Waveform inversion for acoustic VTI media in frequency domain

Zedong Wu and Tariq Alkhalifah, King Abdullah University of Science and Technology

SUMMARY

Reflected waveform inversion (RWI) provides a method to reduce the nonlinearity of the standard full waveform inversion (FWI) by inverting for the background model using a single scattered wavefield from an inverted perturbation. However, current RWI methods are mostly based on isotropic media assumption. We extend the idea of the combining inversion for the background model and perturbations to address transversely isotropic with a vertical axis of symmetry (VTI) media taking into consideration of the optimal parameter sensitivity information. As a result, we apply Born modeling corresponding to perturbations in only for the variable ε to derive the relative reflected waveform inversion formulation. To reduce the number of parameters, we assume the background part of $\eta = \varepsilon$ and work with a single variable to describe the anisotropic part of the wave propagation. Thus, the optimization variables are the horizontal velocity v , $\eta = \varepsilon$ and the ε perturbation. Application to the anisotropic version of Marmousi model with a single frequency of 2.5 Hz shows that this method can converge to the accurate result starting from a linearly increasing isotropic initial velocity. Application to a real dataset demonstrates the versatility of the approach.

INTRODUCTION

Conventional FWI is based on minimizing the misfit between the predicted and observed wavefields at the sensor locations (Virieux and Operto, 2009). However this objective function is far from being convex, especially with respect to the high wavenumber components of the velocity model, which, in the absence of very low frequency, requires a good starting model that is close to the true model to avoid converging to local minimal.

Xu et al. (2012) and Zhou et al. (2012) developed a method partially based on the work of Plessix et al. (1995) of inverting using reflected energy in the course of FWI, namely RWI. The idea is based on using migration to image the reflections in the data. This image serves as the perturbation in the model necessary to create the reflections. As a result, RWI inverts mainly for the background model, like in MVA, without the need for extended images or angle gathers. Alkhalifah and Wu (2016); Wu and Alkhalifah (2015b) suggested a new objective function, which can use the information of both the diving and reflected waves to update the background in the case of isotropic media. They also proposed a scattering angle filter (Wu and Alkhalifah, 2015a) to clean up the gradient and admit only smooth update in the early iterations.

However, the isotropic approximation yields poor results in many regions because of the anisotropic feature of the Earth. Operto et al. (2015); Cheng et al. (2014) showed the potential of multi-parameters anisotropic full waveform inversion.

An additional issue appears when we start to invert for more than one parameter. Castellanos et al. (2015) proved that the Hessian matrix is crucial to correct for the cross-talk and scale differences in the parameter classes. For transversely isotropic media with vertical axis of symmetry (VTI), Gholami et al. (2013) and Alkhalifah and Plessix (2014) analyzed the trade-off based on the radiation patterns. For conventional surface recorded data, where diving waves (transmission) and reflections are both available, a parameterization of the wave equation using the horizontal velocity v_h , and the anisotropy parameters η and ε shows the most promise according to Alkhalifah and Plessix (2014).

In this abstract, we extend the inversion for the background model to VTI media. Starting from the pseudo-acoustic VTI wave equation, we derive the Born modelling formulation and the relative reflected waveform formulation. In order to remove the shear wave artifacts of pseudo-acoustic VTI wave equation, we suggest to apply a simple low pass filter in the wavenumber domain. Applications to the anisotropic Marmousi model and field dataset show the ability of the proposed method.

THEORY

Pseudo-acoustic VTI wave propagation can be described by the following coupled partial differential equations (Duveneck and Bakker (2011); Zhang et al. (2011); Chu and Stoffa (2012); Operto et al. (2014)):

$$(1 + 2\varepsilon)v_v^2 \frac{\partial^2 q}{\partial x^2} + \sqrt{1 + 2\delta}v_v^2 \frac{\partial^2 p}{\partial z^2} + \omega^2 q = f_x, \quad (1)$$

$$\sqrt{1 + 2\delta}v_v^2 \frac{\partial^2 q}{\partial x^2} + v_v^2 \frac{\partial^2 p}{\partial z^2} + \omega^2 p = f_z, \quad (2)$$

in which, $p = p(x, z, \omega)$ and $q = q(x, z, \omega)$ are wavefields, ω is the angular frequency, v_v is the P-wave velocity along the vertical anisotropy symmetry axis, ε and δ are Thomsen's anisotropy parameters, and $f = (f_x, f_z)$ is the source term. After obtaining the wavefield p and q , the total wavefield can be calculated as $p_{total} = \frac{p+2q}{3}$.

Alkhalifah and Plessix (2014) analyzed the trade-off based on the radiation patterns and found that (v, η, ε) offers a practical set necessary to reduce the trade-off and provide reasonable resolution, where $\eta = \frac{\varepsilon - \delta}{1 + 2\delta}$ and $v = \sqrt{1 + 2\varepsilon}v_v$ is the horizontal velocity. With this choice of parameterization, the relative acoustic wave equation is given by:

$$v^2 \frac{\partial^2 q}{\partial x^2} + \frac{v^2}{\sqrt{1 + 2\varepsilon}\sqrt{1 + 2\eta}} \frac{\partial^2 p}{\partial z^2} + \omega^2 q = f_x, \quad (3)$$

$$\frac{v^2}{\sqrt{1 + 2\varepsilon}\sqrt{1 + 2\eta}} \frac{\partial^2 q}{\partial x^2} + \frac{v^2}{1 + 2\varepsilon} \frac{\partial^2 p}{\partial z^2} + \omega^2 p = f_z. \quad (4)$$

Waveform inversion for acoustic VTI media in frequency domain

Based on this forward modeling operator, the standard waveform inversion can be formulated as:

$$\min_{v, \eta, \varepsilon} \frac{1}{2} \left\| \frac{2q+p}{3} - g \right\|_2^2. \quad (5)$$

In order to derive the Born modeling formulation required by reflected waveform inversion, we need to choose a parameter to represent the perturbation. However, according to the trade-off analysis (Alkhalifah and Plessix, 2014), parameter ε is sensitive to small scattering angles. In this case, we only need to perturb ε for Born modeling. The resulting Born modeling formulation can be described as:

$$v^2 \frac{\partial^2 q_1}{\partial x^2} + \frac{v^2}{\sqrt{1+2\varepsilon}\sqrt{1+2\eta}} \frac{\partial^2 p_1}{\partial z^2} + \omega^2 q_1 \quad (6)$$

$$= \frac{v^2 w}{(1+2\varepsilon)\sqrt{1+2\eta}} \frac{\partial^2 p}{\partial z^2}, \quad (7)$$

$$\frac{v^2}{\sqrt{1+2\varepsilon}\sqrt{1+2\eta}} \frac{\partial^2 q_1}{\partial x^2} + \frac{v^2}{1+2\varepsilon} \frac{\partial^2 p_1}{\partial z^2} + \omega^2 p_1 \quad (8)$$

$$= \frac{v^2 w}{(1+2\varepsilon)\sqrt{1+2\eta}} \frac{\partial^2 q}{\partial x^2} + \frac{2v^2 w}{(1+2\varepsilon)^3} \frac{\partial^2 p}{\partial z^2}, \quad (9)$$

in which, w is an independent variable, which is given by the perturbation of $\tilde{\varepsilon} = \sqrt{1+2\varepsilon}$. To reduce the number of parameters, we assume in the background $\varepsilon = \eta$. In summary, we end up with the optimization problem for RWI in VTI media given by,

$$\min_{v, \eta = \varepsilon, w} \frac{1}{2} \left\| \frac{2(q+q_1) + (p+p_1)}{3} - g \right\|_2^2, \quad (10)$$

in which the wavefield (p, q) is the solution of wave equation (3), and (p_1, q_1) is the solution of (6).

Removal of shear wave artifacts

It is well-known that pseudo-acoustic VTI wave propagation suffers from shear wave artifacts. These artifacts become severe especially when the source is located in the anisotropic region, which is the case for the secondary source given by Born modeling. In this case, the removal of the shear wave artifacts becomes important to the success of our reflected waveform inversion. As we know, the dispersion relation for the pseudo-acoustic wave equation is $\omega = v(x, k)k$. However, the shear wave artifacts admit waves with very low velocity, which means the artifacts focus at the high wavenumber components where the real wavefield is zero. More accurately, we can remove the wavefield for which $k \geq \frac{\omega}{v_{min}}$. That is to say, we set

$$p(k) = 0, \text{ and } q(k) = 0, \forall k \geq \frac{\omega}{v_{min}}.$$

In the above formulation, v_{min} is the minimum value of $v(x, k)$ for all x and k . Figure 1a shows the imaginary part of the finite difference solution of the pseudo-acoustic wave equation (3) with $v = 1.8$ km/s, $\eta = \varepsilon = 0.2$ and angular frequency $\omega = 10$ Hz. As we can see, it suffers from the shear wave artifacts. However, there are no shear wave artifacts in Figure 1b after applying the simple filter. Even though we apply a low pass filter in the wavenumber domain, the relative change is only in the gradient. The filter operator works as a preconditioner of the gradient.

Figure 1: (a) The finite difference solution of the wave equation (3). (b) The filtered solution.

NUMERICAL EXAMPLES

Our first example is the anisotropic Marmousi model. To reduce the shear wave artifacts of forward modeling, we add an isotropic water layer above the original model. The exact horizontal velocity is shown in Figure 5a. The exact $\eta = \varepsilon$ is shown in Figure 5b. Since we can only approximate the smooth part of the anisotropic parameter η , the exact η is smoothed and suppressed near the boundary. The same forward modeling operator is used in generating the synthetic data and inversion with angular frequency $\omega = 2.5$ Hz and 0.05 km space sampling in both the vertical and horizontal direction. For each outer iteration, we first update w with the LBFGS method to obtain the true amplitude image. After that, we update the background v and η at the same time. To update the background v with a multi-scale method, we apply a scattering angle filter (Wu and Alkhalifah, 2015a) to the gradient with respect to v with the parameters $n = -4, -3, -2.5, -2, -1.5, -1.0, -0.5, 0$ sequentially. Since the data is only sensitive to the low wavenumber components of η , we apply a scattering angle filter with $n = -3$ to force only smooth updates of η . The inverted horizontal velocity v is shown in Figure 5d, while the inverted $\eta = \varepsilon$ is shown in Figure 5e and the optimal image (w) for the inverted background v and η is shown in Figure 5f. To demonstrate the accuracy of the inverted results, the horizontal velocity v (η) profile at $x = 3$ km is shown in Figure 2a (2b). As we can see from these Figures, the inverted results show reasonable accuracy even starting from a linear increasing initial horizontal velocity, isotropic model and single frequency of 2.5 Hz.

Figure 2: The profile at $x=3$ km of (a) Horizontal velocity and (b) η (Pink curve: Initial model, blue curve: exact model and red curve: inverted model).

Waveform inversion for acoustic VTI media in frequency domain

Figure 3: Well-log comparison (Pink curve: Initial horizontal velocity, blue curve: well-log information and red curve: inverted horizontal velocity).

Next, we apply the proposed approach to a real 2D marine dataset acquired with a variable depth cable (Soubaras and Dowle, 2010). Specifically, the data was acquired with a variable depth streamer, which gives a broader bandwidth through receiver depth notch diversity than conventional flat towed streamer. The increasing cable depths at the intermediate and far offsets, in this case down to 50 m, also boosts the low frequency content, which is very helpful for waveform inversion. Following Díaz et al. (2014), we first take the average of offset and depth of the receiver for all the shots and keep the offset and depth the same for all the shots to reduce the noise. The original dataset has 1824 shots with 18.75 m shot interval. The minimum offset is 169 m, while the receiver interval is 12.5 m. The maximum recording time is about 7 s. We choose one shot from every four for a total of 200 shots in our inversion to reduce the cost. The initial model is isotropic obtained using an isotropic RWI (Wu and Alkhalifah, 2015a) followed by applying a smooth operator. After that, we do reverse time migration (RTM) with a $v(z)$ like almost exact water velocity (Wellington et al., 2015) to obtain a rough water bottom. We set the initial velocity in the water layer as described in Wellington et al. (2015) and keep it fixed throughout the inversion. The medium in the water layer is assumed to be isotropic. The resulting initial velocity model is shown in Figure 6a. Figure 6e shows the corresponding RTM image. We first apply the method to the data with frequencies of 2.9 Hz and 3 Hz, sequentially. For each frequency, we apply a scattering angle filter with $n = -3, -2, -1, 0$ sequentially to the gradient with respect to background horizontal velocity, to implement multi-scale inversion. Since the data is only sensitive to the low wavenumber components of η , a scattering angle filter of $n = -3$ is applied to the gradient with respect to η to force smooth updates of η . We only invert for w considering a maximum offset of 3 km to isolate the perturbation components of w . The inversion is implemented on a space sampling of 37.5 m in both x and z direction with a free surface assumption imposed on the forward modeling operator. Because of the coarse sampling, the receivers are not located on the grid. Interpolation is used to project the wavefields onto the true receiver locations. The resulting horizontal velocity (η and w) for this stage are shown in Figure 6b (6c and 6d). We follow the RWI results with a standard VTI Full waveform inversion (5) with the data frequencies of 4, 5, 6, 8, 10 Hz. During this stage, we keep η fixed and update only the horizontal velocity

v and ε , as data is insensitive to the perturbation component of η using the parametrization given by the horizontal velocity. Thus, we obtain the inverted horizontal velocity shown in Figure 4a. The RTM result with the inverted velocity (Figure 4a) and inverted η (Figure 6c) is shown in Figure 6f. The comparison of the RTM results indicate that the image in Figure 6f is better focused than the image in Figure 6e. Figure 3 shows the well-log comparison of the initial horizontal velocity, inverted horizontal velocity and well velocity at $x = 10.75$ km. This comparison indicates that our inversion procedure is converging.

Figure 4: (a) Inverted horizontal velocity. (b) Inverted ε .

CONCLUSIONS

We formulated a new optimization problem for reflected and full waveform inversion in VTI media. For this implementation of reflected waveform inversion, we only do Born modelling with respect to a perturbation in $\sqrt{1+2\varepsilon}$ to reduce the dependency on large offsets. To reduce the number of parameters, we assume the background $\eta = \varepsilon$. A simple low pass filter in the wavenumber domain is suggested to remove the shear wave artifacts of Born modeling. The proposed method resulted in credible inversions of the Marmousi model and a real marine dataset.

ACKNOWLEDGEMENTS

We thank Bruce VerWest, Esteban Díaz and P.J. Wellington for their insights in the CGG data and Operto Stéphane for some useful discussions. The seismic data shown in this abstract is proprietary to and provided courtesy of CGG. The well-log information is provided by Geoscience Australia. We especially thank Tony Weatherall and Lynda Thomas for reviewing our abstract. We thank KAUST for its support and the SWAG group for collaborative environment.

Waveform inversion for acoustic VTI media in frequency domain

Figure 5: (a) Exact horizontal velocity. (b) Exact $\eta = \epsilon$. (c) Isotropic linear increasing initial velocity. (d) Inverted horizontal velocity. (e) Inverted $\eta = \epsilon$. (f) Inverted w .

Figure 6: (a) The initial velocity. The inverted horizontal velocity (b), η (c) and w (d) of RWI with low frequency data. (e) RTM image of the initial isotropic velocity. (f) RTM image of the inverted horizontal velocity in Figure 4a and inverted $\eta = \epsilon$ in Figure 6d.

EDITED REFERENCES

Note: This reference list is a copyedited version of the reference list submitted by the author. Reference lists for the 2016 SEG Technical Program Expanded Abstracts have been copyedited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

REFERENCES

- Alkhalifah, T., and R.-E. Plessix, 2014, A recipe for practical full-waveform inversion in anisotropic media: An analytical parameter resolution study: *Geophysics*, **79**, no. 3, R91–R101, <http://dx.doi.org/10.1190/geo2013-0366.1>.
- Alkhalifah, T., and Z. Wu, 2016, The natural combination of full and image-based waveform inversion: *Geophysical Prospecting*, **64**, 19–30, <http://dx.doi.org/10.1111/1365-2478.12264>.
- Castellanos, C., L. Metivier, S. Operto, R. Brossier, and J. Virieux, 2015, Fast full waveform inversion with source encoding and second-order optimization methods: *Geophysical Journal International*, **200**, 720–744, <http://dx.doi.org/10.1093/gji/ggu427>.
- Cheng, X., K. Jiao, D. Sun, and D. Vigh, 2014, Anisotropic parameter estimation with full-waveform inversion of surface seismic data: 84th Annual International Meeting, SEG, Expanded Abstracts, <http://dx.doi.org/10.1190/segam2014-0821.1>.
- Chu, C., and P. L. Stoffa, 2012, Pure and pseudo-acoustic VTI wave modeling in the frequency domain using compact finite difference operators: 82th Annual International Meeting, SEG, Expanded Abstracts, <http://dx.doi.org/10.1190/segam2012-0749.1>.
- Díaz, E., Y. Duan, P. Sava, and G. Pratt, 2014, Image-domain and data-domain waveform tomography: A case study: 84th Annual International Meeting, SEG, Expanded Abstracts, 1243–1248.
- Duveneck, E., and P. M. Bakker, 2011, Stable p-wave modeling for reverse-time migration in tilted TI media: *Geophysics*, **76**, no. 2, S65–S75, <http://dx.doi.org/10.1190/1.3533964>.
- Gholami, Y., R. Brossier, S. Operto, A. Ribodetti, and J. Virieux, 2013, Which parameterization is suitable for acoustic vertical transverse isotropic full waveform inversion? Part 1: Sensitivity and trade-off analysis: *Geophysics*, **78**, no. 2, R81–R105, <http://dx.doi.org/10.1190/geo2012-0204.1>.
- Operto, S., R. Brossier, L. Combe, L. Metivier, A. Ribodetti, and J. Virieux, 2014, Computationally efficient three-dimensional acoustic finite-difference frequency-domain seismic modeling in vertical transversely isotropic media with sparse direct solver: *Geophysics*, **79**, no. 5, T257–T275, <http://dx.doi.org/10.1190/geo2013-0478.1>.
- Operto, S., A. Miniussi, R. Brossier, L. Combe, L. Metivier, V. Monteiller, A. Ribodetti, and J. Virieux, 2015, Efficient 3-d frequency-domain mono-parameter full-waveform inversion of ocean-bottom cable data: Application to Valhall in the visco-acoustic vertical transverse isotropic approximation: *Geophysical Journal International*, **202**, 1362–1391, <http://dx.doi.org/10.1093/gji/ggv226>.
- Plessix, R., Y. De Roeck, and G. Chavent, 1995, Automatic and simultaneous migration velocity analysis and waveform inversion of real data using a MBTT/WKBJ formulation: 75th Annual International Meeting, SEG, Expanded Abstracts, <http://dx.doi.org/10.1190/1.1887624>.
- Soubaras, R., and R. Dowle, 2010, Variable-depth streamer — A broadband marine solution: *First Break*, **28**, 89–96.
- Virieux, J., and S. Operto, 2009, An overview of full-waveform inversion in exploration geophysics: *Geophysics*, **74**, no. 6, WCC1–WCC26, <http://dx.doi.org/10.1190/1.3238367>.
- Wellington, P., R. Brossier, S. Garambois, and J. Virieux, 2015, 2D acoustic time domain full waveform inversion — A broadband application in the Carnarvon Basin, Australia: 77th Annual International Conference and Exhibition, EAGE, Extended Abstracts, <http://dx.doi.org/10.3997/2214-4609.201413060>.

- Wu, Z., and T. Alkhalifah, 2015a, Full waveform inversion based on scattering angle enrichment with application to real dataset: 85th Annual International Meeting, SEG, Expanded Abstracts, 1258–1262, <http://dx.doi.org/10.1190/segam2015-5922173.1>.
- Wu, Z., and T. Alkhalifah, 2015b, Simultaneous inversion of the background velocity and the perturbation in full-waveform inversion: *Geophysics*, **80**, no. 6, R317–R329, <http://dx.doi.org/10.1190/geo2014-0365.1>.
- Xu, S., D. Wang, F. Chen, G. Lambare, and Y. Zhang, 2012, Inversion on reflected seismic wave: 82nd Annual International Meeting, SEG, Expanded Abstracts, <http://dx.doi.org/10.1190/segam2012-1473.1>.
- Zhang, Y., H. Zhang, and G. Zhang, 2011, A stable TTI reverse time migration and its implementation: *Geophysics*, **76**, no. 3, WA3–WA11, <http://dx.doi.org/10.1190/1.3554411>.
- Zhou, H., L. Amundsen, and G. Zhang, 2012, Fundamental issues in full waveform inversion: 82nd Annual International Meeting, SEG, Expanded Abstracts, <http://dx.doi.org/10.1190/segam2012-0878.1>.