Scattering angle based filtering via extension in velocity

Vladimir Kazei^{*†}, King Abdullah University of Science and Technology, Saudi Arabia, Ekkehart Tessmer[‡], University of Hamburg, Germany, and Tariq Alkhalifah[†]

SUMMARY

The scattering angle between the source and receiver wavefields can be utilized in full-waveform inversion (FWI) and in reverse-time migration (RTM) for regularization and quality control or to remove low frequency artifacts. The access to the scattering angle information is costly as the relation between local image features and scattering angles has non-stationary nature. For the purpose of a more efficient scattering angle information extraction, we develop techniques that utilize the simplicity of the scattering angle based filters for constant-velocity background models. We split the background velocity model into several domains with different velocity ranges, generating an "extension in velocity". We establish an analytical relation between the proposed extension and conventional RTM images extended in time and show that extending with a few samples in the newly introduced dimension is sufficient to create a scattering angle based filter. Finally the new efficient algorithm is used in a conventional FWI scheme to filter the misfit gradients and improve inversion results.

INTRODUCTION

Conventional multiscale full-waveform inversion (Bunks et al., 1995) faces convergence problems in the case of a lack in low temporal frequencies in the acquired dataset. This problem can be treated by replacing the misfit functional (Van Leeuwen and Mulder, 2010; Bozdağ et al., 2011) or expanding the search space (Symes and Carazzone, 1991; Alkhalifah and Wu, 2014; van Leeuwen et al., 2014). However, these approaches increase the computational cost of inversion. Advanced multi-dimensional optimization techniques like the L-BFGS method (Nocedal, 1980) or the truncated Newton method (Métivier et al., 2014) construct model updates based on the gradients of the misfit and do not exploit directly the values of the functional. Thus instead of or together with modifying misfits one can improve FWI convergence by conditioning the gradients. The linear component of waveform inversion, the gradient, obtained in an imaging-like process, involves correlating two wavefields. One wavefield corresponds to the source and the other corresponds to a wavefield propagated back from a receiver location. The resolution or the smoothness of the resulting gradient, governed by diffraction tomography principles (Devaney, 1984; Mora, 1989; Kazei et al., 2013), is controlled by the angle between the two interacting wavefields (Figure 1) as well as by their local wavelength $\lambda(\mathbf{x})$ (equation 1). Proper selection of the scattering angles can mimic the gradient behavior at lower frequencies ("scale the frequency") by conditioning the gradients at higher frequencies. In particular scattering angle based filters can help FWI to mitigate the local minima of a misfit (Alkhalifah, 2015b; Kazei et al., 2015).

SCATTERING ANGLES & LOCAL WAVENUMBERS

Here we discuss a well-known tool for seismic imaging local resolution analysis — the Ewald's spheres construction (Ewald, 1969). The main outcome from this theory that is used in geophysics (Devaney, 1984; Mora, 1989; Kazei et al., 2013; Zhou et al., 2015) is the simple geometrical relation between the spatial wavenumbers in a local perturbation and the scattering plane wave (Figure 1). This relation is described by equation 1 and is valid within the first-order Born approximation and locally homogeneous media approximations.

$$\mathbf{K} = \frac{2\omega}{v(\mathbf{x})}(\mathbf{s} + \mathbf{r}), \quad |\mathbf{K}| = \frac{4\pi}{\lambda(\mathbf{x})} \cos \frac{\theta}{2}, \quad \lambda = \frac{2\pi v(\mathbf{x})}{\omega}, \quad (1)$$

Here $\mathbf{K}$ is the spatial wavenumber vector of the velocity per-

Figure 1: Scattering angle θ and local wavenumber vector $\mathbf{K}$.

turbation δv that causes a particular scattering event, $\lambda(\mathbf{x})$ is the local wavelength related to the local background velocity $v(\mathbf{x})$, and the source's temporal frequency ω . Since at each iteration in full-waveform inversion the Born approximation is used, the other interpretation of equation 1 is the relation between the wavenumber updated and the incident directions of wavefields ($\mathbf{s}, \mathbf{r}$) from the source and receiver, respectively, that are correlated to obtain the gradient. Equation 1 suggests that filtering the scattering angles θ can be done by imposing conditions on the local spatial wavenumbers $\mathbf{K}$ of the gradients, which depend on the local wavelength $\lambda(\mathbf{x})$. The simplest, yet not the most efficient way, to implement the filtering for high scattering angles is through non-stationary convolution with a filter which has a length proportional to the local velocity of the background model (Kazei et al., 2015).

Figure 2: A filter within the starting model for inversion for Marmousi. The length of the filter (black) is governed by the local wavelength (red) at a desired, lower frequency than that available from the data.

Scattering angle based filtering

THE EFFICIENT FILTERING ALGORITHM

To implement the non-stationary filter we separate the original image into parts with different velocity ranges and process each of them as if the velocity was constant and equal to the average value in the range. We first select a number N which defines the number of domains that the gradient is divided to. This number represents the tradeoff between the efficiency and the quality of the filtering results. The larger the number of domains, the more precisely conditions can be imposed on the scattering angles. We split the image $I(\mathbf{x})$ into N summands based on the background velocity model:

$$I(\mathbf{x}) = \sum_i I_i(\mathbf{x}), \quad I_i(\mathbf{x}) = I(\mathbf{x})U_i(\mathbf{x}), \quad (2)$$

where the U_i functions represent a decomposition of unity

$$\sum_{i=0}^{N-1} U_i(\mathbf{x}) = 1, \quad (3)$$

based on the background velocity distribution

$$U_i(\mathbf{x}) = \begin{cases} \cos^2\left(\frac{\pi(v(\mathbf{x}) - v_i)}{4\Delta v}\right), & \text{if } v(\mathbf{x}) \in [v_i - 2\Delta v; v_i + 2\Delta v] \\ 0, & \text{if } v(\mathbf{x}) \notin [v_i - 2\Delta v; v_i + 2\Delta v]. \end{cases} \quad (4)$$

The sequence of velocities v_i is determined as follows:

$$v_1 = v_{min}, \quad v_{i+1} = v_i + 2\Delta v, \quad \Delta v = \frac{v_{min} - v_{max}}{N}. \quad (5)$$

To obtain the functions U_i for a decomposition of unity the most straight forward way would be to use the characteristic functions for the intervals. Yet this does not lead to good smooth results at intermediate scattering angles due to Gibbs phenomena. Thus, we use the smooth version of a decomposition of unity expressed by equation 4. We show how the model can be split on the example of the slightly smoothed Marmousi model to illustrate the general behavior of functions U_i (Figure 3b-3d). The total number of layers N in this case is taken equal to 3. It shows how complicated domain boundaries of the velocity model in given intervals can be. After the image is decomposed into several areas with different velocities the scattering filter in each of them is applied as a stationary filter as in the case of a constant velocity background:

$$\tilde{I}_i(\mathbf{x}) = F^{-1}w(\mathbf{K}, v_i, \omega)FI_i(\mathbf{x}). \quad (6)$$

Here F and F^{-1} stand for the Fourier and inverse Fourier transforms in space, respectively. To remove the low frequency imaging artifacts in RTM, we can use any function $w(\mathbf{K})$ increasing with absolute $\mathbf{K}$ values. We take a smoothed version of the characteristic function (Khalil et al., 2013):

$$w(\mathbf{K}, v_i, \omega) = \begin{cases} 1, & \text{if } \cos \frac{\theta}{2} > \frac{v_i|\mathbf{K}|}{\omega} \\ 0, & \text{if } \cos \frac{\theta}{2} \leq \frac{v_i|\mathbf{K}|}{\omega} \end{cases} \quad (7)$$

for a given angle θ .

To mitigate the nonlinear parts of the FWI gradient we can choose (Alkhalifah, 2015a):

$$w(\mathbf{K}, v_i, \omega) = \begin{cases} 0, & \text{if } \cos \frac{\theta}{2} > \frac{v_i|\mathbf{K}|}{\omega} \\ 1, & \text{if } \cos \frac{\theta}{2} \leq \frac{v_i|\mathbf{K}|}{\omega} \end{cases} \quad (8)$$

The filtered gradient $\tilde{I}(\mathbf{x})$ is then obtained by stacking the filtered domains via equation 6 parts:

$$\tilde{I}(\mathbf{x}) = \sum \tilde{I}_i(\mathbf{x}). \quad (9)$$

Figure 3: Marmousi model (a) and the corresponding unity decomposition functions U_i (b-d). The model is decomposed into 3 layers: low velocity zone (b), intermediate velocities (c), high velocities (d). Colors denote the weights.

APPLICATION TO FWI

We show the application of the frequency scaling filter on the example of inversion for the Marmousi model (Bourgeois et al., 1991). For modeling we use a pseudo-spectral method (Kosloff and Baysal, 1982; Tessmer, 2011; Virieux et al., 2011) with a 4-th order Taylor expansion in time for time stepping. Pseudo-spectral modeling allows us to keep the model very sparse,

Scattering angle based filtering

having 2 grid points per shortest wavelength, which matches in the low velocity regions the highest possible FWI resolution determined as half of the local wavelength (Virieux and Operto, 2009). We start from the model with velocity linearly increasing in depth (Figure 2). The grid step is chosen equal to 32 m which allows us to apply inversion with frequencies up to 20 Hz. At all sides of the model we attach a 1 km wide sponge absorbing boundary. The synthetic data from 4 shots were acquired at 276 receiver locations at the surface. First, we try to use the conventional multiscale full-waveform inversion (Bunks et al., 1995) of 5 Hz central frequency data. From the first iterations, the parts of the model corresponding to high frequency were updated with the following gradient (Figure 4a) and that probably led us to converge to an inaccurate model as shown in Figure 4c. When applied in time

Figure 4: Gradient of classic FWI (a) and filtered with the new algorithm to mimic the missing frequency of 2.5 Hz (b). FWI results without the filter (c) and using it at first iterations (d).

domain FWI, the non-stationary scattering angle based filter-

ing restricts the gradient local wavenumber content to the one obtained from lower frequencies rather than scattering angles according 1, thus, it is more fair to call it a frequency scaling filter. Convergence can however be achieved by filtering the gradients with the frequency scaling non-stationary filter with length proportional to the local wavelength at a lower frequency than available from the data (Figure 2). The main feature of the non-stationary filtering approach is that it provides a very detailed image of the upper part while the deeper is not updated much due to stronger gradient smearing compared to the regular Gaussian filtering (Figure 4b). This feature is related to all the background models where velocity increases with depth. Inversion of the same data regularized by the frequency scaling filter converges Figure 4d. In general this scattering angle based filtering can be used for scale separation in algorithms combining wave equation migration velocity analysis (WEMVA) and FWI, e.g. reflection waveform inversion RFWI (Alkhalifah, 2015a) or RWI (Zhou et al., 2015) or Tomographic Full-Waveform Inversion (TFWI) (Biondi and Al-momin, 2013) Thus the benefits of this non-stationary scattering angle filter for full-waveform inversion convergence are almost the same as for the conventional one (Alkhalifah, 2015b). Further testing within the advanced FWI frameworks is going to be the next application.

EXTENSION IN VELOCITY VS EXTENSION IN TIME

Filtering for the scattering angle information can also be performed with the aid of extended images. Using the normalized time-lag representation (Khalil et al., 2013), the process of a gradient or an image extension in the frequency domain is given by the following formula

$$I(\mathbf{x}, \zeta) = I(\mathbf{x})e^{-2i\omega \frac{\zeta}{v(\mathbf{x})}}, \quad (10)$$

where $I(\mathbf{x})$ is the image and v is the velocity defined by using the space variable vector $\mathbf{x}$, ζ is the time normalized extension parameter, and ω is the angular frequency. An inherent feature of this modified time-lag (distance units) representation is that the relationship of the scattering angle to the wavenumber of the gradient is free of a velocity (space) dependency. In fact, the scattering angle θ is then given by the following formula:

$$\cos^2 \frac{\theta}{2} = \frac{|\mathbf{K}|^2}{k_\zeta^2}, \quad (11)$$

where $\mathbf{K}$ is the wavenumber vector and k_ζ is the wavenumber (Fourier transform) corresponding to ζ . A four-dimensional Fourier transform of $I(\mathbf{x}, \zeta)$ (three-dimensional in 2D), allows to map $I(\mathbf{K}, k_\zeta)$ to its angle gather equivalent $I(\mathbf{K}, \theta)$, using equation 11. Extending a seismic image in time (Sava and Fomel, 2003) and doing multidimensional Fourier transforms can be computationally overwhelming, especially in 3-D. We consider a monochromatic gradient - sensitivity kernel at 7 Hz for a single pair of source and receiver to explain the procedure and obtain some insights. The conventional L_2 misfit gradient obtained for ray-Born inversion (Beydoun and Mendes, 1989) is considered to isolate the problem of geometrical spreading compensation, which is treated by quasi-Newton methods

Scattering angle based filtering

within the regular full-waveform inversion. Following equation 10 we generate the gradient extension in time shown in Figure 5a. Since the extension is used to filter in the low wavenumbers necessary for FWI convergence we have to consider rather long extensions in ζ which makes it very computationally intensive. This adds a cost of order $N \log N$ to a direct wavenumber filtering of the gradient, where N is the number of samples of the extended axis ζ . In this example, we used 200 ζ samples at a sampling rate of 0.02 km, which also increased our storage requirements 200 times. Fortunately the redundancy visible in the Figure 5a can be exploited to perform filtering more efficiently. After the Fourier transform to $(\mathbf{K}, k_\zeta)$ (Figure 5b) the energy of the image seems to be concentrated in a small range of k_ζ values. In order to explain this observation one can have a look at the image transformed into $(\mathbf{x}, k_\zeta)$ domain (Figure 5c).

$$I(\mathbf{x}, k_\zeta) \propto I(\mathbf{x}) \delta(k_\zeta - \frac{2\omega}{v(\mathbf{x})}), \quad (12)$$

which proves that the images $I(\mathbf{x}, k_\zeta)$ and $I(\mathbf{K}, k_\zeta)$ outside of the range

$$k_\zeta \in \left(\frac{2\omega}{v_{\max}}, \frac{2\omega}{v_{\min}} \right) \quad (13)$$

($v_{\min}$ and $v_{\max}$ are the minimum and maximum velocities in the background model respectively) should be exactly zero. Equation 12 also suggests that in mapping to $I(\mathbf{x}, k_\zeta)$ only certain k_ζ values proportional to the local slowness would matter. This means that extension in k_ζ is similar to splitting the model into the layers based on local slowness for the monochromatic case and only the parametrization differs from the extension in the velocity extension proposed here. Thus, extending a monochromatic image in $k_{\zeta\eta}$ is nothing but extending it in the slowness if we neglect numerical artifacts due to finite and discrete ζ extension, which leads to noise in Figure 5c and filtering algorithms should provide similar results.

CONCLUSIONS

Recognizing that a constant velocity background admits stationary scattering angle filters that can be employed efficiently, we develop an efficient scattering angle filter for complex backgrounds, by extending the image (gradient) over representative constant velocity domains. Often the number of these domain representative constant velocities needed to establish an accurate filter are few, and as a result, the cost of such filtering is much lower than the classic domain extension often used to extract the scattering angle information. For seismic imaging in 3-D, the benefit extends to reduction in memory requirements. An application to inversion for the Marmousi model demonstrated the effectiveness of the approach.

ACKNOWLEDGMENTS

We thank KAUST, the WIT consortium and G-RISC (German-Russian Interdisciplinary Science Center) for support. Vladimir Kazei is also grateful to Prof. Paul Sava of the Colorado School of Mines for discussions on extensions of seismic images.

Figure 5: Monochromatic FWI gradient $I(\mathbf{x}, \zeta)$ at 7Hz extended in normalized time ζ (a), its 3-D spectrum $I(\mathbf{K}, k_\zeta)$, and its 1-D Fourier transform $I(\mathbf{x}, k_\zeta)$ (c).

EDITED REFERENCES

Note: This reference list is a copyedited version of the reference list submitted by the author. Reference lists for the 2016 SEG Technical Program Expanded Abstracts have been copyedited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

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