The linearized inversion of the generalized interferometric multiple imaging

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Summary

The generalized interferometric multiple imaging (GIMI) procedure can be used to image duplex waves and other higher order internal multiples. Imaging duplex waves could help illuminate subsurface zones that are not easily illuminated by primaries such as vertical and nearly vertical fault planes, and salt flanks. To image first-order internal multiple, the GIMI framework consists of three datuming steps, followed by applying the zero-lag cross-correlation imaging condition. However, the standard GIMI procedure yields migrated images that suffer from low spatial resolution, migration artifacts, and cross-talk noise. To alleviate these problems, we propose a least-squares GIMI framework in which we formulate the first two steps as a linearized inversion problem when imaging first-order internal multiples. Tests on synthetic datasets demonstrate the ability to localize subsurface scatterers in their true positions, and delineate a vertical fault plane using the proposed method. We, also, demonstrate the robustness of the proposed framework when imaging the scatterers or the vertical fault plane with erroneous migration velocities.

Introduction

Imaging internally multiple-scattered energy has the potential to reconstruct seismic reflectors, which are poorly illuminated by primary reflection energy such as vertical and nearly vertical faults, and salt flanks. Youn and Zhou (2001) used a model-based estimate of internal multiples in their pre-stack depth migration method. Malcolm *et al.* (2009) proposed a method to image internally multiplied waves in which they migrated prismatic waves and other higher order multiples. Behura *et al.* (2012) used an iterative procedure to image internally scattered and single-scattered waves. Brown and Guitton (2005) and Aldawood *et al.* (2014) proposed linearized inversion frameworks to migrate internal multiples. Other methods for internal multiples migration include full wavefield migration (FWM) (Berkhout, 2012), reverse time migration (RTM) for internal multiples (Liu *et al.*, 2011; Fleury, 2013), and auto-focusing imaging (da Costa Filho *et al.*, 2015).

Recently, Zuberi and Alkhalifah (2014) proposed the generalized interferometric multiple imaging (GIMI) procedure to image first-order and higher order internal multiples. The GIMI migrated images, however, suffer from low spatial resolution, migration artifacts, and cross-talk noise. These effects are caused by the limited recording aperture, limited number of sources and/or receivers, and the band-limited nature of the source wavelet.

Imaging first-order internal multiples using GIMI consists of an extrapolation type datuming, and a cross-correlation type datuming followed by another extrapolation type datuming and the application of the imaging condition. In this abstract, we propose a linearized inversion framework in which the first two datuming steps are formulated as a single least-squares problem. Our goal is to enhance the spatial resolution, and suppress the migration artifacts and cross-talk noise when imaging duplex waves. We apply both the standard and the least squares GIMI frameworks to two synthetic datasets, and compared the images with the least-squares migrated ones using primaries only. Lastly, we demonstrate the robustness of the proposed least-squares GIMI procedure to reconstruct the synthetic models when imaging the recorded data using inaccurate migration velocities.

Theory

Imaging internally multiple-scattered waves using the standard GIMI procedure consists of three datuming steps as follows:

- Extrapolate the surface points (i.e. sources/receivers) from the Earth surface to the subsurface.
- Cross-correlate the extrapolated data with the recorded surface seismic data.
- Extrapolate the surface points (i.e. sources/receivers) from the recording surface to the trial image point and apply the zero-lag cross-correlation imaging condition.

The first two datuming steps, presented here in the frequency domain under the high-frequency asymptotic approximation for simplicity, are defined as follows:

$$d_2(g, x, \omega) = \sum_s \sum_g \left(d(s, g, \omega) e^{-i\omega \tau_{gx}} \right) d^*(s, g, \omega), \quad (1)$$

where $d_2(g, x, \omega)$ is the cross-correlated extrapolated data to the subsurface point x for a particular source s and a particular frequency ω . $d(s, g, \omega)$ is the recorded surface-seismic trace at point g due to a source at point s for a particular frequency ω . In addition, τ_{gx} is the traveltime from the receiver point g to the subsurface point x . The adjoint operation of the datuming steps in Equation (1) is given by:

$$d_a(s, g, \omega) = \sum_x \sum_g [d_2(g, x, \omega) d(s, g, \omega)] e^{+i\omega\tau_{gx}}, \quad (2)$$

Thus, the recorded data on the Earth surface can be predicted by forward extrapolation of the dataset resulting from the convolution of $d_2(g, x, \omega)$ with the recorded surface seismic data $d(s, g, \omega)$. Using the adjoint and forward modeling operators in Equation (1) and (2), the first two GIMI datuming steps can be formulated as one least-squares datuming problem (Xue and Schuster, 2008). The least-squares cross-correlated extrapolated data $\mathbf{d}_2^s$ can be obtained iteratively by solving the corresponding normal equation:

$$\mathbf{L}^\dagger \mathbf{L} \mathbf{d}_2^s = \mathbf{L}^\dagger \mathbf{d}_a, \quad (3)$$

where $\mathbf{L}^\dagger$ denotes the datuming operator in Equation (1), which linearly maps the surface seismic data to the cross-correlated extrapolated data. $\mathbf{L}$ denotes the forward (adjoint $\mathbf{L}^\dagger$) operator in Equation (2) that linearly maps the cross-correlated extrapolated data to the predicted surface seismic data. We use an iterative conjugate-gradient (CG) algorithm to solve the linear system in Equation (3) and obtain the solution after 40 iterations.

The last GIMI step is to apply the following extrapolation-based datuming and zero-lag cross-correlation imaging condition:

$$m(x) = \sum_\omega \sum_g d_2(g, x, \omega) e^{+i\omega\tau_{gx}}, \quad (4)$$

where the zero-lag cross-correlation imaging is obtained by summing the forward extrapolated data over all frequencies. In our proposed least-squares GIMI framework, we use the least-squares solution of Equation (3) as an input to the third GIMI step in Equation (4).

Examples

We tested the feasibility of the proposed least-squares GIMI procedure on two synthetic datasets and compared the results with the standard GIMI procedure results, and the least-squares migration of the primary reflections ones. The two 50 by 100 grid size synthetic models have the following acquisition parameters: 4 ms time sampling rate, $\Delta x = \Delta z = 30$ m, 100 shot points and 100 receiver points along the surface of the Earth, and a Ricker wavelet source function with a dominant frequency of 15 Hz.

The first example consists of two scatterers in a constant background velocity of 2000 m/s and the true reflectivity distribution is shown in Figure 1 (top). The standard GIMI first-order multiples image is plotted in Figure 1 (middle).

The image suffers from low spatial resolution, migration artifacts, and cross-talk noise. These effects are mainly caused by the limited coverage of sources and receivers, the limited bandwidth of the source wavelet, and the cross-talk between recorded primaries and multiples. Therefore, the scatterers are heavily blurred and not accurately localized. To mitigate these effects, we applied the least-squares GIMI method to image the first-order internal multiples and plotted the results in Figure 1 (bottom). The migrated image shows a remarkable enhancement of the spatial resolution of the scatterers as the least-squares GIMI helps suppress the migration artifacts and cross-talk noise, and deconvolve the source wavelet and acquisition fingerprint. The least-squares GIMI framework yields a migrated image that is comparable with the least-squares migration result of primaries only, shown in Figure 2. We then tested the robustness of the least-squares GIMI when imaging with inaccurate velocities. We obtained the solution by imaging with 5% faster and slower migration velocities and plotted the results in Figure 3 (top) and Figure 3 (bottom), respectively. The images shows that the scatterers are slightly defocused and mispositioned, compared with the least-squares GIMI image in Figure 1 (bottom).

The second model consists of two layers with a separating vertical fault plane as shown in Figure 4 (top). We migrated the first-order internal multiples using the standard GIMI framework and plotted the image in Figure 4 (middle). The result shows that the imaging procedure yields a smeared and poorly resolved vertical fault plane due to primarily cross-talk noise and migration artifacts. In fact, the vertical fault is distorted compared with the true fault plane in Figure 4 (top). To rectify these problems, we imaged the first-order multiples using the proposed least-squares GIMI procedure and presented the result in Figure 4 (bottom). The image shows that the fault plane is more resolved and accurately delineated in its true position. For comparison, we plotted the least-squares migrated image of primaries only in Figure 5. It demonstrates that imaging primaries can mainly illuminate horizontal reflectors whereas imaging first-order internal multiples could help delineate the vertical fault plane. We lastly migrated the first-order internal multiples using the proposed least-squares GIMI method with 5% faster and slower migration velocities. The fault plane becomes slightly misplaced and defocused in case we image with the inaccurate migration velocities as shown in Figure 6 (top) and Figure 6 (bottom).

Conclusion

Imaging first-order internal multiples helps delineate seismic reflectors that are vertically aligned such as vertical faults or salt flanks. The standard GIMI procedure can be applied as a three-step datuming and imaging framework to map doubly-scattered waves into seismic images. We

showed that the GIMI images suffer from low spatial resolution due to the presence of cross-talk noise and migration artifacts when imaging prismatic waves. We thus proposed a least-squares GIMI framework in which we formulate the extrapolation-type datuming step and the cross-correlation-type datuming step as a linearized inversion problem. The aim of the least-squares GIMI method is to suppress the cross-talk noise and migration artifacts, and deconvolve the source wavelet and acquisition fingerprint. We demonstrated the applicability of the proposed linearized inversion framework to localize two scatterers and resolve a vertical fault plane. We also showed that the proposed least-squares GIMI procedure is robust in the case of imaging the two scatterers or the vertical fault plane with inaccurate migration velocities.

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Figure 1: Two-scatterer model: the true reflectivity model (top). The standard GIMI image (middle). The least-squares GIMI image (bottom).

Figure 2: Two-scatterer model: the least-squares migration of primaries only.

Figure 3: Two-scatterer model: the least-squares GIMI image using a 5% faster migration velocity (top). The least-squares GIMI image using a 5% slower migration velocity (bottom). The scatterers are shifted upward and downward in the case of imaging the data with the faster and slower migration velocities, respectively.

Figure 4: Fault-plane model: the true velocity and reflectivity distribution (top). The standard GIMI image (middle). The fault plane is distorted and defocused. The least-squares GIMI image (bottom). The least-squares GIMI framework helps suppress the cross-talk noise and migration artifacts, and focus the fault plane in its true position.

Figure 5: Fault-plane model: the least-squares migration of primaries only delineates mainly horizontal reflectors.

Figure 6: Fault-plane model: the least-squares GIMI image using a 5% faster migration velocity model (top). The least-squares GIMI image using a 5% slower migration velocity model (bottom). The fault plane becomes slightly defocused and shifted upward and downward in the case of imaging the data with the faster and slower velocities, respectively.

EDITED REFERENCES

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