while the perturbation in η depends mostly on the horizontal wavenumber. A perturbation in δ is associated primarily with the vertical wavenumber, so δ can be used to fit the near-offset amplitude information. Employing δ helps compensate, to a certain extent, for inadequate amplitude fitting of reflection data in the acoustic approximation. Therefore, we perturb δ and use the Born approximation of the pseudoacoustic wave equation to generate the scattered wavefield.

Kinematically accurate reconstruction of P-wavefields in TI media is provided by a system of two second-order coupled equations (Fletcher et al., 2009; Fowler et al., 2010). Fowler et al. (2010) suggest to describe pseudoacoustic wave propagation by

$$\begin{aligned}\frac{\partial^2 u^p}{\partial t^2} &= V_{\text{hor}}^2 \frac{\partial^2 u^p}{\partial x^2} + V_{p0}^2 \frac{\partial^2 u^q}{\partial z^2}, \\ \frac{\partial^2 u^q}{\partial t^2} &= V_{\text{nmo}}^2 \frac{\partial^2 u^p}{\partial x^2} + V_{p0}^2 \frac{\partial^2 u^q}{\partial z^2},\end{aligned}\quad (1)$$

where $V_{\text{hor}} = V_{\text{nmo}}\sqrt{1+2\eta} = V_{p0}\sqrt{1+2\epsilon}$ is the P-wave horizontal velocity. Both the p- and q-components of the wavefield contain a P-wave with the correct kinematics and a shear-wave “artifact” caused by setting the S-wave vertical velocity to zero. This artifact can be eliminated by placing sources and receivers in a purely isotropic or elliptic ($\eta = 0$) medium (Alkhalifah, 2000; Duveneck et al., 2008).

Representing the parameter δ as the sum of the background value (δ_o) and a perturbation (δ_d), the wavefields can be expressed as $u^p = u^{p0} + u^{p1}$ and $u^q = u^{q0} + u^{q1}$, where u^{p1} and u^{q1} are the perturbations and u^{p0} and u^{q0} are the wavefields for the background medium.

For a small perturbation in δ , equation 1 can be expressed in matrix form as:

$$\mathbf{L} \begin{bmatrix} u^{p1} \\ u^{q1} \end{bmatrix} = \begin{bmatrix} 2\delta_d V_{p0}^2 \left(\frac{\partial^2 u^{q0}}{\partial z^2} \right) \\ 2\delta_d V_{p0}^2 \left(\frac{\partial^2 u^{p0}}{\partial z^2} \right) \end{bmatrix}, \quad (2)$$

$$\text{where } \mathbf{L} = \begin{bmatrix} V_{\text{hor}}^2 \partial_{xx} - \partial_{tt} & V_{p0}^2 \partial_{zz} \\ V_{\text{nmo}}^2 (\partial_{xx}) & V_{p0}^2 \partial_{zz} - \partial_{tt} \end{bmatrix}.$$

Here, we refer to the term $2\delta_d V_{p0}^2$ as the “ δ -image.” The dot product of the δ -image with the squared double-derivative of the q-component of the source wavefield produces secondary sources in the model space. The Born-scattered data (i.e., predicted data) are computed by forward modeling the secondary sources using equation 2. Next, we use this δ -image as our perturbation model and invert only for V_{nmo} to get an updated velocity model; this update is based primarily on short-offset data. In contrast, in existing RWI algorithms the image is obtained from near-offset least-squares migration. This completes the first stage of the inversion.

Perturbation in the parameter η mostly depends on the horizontal wavenumber, so the sensitivity to η increases with the offset-to-depth ratio. Hence, at the second inversion stage, the estimated V_{nmo} is used to generate the perturbation model (image) and invert for η by fitting the far-offset data residuals.

Correlation objective function

The data are generated with the constant-density pseudoacoustic wave equation, which does not properly model reflection amplitudes. In addition, because the actual reflectivity cannot be obtained by cross-correlating the source and receiver wavefields, the amplitude matching of the observed and predicted data using a least-squares objective function may be problematic. The requirement of amplitude matching can be relaxed by using a normalized cross-correlation objective function $\mathbf{C}$ that evaluates the similarity between the observed and Born-modeled data (Choi and Alkhalifah, 2012; Xu et al., 2012a):

$$\mathbf{C} = -\hat{d}_m \cdot \hat{d}_{\text{obs}}, \quad (3)$$

where $\hat{d}_m = \frac{d_m}{\|d_m\|}$ is the normalized Born-simulated data and $\hat{d}_{\text{obs}} = \frac{d_{\text{obs}}}{\|d_{\text{obs}}\|}$ is the normalized observed data.

Adjoint-state method

The adjoint-state method (Tromp et al., 2005; Plessix, 2006) provides an efficient way of computing the derivatives of the objective function with respect to the model parameters. The components of the adjoint-state method are the objective function, state equations, and adjoint equations. The objective function depends on the model parameters through the state equations.

The adjoint-state method involves four main steps:

- (i) Computation of the state variables (forward wavefield) by solving the state equations.
- (ii) Computation of the adjoint source functions.
- (iii) Computation of the adjoint-state variable (adjoint wavefield) by solving the adjoint equations.
- (iv) Computation of the gradient of the objective function.

The problem in hand includes two state equations: the pseudo acoustic wave equation that generates the forward wavefield and the Born approximation of the wave equation for a small perturbation in the parameter δ . RWI can be posed as the following optimization problem:

$$\begin{aligned} &\underset{\mathbf{m}}{\text{minimize}} && J(\mathbf{m}, \mathbf{u}^p, \mathbf{u}^q, \mathbf{u}^{p1}, \mathbf{u}^{q1}), \\ &\text{subject to} && F(\mathbf{m}, \mathbf{u}^p, \mathbf{u}^q) = 0 \text{ \& } F_1(\mathbf{m}, \mathbf{u}^{p1}, \mathbf{u}^{q1}) = 0, \end{aligned} \quad (4)$$

where u^p and u^q are the forward-modeled wavefields and u^{p1} and u^{q1} are the Born-scattered wavefields. The functions F and F_1 are the state equations for the forward-modeled and the Born-simulated data, respectively. The model vector for our problem is defined as $\mathbf{m} = \{V_{\text{nmo}}, \eta, \delta\}$.

Using equation 3 and 4, the Lagrangian can be formulated as:

$$\Lambda = C(\mathbf{x}, t) - \langle Lu_s - f, \lambda_{u_s} \rangle - \langle Lu_1 - Iu_s, \lambda_{u_1} \rangle, \quad (5)$$

where $C(\mathbf{x}, t) = -\frac{d_m}{\|d_m\|} \cdot \frac{d_{\text{obs}}}{\|d_{\text{obs}}\|}$, L is the wave operator, I is the image, u_s and u_1 are the state variables, and λ_{u_s} and λ_{u_1} are the adjoint variables. We generate the source wavefield u_s and the Born-scattered wavefield u_1 by solving the following equations:

$$Lu_s - f = 0, \quad (6)$$

$$Lu_1 - Iu_s = 0. \quad (7)$$

Next, the adjoint variables for both the source and receiver sides are obtained from:

$$L^T \lambda_{u_s} - I \lambda_{u_1} = 0, \quad (8)$$

$$L^T \lambda_{u_1} - \frac{1}{\|d_m\|} (\hat{d}_m (\hat{d}_m \cdot \hat{d}_{\text{obs}}) - \hat{d}_{\text{obs}}) = 0. \quad (9)$$

Here, the adjoint sources are:

$$r_d = \frac{1}{\|d_m\|} (\hat{d}_m (\hat{d}_m \cdot \hat{d}_{\text{obs}}) - \hat{d}_{\text{obs}}). \quad (10)$$

Equation 10 shows that the Born-simulated data are scaled by the dot product of the observed data and the Born-simulated

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data. This controls the amplitude matching depending on the similarity between these two data sets. Finally, the gradient for the model parameters $\mathbf{m}$ is computed from $\partial\Lambda/\partial\mathbf{m} = \partial J/\partial\mathbf{m}$.

Then the source-side gradients for V_{nmo} , η , and δ take the form:

$$\begin{aligned} \frac{\partial J_s}{\partial V_{\text{nmo}}} = & \iint 2 u_{\text{sx}}^p V_{\text{nmo}} (1 + 2\eta) u_{\text{rx}}^p ds dt \\ & + \iint 2 u_{\text{sz}}^q \frac{V_{\text{nmo}}}{(1 + 2\delta)} u_{\text{rz}}^p ds dt \\ & + \iint 2 u_{\text{sx}}^p V_{\text{nmo}} u_{\text{rx}}^q ds dt \\ & + \iint 2 u_{\text{sz}}^q \frac{V_{\text{nmo}}}{(1 + 2\delta)} u_{\text{rz}}^q ds dt, \end{aligned} \quad (11)$$

$$\frac{\partial J_s}{\partial \eta} = \iint 2 u_{\text{sx}}^p V_{\text{nmo}}^2 u_{\text{rx}}^p ds dt, \quad (12)$$

$$\begin{aligned} \frac{\partial J_s}{\partial \delta} = & \iint -2 u_{\text{sz}}^q \frac{V_{\text{nmo}}^2}{(1 + 2\delta)^2} u_{\text{rz}}^q ds dt \\ & + \iint -2 u_{\text{sz}}^q \frac{V_{\text{nmo}}^2}{(1 + 2\delta)^2} u_{\text{rz}}^p ds dt, \end{aligned} \quad (13)$$

where the integration is performed over the sources and time, the subscript “s” denotes the source-side state variables and subscript “r” denotes the adjoint variables. The corresponding expressions for the receiver-side gradient are obtained in a similar way.

Sensitivity kernel

The sensitivity kernel for a particular model parameter is the response in the model space to the data perturbations for a single source and a single receiver. The sensitivity kernel describes the model areas that can provide updates for a particular receiver location. For our problem, the source-side sensitivity kernel is the cross-correlation of the source wavefield with the adjoint-source wavefield, whereas the receiver-side sensitivity kernel is the cross-correlation of the residual receiver wavefield with the demigrated source wavefield. The data residuals are back-propagated along the “rabbit-ear” wavepath to generate the sensitivity kernel. Model of the parameters δ , V_{nmo} and η are shown in Figure 1 (a), 3 (a), and 4 (a), respectively. The sensitivity kernels for the parameters V_{nmo} , η , and δ are shown in Figure 1.

SYNTHETIC EXAMPLE

The algorithm is tested on the layered VTI model in Figure 1. The horizontal and vertical grid spacing is 25 m. The data are excited by 16 sources positioned at 50 m intervals on the surface with a maximum source-receiver offset of 7.5 km. The source signal is a Ricker wavelet with a central frequency of 5 Hz. There are no diving waves in this model, and we use only reflections by muting the direct arrivals in the recorded data. The sources and receivers are embedded in a thin isotropic layer to suppress the shear-wave artifact. The modeled wavefield is computed for a smooth background medium free from reflections, and the adjoint sources are injected back into the

Figure 1: (a) Model of the parameter δ with a source at 2 km, receiver at 5.5 km, and a scatterer at $x = 3.75$ km and $z = 1.025$ km. The sensitivity kernels for (b) V_{nmo} , (c) η , and (d) δ .

model to generate the adjoint wavefield. For both the forward and the adjoint wavefield extrapolation, we use a finite difference algorithm developed within the framework of **MADA-GASCAR**.

The gradients in this example are obtained by perturbing δ , while V_{nmo} and η are kept constant. Note that the gradient illuminates even the third (deepest) layer without applying any illumination compensation (Figure 2). The gradient for η (equation 12) depends on the background V_{nmo} and the first lateral derivatives of the forward and adjoint wavefields. Therefore, there is a trade-off between V_{nmo} and η in the horizontal direction. The gradient for δ (equation 13) is controlled by the background V_{nmo} , δ and the first vertical derivatives of the forward and adjoint wavefields, which causes a trade-off between V_{nmo} and δ in the vertical direction. The gradient for V_{nmo} (equation 11) is dependent on the NMO velocity itself, η , δ and the first derivatives of the forward and adjoint wavefields in both the horizontal and vertical directions. Because the gradient for V_{nmo} contains δ , we invert for V_{nmo} first.

The initial NMO velocity is taken equal to the velocity in the first layer, whereas the initial $\delta = \eta = 0$. As described above, first we fix the δ -image and invert for V_{nmo} . The algorithm was able to recover V_{nmo} in the second and the third layers, although the velocity in the deepest layer is somewhat overestimated (Figure 3). At the second stage, the inverted V_{nmo} is used to generate the image and invert only for η by fitting the data residuals at the far offsets. The estimated η in the first layer is accurate because the offset-to-depth ratio for its bottom is large ($x/z = 7$), whereas the inverted η -values in the second and third layer are slightly distorted. Even better estimates of η could be obtained if the model contained dipping reflectors. Clearly, the hierarchical inversion approach makes it possible to handle the nonlinearity of the objective function and the trade-offs between the model parameters.

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Figure 2: Gradients of the objective function (equation 3) with respect to (b) V_{nmo} , (c) η , and (d) δ for a small perturbation in the δ -model from Figure 1(a).

Figure 4: Second stage of the inversion: (a) the actual η -field, (b) the initial η , and (c) the inverted η .

Figure 3: First stage of the inversion: (a) the actual V_{nmo} -field, (b) the initial V_{nmo} , and (c) the inverted V_{nmo} .

CONCLUSIONS

We proposed a hierarchical two-stage RWI (reflection waveform inversion) algorithm to reduce the nonlinearity of the objective function and mitigate the tradeoffs between the model parameters for VTI media. At the first stage of RWI, the δ -image is employed to estimate V_{nmo} from near-offset data. Then we use the updated NMO velocity to generate the image which includes the far-offset velocity information needed to constrain η . An example for a layered VTI model confirms that the algorithm can resolve the interval V_{nmo} and η from reflection waveforms. As is the case in moveout analysis, either dipping interfaces or large offset-to-depth ratios are required to estimate η with sufficient accuracy.

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EDITED REFERENCES

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