Micro-seismic imaging using a source independent full waveform inversion method

Hanchen Wang and Tariq Alkhalifah, King Abdullah University of Science and Technology

SUMMARY

Using full waveform inversion (FWI) to locate microseismic and image microseismic events allows for an automatic process (free of picking) that utilizes the full wavefield. However, waveform inversion of microseismic events faces incredible nonlinearity due to the unknown source location (space) and function (time). We develop a source independent FWI of microseismic events to invert for the source image, source function and the velocity model. It is based on convolving reference traces with the observed and modeled data to mitigate the effect of an unknown source ignition time. The adjoint-state method is used to derive the gradient for the source image, source function and velocity updates. The extended image for source wavelet in z axis is extracted to check the accuracy of the inverted source image and velocity model. Also the angle gather is calculated to see if the velocity model is correct. By inverting for all the source image, source wavelet and the velocity model, the proposed method produces good estimates of the source location, ignition time and the background velocity for part of the SEG overthrust model.

INTRODUCTION

Hydraulic fracturing is one of the most commonly used methods in oil and gas extraction, especially in those reservoirs with dense rock such as shale. Usually, water or other liquids are injected into the reservoir area and to create high pressure conditions, which eventually crack the dense rock in order to make the oil and gas flow more freely. The fracking process causes micro-seismic events, which can be monitored through sensors in a well or on the earth surface. Through the monitoring, the locations of the micro-seismic events can be estimated, which in turn can help us monitor the fracturing process. Such information helps the engineers optimize their injection strategy. Thus, locating the micro-seismic event is important to the hydraulic fracturing process. There are many ways to find the micro-seismic source locations. Some methods are based on traveltimes picks of both P-wave and S-wave arrivals (Eisner et al., 2009; Waldhauser and Ellsworth, 2000). However, traveltimes based methods suffer in noisy data resulting in errors in picking the arrivals, which will eventually lead to incorrect micro-seismic source locations. Other methods are based on migration techniques, such as time reverse migration (Artman et al., 2010). These methods use not only the traveltimes, but also the waveform information. If a velocity model is given, one can use the wave equation to back propagate the observed data and find the sources in the time reversed wavefield. Migration based methods require a good velocity model so as to correctly focus the energy, as well as, a dense and uniform acquisition system. Sava (2011) used an interferometric method to breakdown the limits. However, he assumed the onset of the sources are known, which is also a

big problem in micro-seismic imaging. Another sort of micro-seismic imaging methods are based on full waveform inversion (FWI). Kamei et al. (2014) updated the velocity model via an FWI algorithm, but they assumed the velocity change only happened in fluid injection area. The rest of the velocity model remained the same and was given as the initial velocity model. Kaderli et al. (2015) used an FWI based technique to update the spatial and temporal components of the source. In their paper, a homogeneous model is given and used, which is not satisfied in real life. In this abstract, a gradient based FWI method is used to update the spatial component of the source and a source-independent FWI approach is used to update the velocity model. The source-independent FWI approach works well with any given source wavelet, as it is based on convolved wavefields (Choi and Alkhalifah, 2011). An extended image method is also used to check whether the inverted velocity and source image are good or not. By inverting for the source spatial and time components and the velocity model, the method is able to find the source correctly even with an initial velocity model that is very different from the true one. Application on the SEG overthrust model demonstrates these features and the angle domain common image gather (ADCIG) is computed to show the effectiveness of the approach.

THEORY

The acoustic wave propagation in a constant density 2D medium obeys the following wave equation:

$$\frac{1}{c^2(x,z)} \frac{\partial^2 u(x,z,t)}{\partial t^2} - \nabla^2 u(x,z,t) = f(x,z)w(t) \quad (1)$$

where $u(x,z,t)$ is the acoustic wavefield in 2D media described location wise by x laterally, and z in depth, $c(x,z)$ is the velocity, while $f(x,z)$ and $w(t)$ are source components in space and time, t , respectively. This decomposition of the source assumes that the time and space dependency are decoupled, which may be a reasonable approximation considering our source independent implementation. We usually start our investigation with a crude guess or evaluation of the initial velocity model, $c_0(x,z)$. We can estimate an initial $w(t)$ with the available initial model using time reversal migration. Thus, through an iterative inversion we update these initial guesses of the microseismic event and medium. Considering the observed data of the microseismic events on the Earth surface (or a well), d^{obs} , we can devise an iterative inversion by defining the optimization problem based on the classic least-square minimization of

$$E(f,c) = \frac{1}{2} \|d^{pred} - d^{obs}\|, \quad (2)$$

with respect to the source image, the source wavelet and the velocity, where d^{pred} is the predicted data generated from the current velocity model. The general update direction, given first by the gradient with respect to f , courtesy of the adjoint-

state method, is as follows:

$$\frac{\partial E}{\partial f} = \int_0^T w(t) \cdot u_r(x, z, t) dt, \quad (3)$$

where u_r is the adjoint (residual) wavefield. The gradient with respect to w is,

$$\frac{\partial E}{\partial w} = \int_x \int_z f(x, z) \cdot u_r(x, z, t) dx dz, \quad (4)$$

The gradient with respect to c is, thus,

$$\frac{\partial E}{\partial c} = \int_0^T u_s(x, z, t) \cdot u_r(x, z, t) dt, \quad (5)$$

where u_s is the source wavefield generated from the current velocity model (Plessix, 2006). In this algorithm, we assume $w_0(t)$ is given by the time reversed method (specifically its time). The observed data is simply back propagated into the initial velocity model and then apply the maximum energy imaging condition. Then extract the wavefield at the maximum energy point, and it will be our initial wavelet $w_0(t)$. But the initial guess of $w(t)$ is certainly far from the true one, especially the ignition time, due to the velocity model misfit. Directly using it in the inversion is not acceptable because a small shift in the ignition time will lead to a critical cycle skipping in the data, which is one of the server problems in full waveform inversion. To avoid this problem, we utilize a convolution objective function (Choi and Alkhalifah, 2011), which is reasonably source independent. In this case, the objective function is given by:

$$E = \sum_i^{nr} \|r_i\|^2 = \sum_i^{nr} \left\| d_i^{pred} * d_{ref}^{obs} - d_i^{obs} * d_{ref}^{pred} \right\|^2, \quad (6)$$

where d_i means the i^{th} trace both in the predicted data and in the observed data and the d_{ref} means a reference trace. The symbol $*$ is the convolution operator. In this new objective function, a new source wavelet is convolved in both terms equally, which means the effect of the delay in the source wavelet $w(t)$ is muted. The gradient of the convolution objective function is calculated by taking the derivative of Equation 6 with respect to the model parameters, $m(x, z) = \{f(x, z), c(x, z)\}$.

The gradient can then be written as:

$$\frac{\partial E}{\partial m} = \sum_i^{nr} \left[\left(\frac{\partial d_i^{pred}}{\partial m} * d_{ref}^{obs} \right) \cdot r_i - \left(d_i^{obs} * \frac{\partial d_{ref}^{pred}}{\partial m} \right) \cdot r_i \right]. \quad (7)$$

Thus, the gradient involves calculating the adjoint wavefields with the adjoint sources:

$$R_i^{(1)} = d_{ref}^{obs} \otimes \left(d_i^{pred} * d_{ref}^{obs} - d_i^{obs} * d_{ref}^{pred} \right), \quad (8)$$

at the i^{th} receiver position, and

$$R_i^{(2)} = d_i^{obs} \otimes \left(d_i^{pred} * d_{ref}^{obs} - d_i^{obs} * d_{ref}^{pred} \right), \quad (9)$$

at the reference trace position, where $\otimes$ is the cross-correlation operator. The two cross-correlated seismograms are back-propagated

at the same time in order to reduce the computational cost (equivalent to FWI). Although the velocity update is independent of the source wavelet, we can still update the source wavelet with Equation 4 in order to have benefits in source imaging. The whole inversion uses a nested approach, i.e. 1. inverting for the source image $f(x, z)$ for several iterations; 2. inverting for the background velocity $c(x, z)$ for several iterations, and 3. updating the source function $w(t)$. Repeating the steps above until all the three unknowns are converged. When people doing migration, the extended image is a powerful tool to check if the migration velocity is correct (Sava and Fomel, 2006). In this research, a similar extended imaging condition is applied. Instead of extend the time axis in the imaging condition, the depth axis Z is extended when calculating for source wavelet $w(t)$. Then Equation 4 can be modified as,

$$\nabla_w E(t, h) = \int_x \int_z f(x, z - h) \cdot u_r(x, z + h, t) dx dz, \quad (10)$$

where h is the offset in z direction. The reason of extending the depth axis is that the earth is mainly varying in the Z direction. This will lead to larger imaging errors in Z direction than that in X direction. So extend the Z axis will have larger difference between the images by correct and incorrect velocity models. For demonstration, a homogeneous medium example is tested to show the extended image for $w(t)$ with correct and incorrect velocity model. We can see that the maximum amplitude

Figure 1: Extended image for $w(t, h)$ with (a) true (2km/s), (b) 10% slower and (c) 10% higher velocity models.

Micro-seismic imaging with source independent FWI

point, which is also the crossing point, is located at $h = 0$ (zero offset trace) when the velocity is correct. But the maximum amplitude point is shifted to $h = \pm 0.25$ when velocity is incorrect. With this modified extended imaging condition, we may check both the velocity model and the source image are correct or not. Angle domain common image gathers (ADCIG) is another tool that can show whether the migration velocity is correct (Sava and Fomel, 2003; Biondi and Tisserant, 2004). If the migration velocity is correct, the reflectors in the ADCIG will be flat and located at the right depth. If the migration velocity is incorrect, the reflectors will be curved upward (lower velocity) or downward (higher velocity). In micro-seismic cases, the unknown sources can be considered as a scatter, which shares the similar concept with the reflectors in migration. So simply picking the common image point at the source location. The angle in the angle gather refers to the open angle between the transmission ray and the vertical direction. If the velocity is good enough, the ADCIG should be flat as well. In the following example, the initial $w(t)$ is obtained by simply back propagating the recorded surface data and extracting the $w(t)$ corresponding to the maximum energy time and location, as mentioned above. The initial $f(x, z)$ is set to be 0 at all points in order to be consistent with the source gradient, which is Equation 3. There are more innovative ways to estimate an initial f and w .

EXAMPLES

A piece of the SEG overthrust model (fault zone) is used to represent what we refer to as the true velocity model, shown in Figure 2, for our tests. The model has 400×180 grid points with a $20m$ spatial interval in both x and z directions. The initial velocity model is linearly increasing with depth, shown in Figure 3. The time interval is $1ms$. Every grid point on the top surface is assumed to be a receiver. Absorbing boundary condition is assumed on all the four boundaries. In the ex-

Figure 2: The true velocity model.

periment, a single source case is assumed. A Ricker's wavelet of $6Hz$ peak frequency is set to be the unknown source function. The source is located at $\{4.0km, 2.2km\}$ in the model. The starting $f_0(x, z)$ is zero everywhere. In another word, no priori information of the source spatial component is given. Using the time reversal method mentioned above, the initial source wavelet $w(t)$ is obtained by reversing the wavefield at

Figure 3: The initial velocity model.

the maximum energy point along time axis, shown as the red line in Figure 6. We can see the onset is later than the true one (blue). With the nested approach, the misfit reached its minimum after repeating the steps for 25 times. The inverted $c(x, z)$ and $f(x, z)$ are shown in Figure 4 and Figure 5. The inverted velocity model shows very low resolution because of the lack of reflection energy in the wavefield. The micro-seismic events are buried in the earth while the receivers are on the earth surface. Most of the energy in the observed data are transmission waves. In FWI, reflection waves update the high wavenumber parts of the model, while transmissions update the low wave number parts. So the gradient for the velocity updates is mostly focused on the smooth background. We may obtain higher resolution for higher frequency events, beyond our $6Hz$ peak frequency Ricker wavelet. Although the inverted velocity model is very smooth, it still shows reasonably good structures of the model, such as the fault zone, which is about $2.5km$ deep. Also the sea bottom is accurate in depth. The artifacts around the source is mainly caused by the single event case. Using multi-sources may mitigate it. The inverted source image in Figure 5 shows an accurate source location at $(4.0km, 2.2km)$. There are some weak energy in the source image that is not focused to the source point. One can either choose to ignore them or set some kind of constrain, for example Laplacian filter, to reduce the influence. If higher frequency wavelet is used, the image should be sharpened in space because the width of the source image depends on the wavelength of the source wavelet. The inverted source

Figure 4: The inverted velocity model.

Micro-seismic imaging with source independent FWI

Figure 5: The inverted source image (spatial component).

wavelet is shown as the pink line in Figure 6. The true one (blue) and the initial one (red) are also shown in the figure as comparison. We can easily find that the ignition time of the initial source wavelet is not accurate, which is shifted backward to 0.35s while the true ignition time is 0.2s. After the inversion, the inverted source wavelet (pink) is ignited at the exactly correct time as the true one. Although there are some small difference in shapes between the inverted one and the true one, it is still good enough for the calculation of $f(x, z)$ and $c(x, z)$. The difference may be caused by the remaining misfit of the model parameter m . Figure 7 shows the vertical extended image for $w(t)$ after the inversion. The maximum amplitude point is at the right place ($h = 0$ and $t = 0.2s$), which also shows the effectiveness of the approach.

To show how good the inverted velocity model is, the ADCIG at the source position is extracted, as mentioned in the previous section. Figure 8a and Figure 8b show the ADCIG of the initial velocity model and the inverted velocity model, respectively. In Figure 8a the scatters are highly curved upward. It means the initial velocity is much slower than the true one. On the other hand, we may easily say that the image in Figure 8b is generally flat and the depth is correctly located at 2.2km, which means the inverted model is much more accurate.

Figure 6: Comparison between source functions, red: initial, blue: true and pink: inverted.

Figure 7: Extended image of $w(t, h)$.

Figure 8: ADCIG at the source position of the (a) initial and (b) inverted velocity model.

CONCLUSIONS

We develop an approach to invert for the source location image, the source wavelet and the velocity model using recorded microseismic events setup. To mitigate the role of the source wavelet, which is usually inaccurate in shape or time, we utilize a source independent objective function based on the convolution of our recorded traces with a reference trace. The inversion for all three unknowns yielded convergent results on a model inspired by the SEG overthrust model, in spite starting with an initial linearly increasing velocity model. The extended image method is used to check the accuracy of the velocity model and the source image. The ADCIG also showed the effectiveness of our approach.

ACKNOWLEDGEMENTS

We thank KAUST for sponsoring this research. We also thank to the team of SWAG for their help during the research.

EDITED REFERENCES

Note: This reference list is a copyedited version of the reference list submitted by the author. Reference lists for the 2016 SEG Technical Program Expanded Abstracts have been copyedited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

REFERENCES

- Artman, B., I. Podladtchikov, and B. Witten, 2010, Source location using time-reverse imaging: *Geophysical Prospecting*, **58**, 861–873, <http://dx.doi.org/10.1111/j.1365-2478.2010.00911.x>.
- Biondi, B., and T. Tisserant, 2004, 3D angle-domain common-image gathers for migration velocity analysis: *Geophysical Prospecting*, **52**, 575–591, <http://dx.doi.org/10.1111/j.1365-2478.2004.00444.x>.
- Choi, Y., and T. Alkhalifah, 2011, Source-independent time-domain waveform inversion using convolved wavefields: Application to the encoded multisource waveform inversion: *Geophysics*, **76**, no. 5, R125–R134, <http://dx.doi.org/10.1190/geo2010-0210.1>.
- Eisner, L., P. M. Duncan, W. M. Heigl, and W. R. Keller, 2009, Uncertainties in passive seismic monitoring: *The Leading Edge*, **28**, 648–655, <http://dx.doi.org/10.1190/1.3148403>.
- Kaderli, J., M. D. McChesney, and S. E. Minkoff, 2015, Microseismic event estimation in noisy data via full waveform inversion: 85th Annual International Meeting, SEG, Expanded Abstracts, 1159–1164, <http://dx.doi.org/10.1190/segam2015-5867154.1>.
- Plessix, R.-E., 2006, A review of the adjoint-state method for computing the gradient of a functional with geophysical applications: *Geophysical Journal International*, **167**, 495–503, <http://dx.doi.org/10.1111/j.1365-246X.2006.02978.x>.
- Sava, P., 2011, Micro-earthquake monitoring with sparsely sampled data: *Journal of Petroleum Exploration and Production Technology*, **1**, 43–49.
- Sava, P., and S. Fomel, 2006, Time-shift imaging condition in seismic migration: *Geophysics*, **71**, no. 6, S209–S217, <http://dx.doi.org/10.1190/1.2338824>.
- Sava, P. C., and S. Fomel, 2003, Angle-domain common-image gathers by wavefield continuation methods: *Geophysics*, **68**, 1065–1074, <http://dx.doi.org/10.1190/1.1581078>.
- Waldhauser, F., and W. L. Ellsworth, 2000, A double-difference earthquake location algorithm: Method and application to northern Hayward fault, California: *Bulletin of the Seismological Society of America*, **90**, 1353–1368, <http://dx.doi.org/10.1785/0120000006>.