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The Static and Dynamic Behavior of MEMS Arch Resonators near Veering and the Impact of Initial Shapes

A.Z. Hajjaj, N. Alcheikh, and M.I. Younis^a

*Physical Science and Engineering Division, King Abdullah University of Science and Technology, Thuwal,
23955-6900, Saudi Arabia*

ABSTRACT

We investigate experimentally and analytically the effect of initial shapes, arc and cosine wave, on the static and dynamic behavior of microelectromechanical systems (MEMS) arch resonators. We show that by carefully choosing the geometrical parameters and the initial shape of the arch, the veering phenomenon (avoided-crossing) among the first two symmetric modes can be strongly activated. To demonstrate this, we study electrothermally tuned and electrostatically driven initially curved MEMS resonators. Upon changing the electrothermal voltage, we demonstrate high frequency tunability of arc resonators compared to the cosine-configuration resonators for the first and third resonance frequencies. For arc beams, we show that the first resonance frequency increases up to twice its fundamental value and the third resonance frequency decreases until getting very close to the first resonance frequency triggering the veering phenomenon. Around the veering regime, we study experimentally and analytically the dynamic behavior of the arc beam for different electrostatic loads. The analytical study is based on a reduced order model of a nonlinear Euler-Bernoulli shallow arch beam model. The veering phenomenon is also confirmed through a finite-element multi-physics and nonlinear model.

Keywords: Arch, nonlinearity, vibrations, micro and nano systems, veering, near-crossing, electrothermal and electrostatic actuation

^a Corresponding author Electronic mail: Mohammad.Younis@kaust.edu.sa

1. Introduction

Bistable structures have been the center of focus for several applications, such as energy harvesting [1], memory [2], logic [3], and filtering [4]. Bistable structures at the micro and nano scale can be realized by intentional fabrication (arches) [3-4] or by tuning a control parameter, as in buckled structures [5].

Several actuation mechanisms have been used to tune the behavior of bistable structures through axial loads, such as electrostatic [6], piezoresistive [7], and electrothermal [4-5]. The static and dynamic buckling behavior of different electrothermally actuated bistable structures have been investigated, such as U-shaped structures [8], V-shaped structures [9], and clamped-clamped structures [10-11]. Thermal actuation is also used to tune the resonance frequencies of bridge structures [5, 12] through controlling their stiffness.

The variation of the axial load, which accordingly changes the curvature of arch structures, can lead to coupling among the different modes of vibration, which can be linear in the case of veering [4] or nonlinear in the case of internal resonance [13]. Those modes can be coupled mechanically [14], electrically [15], among the structure itself [16], and through internal resonances [15, 17]. In a recent work [4], the veering phenomenon among the first two vibrational modes of an electrothermally tuned arch beam was exploited and demonstrated experimentally for filtering applications.

The phenomena of frequency crossing and veering [18-31] have long attracted attention in the classical structural dynamics. When two frequencies approach each other as varying a control parameter, they either crossover or veer from each other. Crossing occurs mostly among two frequencies of symmetric and antisymmetric modes. On veering (avoided-crossing), on the other hand, two frequencies get close to each other, as changing a control parameter, and then deviate away from each other. Mathematically, it is due to the linear coupling among the two involved modes. In the veering regime, both modes get affected by the shape of each other (hybridization) and then each one continues along the path that the other would have taken if they would to cross. Veering has been reported to occur among symmetric and anti-symmetric modes in cable-spring system [22, 24-25], plates [19], and curved beams [23]. It was also shown to occur among the symmetric modes of curved beams [18], sagged cables [24-25], CNTs [31], and curved cylinders [27]. Also it was demonstrated in discrete systems in spring [28-29], pendulums [21].

Among the early works, Petyt and Fleischer [18] investigated using the finite element method the variation of the frequencies of circular curved beams as varying the subtended angle. They showed the crossover of symmetric and antisymmetric frequencies and the veering among the symmetric frequencies.

The veering terminology was first dubbed by Leissa [19] who indicated that veering can occur among two degenerate modes of a membrane due to numerical discretization errors. On the other hand, Perkins and Mote [20] have proven that the avoid-crossing can physically occur in continuous and discrete systems. Lacarbonara *et al.* [30] studied the nonlinear vibrations of a hinged-hinged beam accounting for the interaction among the first two modes (first symmetric and antisymmetric). Both crossover and veering phenomena are shown as varying the stiffness of the torsional spring at the end of one boundary.

Veering has been reported recently on micro and nano curved structures and arch beam [4, 31]. The static and dynamic behavior of arch beams under electrostatic actuation is well investigated in the literature [31-39]. At the nanoscale, many studies have investigated the static and dynamic behavior of clamped-clamped CNTs (slack). Sazonova *et al.* [32] reported experimental investigations showing veering and indicated the importance of slack on the dynamical behavior of CNTs.

In previous works, we demonstrated the tunability of arch shaped beams [5-6] with various actuation mechanisms, electrothermal and electrostatic, and for various applications, such as logic [3], memory [40] and filtering [4]. These investigations were mainly experimental. Also, the static and dynamics of arches under electrostatic excitation have been investigated [37-38, 41-43]; however assuming idealized buckled beam configuration as the initial shape. No veering has been reported in these studies. One can note that despite the extensive work on the mechanics of arches, veering of arches has been rarely reported, and with no experimental presented data. There is a lack for consistent analytical and experimental work into the veering phenomenon, and particularly, on the forced vibration response of arches before, at, and after veering. Most of the theoretical studies of arch beams assume an initial cosine-wave shape (similar to the buckled configuration) except for few studies. For example, the static and dynamics of curved cables/beams of an arc shape were investigated in [43-48]. Also, in [43], the exact profile of a micromachined arch after fabrication, as scanned optically, was used in the model and was shown to yield better agreement with the experimental data.

In this work, we aim to study analytically and experimentally the effect of the arch initial shape on the activation of the veering phenomenon. We aim to investigate the variation of the resonance frequencies and specially the dynamic response of the system under consideration around the veering range (before, on, and after veering of the first two symmetric modes) while applying a DC bias, an AC electrostatic force, and a constant electrothermal voltage.

The rest of the paper is organized as follows. The nonlinear Euler-Bernoulli beam equation combined with the heat conduction equation is solved in Section 2. The experimental setup and results are presented in Section 3. A discussion of the variation of the eigenvalues is presented in Section 4. The dynamic response of the curved beam under electrostatic forcing and at a constant electrothermal load is reported in Section 5. Finally, the main conclusions are summarized in Section 6.

2. Problem Formulation

The device under consideration is made of silicon and consists of an initially a curved clamped-clamped beam, Fig. 1, of an initial shape $\hat{w}_0(\hat{x})$ governed by Eq. (1.a) for a cosine wave (buckled) shape and Eq. (1.b) for an arc shape:

$$\hat{w}_0(\hat{x}) = -\frac{l}{2}\hat{b}_0\left(1 - \cos\left(2\pi\frac{\hat{x}}{l}\right)\right) \quad (1.a)$$

$$\hat{w}_0(\hat{x}) = \left(\hat{b}_0 - R + \sqrt{R^2 - \left(\hat{x} - \frac{l}{2}\right)^2}\right)\left(u[l + \hat{x}] - u[\hat{x}]\right) \quad (1.b)$$

where $\hat{x}$ is the position along the microbeam and $\hat{b}_0$ represents the rise at the mid-point of the arch. $u(\hat{x})$ and R represent the Heaviside function and the radius of the arc, respectively. The curved beam has Young's modulus E , material density ρ , length l , width b , and thickness h . The cross section area of both configurations is assumed to be rectangular $A = bh$ with a moment of inertia given by $I = bh^3/12$. The curved beam is separated from a stationary electrode with a gap width d . The curved beam is actuated electrostatically by a DC polarization voltage V_{DC} and an AC harmonic voltage of amplitude V_{AC} and frequency $\hat{\Omega}$ and is subjected to a viscous damping of coefficient $\hat{c}$. The electrothermal voltage V_{TH} is applied between the anchors of the curved beam inducing a current I_{TH} passing through the beam that heats up it and controls its internally induced axial stress.

Fig. 1: Schematic of an electrothermally actuated clamped-clamped shallow arch (a) cosine wave (b) arc shape. (c) The profile of both cosine wave and arc configurations highlighting their difference.

Referring to [12] and the Fourier's law, the equation governing the average temperature across the cross-section of the curved beam $T[\hat{x}]$, induced by the electrical current I_{TH} , is given by

$$-\frac{d}{d\hat{x}}\left(k(T)\frac{dT}{d\hat{x}}\right)=J^2\rho_e(T) \quad (2)$$

where $k(T)$ is the thermal conductivity of the microbeam material, which has a nonlinear dependence on temperature. $\rho_e(T)$ denotes the electrical resistivity, which is assumed to have linear dependence on temperature for the doped silicon. The ends of the beam are assumed to be at room temperature T_a . The current density, J , can be written as a function of the DC electrothermal voltage V_{TH} , $J=V_{TH}/\rho_e(T)l$. Therefore, the Fourier's law equation can be written as

$$-\frac{d}{d\hat{x}}\left(k(T)\frac{dT}{d\hat{x}}\right)=\frac{V_{TH}^2}{\rho_e(T)l^2} \quad (3)$$

Thus, the variation of the temperature along the microbeam induces a compressive stress, $\hat{S}_{TH}$, given by

$$\hat{S}_{TH}=\frac{EA}{l}\int_0^l\alpha(T)(T[\hat{x}]-T_a)d\hat{x} \quad (4)$$

where $\alpha(T)$ is the coefficient of thermal expansion, which is assumed to be dependent on temperature.

The thermal and electrical parameters are given in Table 1.

Table 1. Thermal and electrical properties of the silicon microbeam.

Parameters	Expression/Value
$k(T)$ [49]	$k(T)=\frac{2.7}{-2.2\times 10^{-11}T^3+9\times 10^{-8}T^2-10^{-5}T+0.014}$
$\rho_e(T)$ [49]	$\rho_e(T)=\rho_0[l-\eta(T-T_a)]$
ρ_0 (electrical resistivity at room temperature T_a) [12]	$1.14\times 10^{-4}\Omega\text{m}$
η (resistivity temperature coefficient) [12]	$2.28\times 10^{-3}\text{K}^{-1}$
$\alpha(T)$ [50]	$\alpha(T)=\left\langle 3.75\times\left(1-e^{\{-5.88\times 10^{-5}(T-125)\}}\right)+5.548\times 10^{-4}T\right\rangle\times 10^{-6}$

The governing equation of motion of the curved beam under consideration, Fig. 1, describing its transverse deflection $\hat{w}(\hat{x}, \hat{t})$ in space and time $\hat{t}$ is written as [37, 51]

$$\rho b h \frac{\partial^2 \hat{w}}{\partial \hat{t}^2} + EI \frac{\partial^4 \hat{w}}{\partial \hat{x}^4} + \hat{c} \frac{\partial \hat{w}}{\partial \hat{t}} = \left(\frac{\partial^2 \hat{w}}{\partial \hat{x}^2} + \frac{d^2 \hat{w}_0}{d \hat{x}^2} \right) \left[\hat{N} + \frac{EA}{2l} \int_0^l \left(\left(\frac{\partial \hat{w}}{\partial \hat{x}} \right)^2 + 2 \frac{\partial \hat{w}}{\partial \hat{x}} \frac{d \hat{w}_0}{d \hat{x}} \right) d \hat{x} \right] + \frac{l}{2} \frac{(V_{DC} + V_{AC} \cos(\hat{\Omega} \hat{t}))^2}{(d - \hat{w} - \hat{w}_0)^2} \quad (5)$$

The microbeam is subjected to the following fixed-fixed boundary conditions:

$$\hat{w}(0, \hat{t}) = \hat{w}(l, \hat{t}) = 0 \text{ and } \left. \frac{\partial \hat{w}}{\partial \hat{x}} \right|_{(0, \hat{t})} = \left. \frac{\partial \hat{w}}{\partial \hat{x}} \right|_{(l, \hat{t})} = 0 \quad (6)$$

The term $\hat{N} = \hat{N}_0 - \hat{S}_{TH}$ represents the tensile axial load, where $\hat{N}_0$ is arising from the fabrication process and $\hat{S}_{TH}$ denotes the thermal compressive stress given by Eq. (4). ϵ presented the dielectric constant of the medium.

For convenience, we introduce the nondimensional variables

$$w = \frac{\hat{w}}{d}; x = \frac{\hat{x}}{l}; t = \frac{\hat{t}}{T_S}; w_0 = \frac{\hat{w}_0}{d} \text{ and } b_0 = \frac{\hat{b}_0}{d} \quad (7)$$

where $T_S = \sqrt{\rho b h l^4 / EI}$ is a time scale. Substituting Eq. (7) into Eq. (5) and Eq. (6), we obtain the nondimensional equation of motion of the beam

$$\frac{\partial^2 w}{\partial t^2} + \frac{\partial^4 w}{\partial x^4} + c \frac{\partial w}{\partial t} = \left(\frac{\partial^2 w}{\partial x^2} + \frac{d^2 w_0}{d x^2} \right) \left[N + \alpha_1 \int_0^l \left(\left(\frac{\partial w}{\partial x} \right)^2 + 2 \frac{\partial w}{\partial x} \frac{d w_0}{d x} \right) dx \right] + \alpha_2 \frac{(V_{DC} + V_{AC} \cos(\Omega t))^2}{(l - w - w_0)^2} \quad (8)$$

subjected to the nondimensional boundary conditions

$$w(0, t) = w(1, t) = 0 \text{ and } \left. \frac{\partial w}{\partial x} \right|_{(0, t)} = \left. \frac{\partial w}{\partial x} \right|_{(1, t)} = 0 \quad (9)$$

The parameters appearing in Eq. (8) are defined as

$$\alpha_1 = 6 \left(\frac{d}{h} \right)^2; N = N_0 + S_{TH}; N_0 = \frac{l^2}{EI} \hat{N}_0; S_{TH} = \frac{l^2}{EI} \hat{S}_{TH}; \alpha_2 = \frac{6 \epsilon l^4}{E h^3 d^3}; c = \frac{l^4}{E I T} \hat{c} \text{ and } \Omega = T_S \hat{\Omega} \quad (10)$$

3. Experimental Setup

The experimental validation was conducted on intentionally fabricated arc beams with specific initial shapes. The arc beams were fabricated by MEMCAP [52], from SOI wafers with highly conductive Si device layer. To determine the resonance frequencies as well as the frequency response of the structures, we use a stroboscopic video microscopy from Polytec [53], Fig. 2.

Fig. 2: Experimental setup.

The arc beams are actuated electrothermally by passing a DC current, I_{TH} , through them, Fig. 3. Also, they are actuated electrostatically to excite the structure into vibration. Three case studies presented in Table 2 will be investigated.

Fig. 3: An SEM image of the arch beam showing in schematic the electrothermal and electrostatic actuation circuits.

Table 2. Geometrical properties of the silicon microbeam.

Quantity	Arc Beam 1	Arc Beam 2	Arc Beam 3
Length (μm)	600	600	800
Thickness (μm)	2	2	2
Width (μm)	25	25	25
Initial rise at the mid-point (μm)	2	3	2.6
Gap (μm)	7	8	8

4. Eigenvalue Problem

In this section, we examine the variation of the resonance frequencies of both shape configurations, under consideration, as a function of the compressive load induced by the thermal voltage. Hence, the electrostatic force term is dropped from the equations. The deflection of the arch under the electrothermal force is split into a static deflection $w_s(x)$ due to V_{TH} and a small dynamic deflection $w_d(x, t)$ [37, 51].

The static equation is governed by

$$\frac{d^4 w_s}{dx^4} = \left(\left(\frac{d^2 w_s}{dx^2} + \frac{d^2 w_0}{dx^2} \right) \left[N + \alpha_l \int_0^l \left(\left(\frac{dw_s}{dx} \right)^2 + 2 \frac{dw_s}{dx} \frac{dw_0}{dx} \right) dx \right] \right) \quad (11)$$

with the associated boundary conditions

$$w_s(0) = w_s(1) = 0 \text{ and } \left. \frac{dw_s}{dx} \right|_{x=0} = \left. \frac{dw_s}{dx} \right|_{x=1} = 0 \quad (12)$$

The linearized equation of motion describing the small dynamic behavior of the curved beam around the new static configuration induced by V_{TH} and governed by Eq. (11) is derived by substituting $w(x, t) = w_s(x) + w_d(x, t)$ into Eq. (5) and dropping the terms representing the equilibrium position, the electrostatic force, and the nonlinear terms. The outcome equation becomes

$$\frac{\partial^2 w_d}{\partial t^2} + \frac{\partial^4 w_d}{\partial x^4} = \left[N + \alpha_I \int_0^l \left(\left(\frac{dw_s}{dx} \right)^2 + 2 \frac{dw_s}{dx} \frac{dw_0}{dx} \right) dx \right] \frac{\partial^2 w_d}{\partial x^2} + 2\alpha_I \int_0^l \left(\frac{dw_s}{dx} + \frac{dw_0}{dx} \right) \frac{\partial w_d}{\partial x} dx \left(\frac{d^2 w_s}{dx^2} + \frac{d^2 w_0}{dx^2} \right) \quad (13)$$

with the associated boundary conditions

$$w_d(0, t) = w_d(1, t) = 0 \text{ and } \left. \frac{\partial w_d}{\partial x} \right|_{x=0, t} = \left. \frac{\partial w_d}{\partial x} \right|_{x=1, t} = 0 \quad (14)$$

We resort to the Galerkin discretization to represent the dynamic deflection $w_d(x, t)$ and to solve the eigenvalue problem of the curved beam under the DC thermal voltage [51]. Toward this, we let

$$w_d(x, t) = \sum_{i=0}^n u_i(t) \phi_i(x) \quad (15)$$

where $u_i(t)$ ($i=0, 1, 2, \dots, n$) denotes the nondimensional modal coordinates and $\phi_i(x)$ ($i=0, 1, 2, \dots, n$) denotes the mode shape of the unactuated straight clamped-clamped beam.

Then, we substitute Eq. (15) into Eq. (13), multiplying the outcome by the mode shape ϕ_j and integrating over the beam domain (from 0 to 1), which yields the below equation [51]

$$\ddot{u}_j = - \int_0^l \phi_j \left(\sum_{i=0}^n u_i \phi_i^{(iv)} \right) dx + \left[N + \alpha_I \int_0^l \left(\left(\frac{dw_s}{dx} \right)^2 + 2 \frac{dw_s}{dx} \frac{dw_0}{dx} \right) dx \right] \int_0^l \phi_j \left(\sum_{i=0}^n u_i \phi_i'' \right) dx + 2\alpha_I \int_0^l \left(\frac{dw_s}{dx} + \frac{dw_0}{dx} \right) \sum_{i=0}^n u_i \phi_i' dx \int_0^l \phi_j \left(\frac{d^2 w_s}{dx^2} + \frac{d^2 w_0}{dx^2} \right) dx \quad (16)$$

Using five symmetric modes [12], we compute the Jacobian of the system of the five obtained equations, for each V_{TH} , and find the corresponding eigenvalues and mode shapes. Then, we compute the resonance frequencies of the resonators, at constant V_{TH} , by taking the square root of these eigenvalues.

Fig. 4 shows the variation of the first two symmetric resonance frequencies of the curved beams while tuning V_{TH} , experimentally and analytically for the two configurations. One can note that for all the case studies, the arc configuration shows a good agreement with the experimental results compared to the classically assumed cosine-wave (buckled) configuration. As shown in Figs. 4(a), 4(b) and 4(c), for the buckled configuration, the first resonance frequency increases while increasing V_{TH} and then slows down, almost flattens, as it gets close and passes the third resonance frequency. The third resonance frequency decreases slightly with increasing V_{TH} then starts to increase when getting close to the first resonance frequency. As noted, the first and third resonance frequencies never get too close to each other except for arc beam 3, Fig. 4(c).

On the other hand, and as proven experimentally, for the arc configuration, higher tunability is achieved for both resonance frequencies. The first resonance frequency increases as high as twice the fundamental frequency at zero

electrothermal voltage. The third resonance frequency decreases and gets much closer to the first resonance frequency, where both modes deviate from each other. Fig. 4(c) shows that for arc beam 3 the first and third resonance frequencies get very close at a critical electrothermal voltage. Then, they alter their directions, and each frequency continues along the path that the other frequency would have taken if they crossed, i.e., the first resonance frequency decreases while the third resonance frequency increases. This demonstrates the veering phenomenon (avoided-crossing), which is a mechanical way to linearly couple the two involved modes.

Fig. 4: The variation of the first two symmetric resonance frequencies while varying the electrothermal voltage of the curved beams for (a) Arc beam 1, (b) Arc beam 2, and (c) Arc beam 3.

To further verify the avoided-crossing behavior for arc beam 3, we analytically study the variation of the resonance frequency as varying the compressive load for the same beam but with different thicknesses, Fig. 5. Fig. 5(a) indicates that veering cannot be activated for cosine-wave configuration. Fig. 5(b) shows that for a specific thickness the third resonance frequency decreases until getting very close to the first resonance frequency. The figure verifies the avoided-crossing behavior. This presents a way to choose the geometric parameters of such curved beams carefully to activate or avoid the veering phenomenon depending on the targeted application.

Fig. 5: The variation of the first two symmetric resonance frequencies of Arc beam 3 for different thicknesses and while changing the nondimensional compressive stress. (a) Cosine wave shape. (b) Arc shape.

For arc beam 3, we investigate the variation of the different mode shapes around veering by solving the mode shapes problem associated with Eq. (16). Table 3 shows the variation of the first two symmetric modes as varying the electrothermal voltage. One can note the interchange of mode shapes among the first and third vibrational modes at veering. This explains the increase in sensitivity, after veering, of the third mode compared to the first mode.

Table 3. The first two symmetric mode shapes for various electrothermal voltages corresponding to Fig. 4c.

V_{TH} (V)	Mode 1	Mode 3
0		
3.5		
4.25		
4.5

In order to prove the validity of the different assumptions made by the analytical model, we conduct a 3D multi-physics finite-element FE simulation using the commercial finite element software COMSOL [54]. The arc shape (arc beam 3) is implemented in the model since it better describes the behaviors of the fabricated curved beams. For the different physical domains in the developed model, the Solid Mechanics, Electric Currents, and Heat Transfer interfaces are implemented. For the Electric Currents module, an electrical potential and a ground were defined on top of the anchors to allow passing an electrical current through a conductor and to simulate the Joule's heating. The detailed FE procedure is presented in [12]. A good agreement is shown in Fig. 6(a) among the experimental, analytical, and finite element results. The first and third mode shapes for different electrothermal voltages are depicted in Figs. 6(b), 6(c) and 6(d), and 6(e). A good agreement is shown among the mode shapes and those in Table 3.

Fig. 6: (a) The variation of the first two symmetric resonance frequencies while varying the electrothermal voltage of arc beam 3 experimentally, analytically, and using a finite element model. (b)-(e): The first three mode shapes of the arc beam for different electrothermal voltage obtained by the FE model for (b) $V_{TH}=0V$, (c) $V_{TH}=4.625V$, (d) $V_{TH}=4.75V$, and (e) $V_{TH}=9V$.

5. Dynamic Analysis

Next, we further analyze the forced vibration response near the veering phenomenon of arc beam 3. While exciting the beam electrostatically, we study the dynamic response before, at, and after veering; i.e., for different values of electrothermal voltage.

Fig. 7 shows different frequency responses, obtained experimentally, of the arc beam for various electrothermal voltages and different excitation electrostatic voltages. Fig. 7(a) shows the dynamic response before veering at $V_{TH}=3.5V$. The amplitude of vibration of the first mode is higher than the third mode, as expected since it is more sensitive than the third mode to the electrostatic forcing. As increasing the electrostatic voltage, the first mode exhibits softening (dominated by the quadratic nonlinearity coming from the beam curvature and the electrostatic force) and the third mode exhibits hardening (dominated by the cubic nonlinearity coming from mid-plane stretching; where the quadratic nonlinearity seems weaker here).

Getting much closer to the veering regime, $V_{TH}=4V$, the amplitude of vibration of the third mode starts to increase compared to the one of the first mode as increasing the electrostatic force, as shown in Fig. 7(b). This suggests that the third mode starts to take energy from the first mode. At veering, $V_{TH}=4.5V$, both modes, first and third, start to exchange energy. Fig. 7(c) displays equal amplitude of vibration for both modes for different electrostatic voltages (i.e., same sensitivity to the electrostatic force for both modes).

After the veering zone, the third mode starts to be more sensitive than the first mode as shown in Fig. 7(d) ($V_{TH}=5V$) and Fig. 7(e) ($V_{TH}=6V$). Figs. 7(d) and 7(e) demonstrate that after veering the third mode takes the nonlinear properties of the first mode before veering. Indeed, it starts to exhibit softening behavior for high electrostatic forcing instead of hardening behavior. That means that the cubic nonlinearity starts to dominate the quadratic nonlinearity after veering. One can note that, for the same applied electrostatic force, the maximum amplitude of vibration at the third resonance frequency at $V_{TH}=5V$ is higher than at $V_{TH}=4.5V$, contrary to what is expected since the stiffness and the rise at mid-point increase more by increasing the electrothermal voltage. Figs. 7(d) and 7(e) demonstrate that the response of the first mode has weakened after veering. The first mode shows also signs of hardening behavior in this regime.

(a)

(b)

Fig. 7: Frequency response of arc beam (3) under different electrostatic loads for different constant electrothermal voltages. (a) $V_{TH}=3.5V$ (before veering), (b) $V_{TH}=4V$ (close to veering), (c) $V_{TH}=4.5V$ (on veering), (d) $V_{TH}=5V$ (after veering) and (e) $V_{TH}=6V$ (after veering).

To simulate the dynamic response of the arc beam under electrostatic forcing and at a constant electrothermal voltage, we discretize Eq. (8) using the Galerkin procedure, which yields a reduced order model (ROM). To do so, the transverse deflection of the arc beam is written as [51]

$$w(x,t) = \sum_{i=0}^n q_i(t) \varphi_i(x) \quad (17)$$

where $q_i(t)$ ($i=0 \dots n$) are the nondimensional modal coordinates and $\varphi_i(x)$ ($i=0 \dots n$) are the mode shapes obtained either by solving the eigenvector problem associated with Eq. (16) at a constant electrothermal voltage or those of the unactuated straight beam.

Following [437, 51], we first multiply Eq. (8) by $(1 - w - w_0)^2$ in order to reduce the computational costs (integration of a numerator term is much less expensive than a dominator term). Then, by substituting Eq. (17) into Eq. (8), multiplying by $\varphi_j(x)$ and integrating along the arc beam, this yields n algebraic equation in terms of $q_i(t)$

$$\sum_{i=0}^n M_{ij} \ddot{q}_i(t) + \sum_{i=0}^n c_{ij} \dot{q}_i(t) + \sum_{i=0}^n K_{ij} q_i(t) = F m_j + F e_j(t) \quad \forall (j = 0 \dots n) \quad (18)$$

where

$$\begin{aligned}
 M_{ij} &= \int_0^l \left[\varphi_i(x) \varphi_j(x) (l - w(x) - w_0(x))^2 \right] dx \\
 c_{ij} &= c \int_0^l \left[\varphi_i(x) \varphi_j(x) (l - w(x) - w_0(x))^2 \right] dx \\
 K_{ij} &= \int_0^l \left[\varphi_i^{(iv)}(x) \varphi_j(x) (l - w(x) - w_0(x))^2 \right] dx
 \end{aligned} \tag{19}$$

$$\begin{aligned}
 Fe_j(t) &= \alpha_2 (V_{DC} + V_{AC} \cos(\Omega t))^2 \int_0^1 \varphi_j(x) dx \\
 Fm_j &= \alpha_1 \Gamma \int_0^l \left[\varphi_j(x) \left(\sum_{i=0}^n q_i(t) \varphi_i''(x) + w_0''(x) \right) (l - w(x) - w_0(x))^2 \right] dx \\
 \Gamma &= N + \int_0^l \left(\sum_{i=0}^n q_i(t) \varphi_i'(x) \right)^2 dx + 2 \int_0^l \left(\sum_{i=0}^n q_i(t) \varphi_i'(x) \right) w_0'(x) dx
 \end{aligned}$$

Starting by computing the integrals of Eq. (19) and then by using the Runge-Kutta technique, the dynamic response of the arc beam is obtained by time-integrating Eq. (18). Using four symmetric mode shapes (by solving the eigenvector problem in Eq. (16)), good agreement is reported in Figs. 8(a) and 8(b) between the simulations and the experimental results for $V_{TH}=3.5V$ ($V_{DC}=15V$, $V_{AC}=15V$) and $V_{TH}=4V$ ($V_{DC}=10V$, $V_{AC}=10V$), respectively. In Figs. 8(a) and 8(c), we show that using either four exact mode shapes of the arc beam under electrothermal voltage or using five symmetric mode shapes of unactuated straight beam yields the same result. For $V_{TH}=4V$, Fig. 8(c), increasing more the electrostatic force, we start to have a mismatch between the experimental and the analytical results around the first mode. On the other hand, the ROM was able to detect the response of the system around the third mode accurately, Figs. 8(c) and 8(d). Increasing more the electrothermal voltage $V_{TH}=5V$ (after veering regime), Fig. 8(d), the ROM was able to detect the change of the nonlinear behavior of the third mode from hardening to softening behavior while failing to accurately predict the response around the first mode.

Fig. 8: Analytical and experimental frequency response curves of arc beam (3) under electrostatic force and different constant electrothermal voltages. (a) $V_{TH}=3.5V$, $V_{DC}=15V$ and $V_{AC}=15V$, (b) $V_{TH}=4V$, $V_{DC}=10V$ and $V_{AC}=10V$, (c) $V_{TH}=4V$, $V_{DC}=15V$ and $V_{AC}=15V$, (d) $V_{TH}=5V$, $V_{DC}=20V$ and $V_{AC}=20V$. The assumed exact mode shapes are obtained by solving the eigenvector problem associated with Eq. (16) at a constant electrothermal voltage and SB (straight beam) mode shapes refers to those of a straight unactuated beam.

One can note that the ROM could not accurately capture the dynamic behavior near and after veering. One reason might be that the ROM does not take into consideration the contribution of the out-of-plane and rotational modes in the response, which experimentally are observed to affect the response. We have noticed high sensitivity of such modes as increasing the electrothermal voltage as seen during the experiment using the high speed camera. In addition to the fabrication imperfections, another potential source of error is the clamping condition that is assumed in the model to be with zero slopes. Experimental images suggest that this might not be accurate. Other techniques can be employed to demonstrate theoretically the veering phenomenon, mainly the multiple scale methods [30], which will be considered in future work.

6. Conclusions

In this paper, we showed that by choosing carefully the geometric parameters of an initially curved arc beam, the veering phenomenon can be activated. High tunability for the first two symmetric resonance frequencies is shown as varying the compressive load induced by the applied electrothermal voltage. We studied theoretically, using the reduced order model, and experimentally the dynamic behavior of such resonator before, at, and after veering. In the veering regime, the first and third modes exchange energy. After veering, we demonstrated that the third mode takes the nonlinear behavior of the first mode before veering and starts to be more sensitive than the first mode. We showed that the ROM can capture most of the dynamical behavior; however suffers near the veering regime, potentially due to the activation of the rotational modes near this regime.

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- ❖ The effect of the initial shape, arc and cosine wave, is investigated theoretically and experimentally on the static and dynamic behavior of microelectromechanical systems (MEMS) resonators.
- ❖ An electrothermally tuned and electrostatically driven initially curved MEMS resonators is used to study the activation of veering phenomenon.
- ❖ By carefully choosing the geometrical parameters and the shape of the initial curvature for arc beams, the first resonance frequency increases up to twice its fundamental value and the third resonance frequency decreases until getting very close to the first resonance frequency triggering the veering phenomenon.
- ❖ Around the veering regime, the dynamic behavior of the arc beam for different electrostatic loads is investigated experimentally and analytically. The analytical study is based on a reduced order model of a nonlinear Euler-Bernoulli shallow arch beam model.