A nonlinear approach of elastic reflection waveform inversion

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SUMMARY

Elastic full waveform inversion (EFWI) embodies the original intention of waveform inversion at its inception as it is a better representation of the mostly solid Earth. However, compared with the acoustic P-wave assumption, EFWI for P- and S-wave velocities using multi-component data admitted mixed results. Full waveform inversion (FWI) is a highly nonlinear problem and this nonlinearity only increases under the elastic assumption. Reflection waveform inversion (RWI) can mitigate the nonlinearity by relying on transmissions from reflections focused on inverting low wavenumber components of the model. In our elastic endeavor, we split the P- and S-wave velocities into low wavenumber and perturbation components and propose a nonlinear approach to invert for both of them. The new optimization problem is built on an objective function that depends on both background and perturbation models. We utilize an equivalent stress source based on the model perturbation to generate reflection instead of demigrating from an image, which is applied in conventional RWI. Application on a slice of an ocean-bottom data shows that our method can efficiently update the low wavenumber parts of the model, but more so, obtain perturbations that can be added to the low wavenumbers for a high resolution output.

INTRODUCTION

Acoustic wavefield inversion of P-waves has been the focus of research and inversion implementations over the past few years (Tarantola, 1986; Xu et al., 2012; Alkhalifah and Wu, 2015). With the rise of full waveform inversion (FWI) and its possible use in delineating the reservoir, the elastic description of the Earth is drawing a lot of attention. The conception of elastic FWI (EFWI) by Tarantola (1986) allowed its later application to real data problems by Mora (1987) and Crase et al. (1990) with mixed results, as the approach was still under development (Wang et al., 2015). FWI is highly nonlinear because of the complexity of the Earth reflectivity and the nonlinearity only increases in the case of inverting multi-component data for P- and S-wave velocities. In the absence of a good initial guess of the background model, the optimization problem using gradient methods is not capable of providing low wavenumber updates. Thus, gradient based inversion methods tend to converge to local minima (Tarantola, 1986; Alkhalifah and Wu, 2015). Xu et al. (2012), based on the work of Plessix et al. (1995), suggested using the recorded reflections to predict the P-wave reflectivity using a migration/demigration process, referred to as RWI. In this case, the objective is to find the long wavelength components of the velocity model necessary to image such reflections. This approach is a good stepping stone to FWI, as FWI needs a reasonable (depending on the minimum frequency) description of the kinematics of the wavefield embedded in the initial velocity model. RWI can be implemented either in time or frequency domain with differ-

ent workflows (Wang et al., 2013; Wu and Alkhalifah, 2014), but one thing is in common in the two implementations that the demigrated reflections are highly sensitive to the image. In order to achieve a better fit of the data, the image requires a least square type of optimization. Wu and Alkhalifah (2015), thus, proposed a new optimization problem, where they invert for both the background model and perturbations. For EFWI, which is a multi-parameter inversion problem, an accurate description of the kinematic component of the P- and S-wavefields is needed to mitigate the high nonlinearity. Considering the higher wavenumbers of the S-wave model, optimizing low wavenumber components for the S-wave velocity is even more crucial in avoiding EFWI converging to local minima. The task of finding this long wavelength S-wave velocity will be left to our newly proposed RWI.

In this abstract, we split the P- and S-wave velocities into background and perturbation (reflectivity) parts and solve the nonlinear inversion problem for both of them. First we invert for the perturbation components. To generate scattered P- and S-wavefields, we introduce an equivalent stress source based on the inverted perturbation. The scatterers are no longer regarded as a single scatterer, thus no Born approximation limitation is assumed in the perturbed model. The final inverted model is composed of the optimized perturbation and the background. Compared with regular EFWI, those results show even higher resolution.

THEORY

In conventional FWI, we solve the optimization problem by minimizing the L-2 norm distance between modeled data and recorded data

$$\min E(\mathbf{m}) = \sum_r \frac{1}{2} \int_t |\mathbf{u}(\mathbf{x}_r, t) - \mathbf{d}(\mathbf{x}_r, t)|^2, \quad (1)$$

where $\mathbf{d}(\mathbf{x}_r, t)$ is the multi-component data observed at receiver location $\mathbf{x}_r$, $\mathbf{u}$ is the modeled wavefield. Thus, we search for the model, $\mathbf{m}$, that can represent the elastic moduli or any other parameterization of the elastic medium, which minimizes E . If we don't concern ourselves with density (does not effect the kinematics), setting it constant, the gradient related to the elastic parameters, λ and μ , in isotropic media is given by Crase et al. (1990):

$$\delta\lambda = - \sum_s \int_t dt \left(\frac{\partial}{\partial x} u_x + \frac{\partial}{\partial z} u_z \right) \left(\frac{\partial}{\partial x} \Psi_x + \frac{\partial}{\partial z} \Psi_z \right), \quad (2)$$

$$\begin{aligned} \delta\mu = & - \sum_s \int_t dt 2 \left(\frac{\partial}{\partial x} u_x \frac{\partial}{\partial x} \Psi_x + \frac{\partial}{\partial z} u_z \frac{\partial}{\partial z} \Psi_z \right) \\ & + \left(\frac{\partial}{\partial z} u_x + \frac{\partial}{\partial x} u_z \right) \left(\frac{\partial}{\partial z} \Psi_x + \frac{\partial}{\partial x} \Psi_z \right), \end{aligned} \quad (3)$$

where u_z and u_x are the modeled z and x displacement wavefields, respectively. Also, Ψ_z and Ψ_x are the z and x compo-

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nents of the adjoint wavefield. Based on the concept of reflection waveform inversion (RWI), the model is separated into low and high wavenumber components, and thus we can focus the optimization on the low wavenumber part, the background. We split the model, the square of P -wave velocity α and S -wave wave velocity β , into a background (α_0, β_0) and perturbation $(\delta\alpha, \delta\beta)$. The displacement field is accordingly split into a component propagating in the background model $\mathbf{u}$ and those resulting from the perturbation $\delta\mathbf{u}$. According to Hooke's law that relates stress and strain, we define the background and perturbation of the stress components as

$$\begin{aligned}\tau_{xx} + \delta\tau_{xx} &= (\alpha_0 + \delta\alpha) \left(\frac{\partial u_x}{\partial x} + \frac{\partial \delta u_x}{\partial x} \right) \\ &\quad + (\alpha_0 + \delta\alpha - 2\beta_0 - 2\delta\beta) \left(\frac{\partial u_z}{\partial z} + \frac{\partial \delta u_z}{\partial z} \right), \\ \tau_{zz} + \delta\tau_{zz} &= (\alpha_0 + \delta\alpha - 2\beta_0 - 2\delta\beta) \left(\frac{\partial u_x}{\partial x} + \frac{\partial \delta u_x}{\partial x} \right) \\ &\quad + (\alpha_0 + \delta\alpha) \left(\frac{\partial u_z}{\partial z} + \frac{\partial \delta u_z}{\partial z} \right), \\ \tau_{xz} + \delta\tau_{xz} &= (\beta_0 + \delta\beta) \left(\frac{\partial (u_x + \delta u_x)}{\partial z} + \frac{\partial (u_z + \delta u_z)}{\partial x} \right).\end{aligned}\quad (4)$$

The background wavefield and model (α_0, β_0) satisfy the elastic wave equation, which can be substituted in the above equation, resulting in

$$\begin{aligned}\delta\tau_{xx} &= (\alpha_0 + \delta\alpha) \frac{\partial \delta u_x}{\partial x} + (\alpha_0 + \delta\alpha - 2\beta_0 - 2\delta\beta) \frac{\partial \delta u_z}{\partial z} + \tilde{\tau}_{xx}, \\ \delta\tau_{zz} &= (\alpha_0 + \delta\alpha - 2\beta_0 - 2\delta\beta) \frac{\partial \delta u_x}{\partial x} + (\alpha_0 + \delta\alpha) \frac{\partial \delta u_z}{\partial z} + \tilde{\tau}_{zz}, \\ \delta\tau_{xz} &= (\beta_0 + \delta\beta) \left(\frac{\partial \delta u_x}{\partial z} + \frac{\partial \delta u_z}{\partial x} \right) + \tilde{\tau}_{xz}\end{aligned}\quad (5)$$

where $\tilde{\tau}_{ij}$ is regarded as the equivalent stress source term, similar to the equivalent force derived by Wu and Zheng (2014) in the acoustic case. It is, thus, given by $\tilde{\tau}_{xx} = \delta\alpha \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} \right) - 2\delta\beta \frac{\partial u_z}{\partial z}$, $\tilde{\tau}_{zz} = \delta\alpha \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} \right) - 2\delta\beta \frac{\partial u_x}{\partial x}$, $\tilde{\tau}_{xz} = \delta\beta \left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} \right)$. In conventional RWI, reflections in the modeled data are produced by a demigration process. Thus, the data residuals in this case measure mostly the focusing ability of the velocity model, which is needed to guarantee a correct phase of the reflection. To reproduce the accurate amplitude, the inverted perturbation (migration) must fit the data amplitude as well. Thus, we invert for the perturbations and background in a nested approach instead of only inverting for background and following up with EFWI. In fact, our objective function is specifically given by

$$\min E_{RWI}(\mathbf{m}) = \sum_r \frac{1}{2} \int_t |\mathbf{u}(\mathbf{x}_r, t) + \delta\mathbf{u}(\mathbf{x}_r, t) - \mathbf{d}(\mathbf{x}_r, t)|^2, \quad (6)$$

In our applications, we consider using an explosive source, which mainly produces pressure waves. The multi-component acquisition, is thus, dominated by P -waves and at larger offsets includes converted P - to S -waves. We can apply our equivalent stress source using the inverted velocity perturbations and the incident displacement field. After obtaining the scattered P - and S -wavefields, we calculate the gradient of the objective function in equation (6) with respect to (α_0, β_0) and $(\delta\alpha, \delta\beta)$

as follows:

$$\begin{aligned}\nabla E_{\alpha_0} &= - \sum_s \int_t dt \left(\frac{\partial}{\partial x} u_x + \frac{\partial}{\partial z} u_z \right) \left(\frac{\partial}{\partial x} \Psi_x + \frac{\partial}{\partial z} \Psi_z \right) \\ &\quad + \left(\frac{\partial}{\partial x} u_x + \frac{\partial}{\partial z} u_z \right) \left(\frac{\partial}{\partial x} \delta \Psi_x + \frac{\partial}{\partial z} \delta \Psi_z \right) \\ &\quad + \left(\frac{\partial}{\partial x} \delta u_x + \frac{\partial}{\partial z} \delta u_z \right) \left(\frac{\partial}{\partial x} \tilde{\Psi}_x + \frac{\partial}{\partial z} \tilde{\Psi}_z \right), \\ \nabla E_{\delta\alpha} &= - \sum_s \int_t dt \left(\frac{\partial}{\partial x} u_x + \frac{\partial}{\partial x} \delta u_x + \frac{\partial}{\partial z} u_z + \frac{\partial}{\partial z} \delta u_z \right) \\ &\quad \left(\frac{\partial}{\partial x} \tilde{\Psi}_x + \frac{\partial}{\partial z} \tilde{\Psi}_z \right), \\ \nabla E_{\beta_0} &= - \sum_s \int_t dt 2 \left(\frac{\partial}{\partial x} u_x \frac{\partial}{\partial x} \Psi_x + \frac{\partial}{\partial z} u_z \frac{\partial}{\partial z} \Psi_z \right) \\ &\quad + \left(\frac{\partial}{\partial z} u_x + \frac{\partial}{\partial x} u_z \right) \left(\frac{\partial}{\partial z} \Psi_x + \frac{\partial}{\partial x} \delta \Psi_x + \frac{\partial}{\partial x} \Psi_z + \frac{\partial}{\partial x} \delta \Psi_z \right) \\ &\quad + 2 \left(\frac{\partial}{\partial x} u_x \frac{\partial}{\partial x} \delta \Psi_x + \frac{\partial}{\partial z} u_z \frac{\partial}{\partial z} \delta \Psi_z \right) \\ &\quad + 2 \left(\frac{\partial}{\partial x} \delta u_x \frac{\partial}{\partial x} \tilde{\Psi}_x + \frac{\partial}{\partial z} \delta u_z \frac{\partial}{\partial z} \tilde{\Psi}_z \right) \\ &\quad + \left(\frac{\partial}{\partial z} \delta u_x + \frac{\partial}{\partial x} \delta u_z \right) \left(\frac{\partial}{\partial z} \tilde{\Psi}_x + \frac{\partial}{\partial x} \tilde{\Psi}_z \right), \\ \nabla E_{\delta\beta} &= - \sum_s \int_t dt 2 \left(\frac{\partial}{\partial x} u_x \frac{\partial}{\partial x} \tilde{\Psi}_x + \frac{\partial}{\partial x} \delta u_x \frac{\partial}{\partial x} \tilde{\Psi}_x \right) \\ &\quad + 2 \left(\frac{\partial}{\partial z} u_z \frac{\partial}{\partial z} \tilde{\Psi}_z + \frac{\partial}{\partial z} \delta u_z \frac{\partial}{\partial z} \tilde{\Psi}_z \right) \\ &\quad + \left(\frac{\partial}{\partial z} u_x + \frac{\partial}{\partial x} \delta u_x + \frac{\partial}{\partial x} u_z + \frac{\partial}{\partial x} \delta u_z \right) \left(\frac{\partial}{\partial z} \tilde{\Psi}_x + \frac{\partial}{\partial x} \tilde{\Psi}_z \right),\end{aligned}\quad (7)$$

where Ψ is the adjoint displacement field propagated in background model, $\tilde{\Psi}$ is the adjoint wavefield that satisfies the elastic wave equation with assumed true model $(\alpha_0 + \delta\alpha, \beta_0 + \delta\beta)$, $\delta\Psi$ is the field produced by perturbing $\tilde{\Psi}$ at the inverted high wavenumber model components. Here, the diving waves are generated and included in the inversion.

APPLICATION TO REAL DATA

We apply our approach on a 2D slice with a two-component dataset extracted from a real 3D marine dataset of an OBS survey. In the 3D OBS dataset, 12 cables were used, with 400m separation. Each cable contained 240 geophones, separated by 25m, so the total cable length is 6km. The survey is located in the Volve field, a Middle Jurassic oil field in the southern part of Viking Graden in the gas/condensate rich Sleipner area. OBS was chosen over marine streamer acquisition to take advantage of expected improved water-layer multiple attenuation by combining geophone and hydrophone measurements, and for the superior illumination properties given by its wider azimuth range (Szydluk et al., 2007). The 2D dataset is extracted in the inline direction from the middle of 3D dataset. The chosen 33 sources are evenly distributed within the receiver spread (at total 6km). We apply power gain to the data to match the energy of 2D modeling, and a 3-10Hz Butterworth filter to the frequency band. The source wavelet we use is a regularized early reflection. We start the inversion with linearly increasing initial models to test our method, which are shown in Figure 1.

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Figure 1: (a) The initial P -wave velocity. (b) The initial S -wave velocity.

The initial models include a water layer, in which the P wave and S -wave velocities are set to be 1.49 km/s and 0 km/s, respectively. The model size is 6 km in the horizontal direction and 3.75 km in depth. For inverting the perturbation parts of the model, we use the L2-norm misfit function on near offset data. The inverted perturbation $\delta\alpha$ and $\delta\beta$ are supposed to image reflections, which can match those events in the recorded data. For background updates, we utilize the global correlation objective function (Choi and Alkhalifah, 2012) with far offset data. The estimated background of P - and S -wave velocities using our nested approach are shown in Figures 2a and 3a, respectively. The inversion tends to admit reasonable long wavelength inverted models. Adding the perturbation to the inverted background, we obtain the models shown in Figures 2b and 3b. For comparison, we apply regular EFWI starting from the linearly increasing velocities in Figure 1 and obtain the models shown in Figures 2c and 3c. We note that the models obtained using our method include a better estimation of the low wavenumber components (the background), especially for the S -wave velocity, meanwhile they end up with even higher resolution. However, because the linearly increasing model is usually far away from the real solution, we utilize the model given by the data provider and smooth it five times with a window size of 500m by 500m, resulting in the models shown in Figures 4a and 4b. Figures 5a and 5b show the inverted model $\alpha_0 + \delta\alpha$ and $\beta_0 + \delta\beta$. The optimized perturbation parts of P and S -waves are shown in Figure 6. The inverted P and S -wave velocity models have consistent flat layers at the shallow area and generally similar structure at the deeper parts. Both of them indicate some unconformity between 2.3 km and 2.6 km depth. In the inverted P -wave velocity we can see indications of a low velocity zone at 2.7 km depth.

CONCLUSIONS

In our elastic RWI, we implement a nonlinear optimization for low and high wavenumber model components of the P - and S -wave velocities in a nested approach. The low wavenumber components are attainable from the modeled reflections and the diving waves. Compared with regular RWI, admitting low wavenumber models good enough for a subsequent EFWI, we obtain high-resolution results (cheaper) by adding the inverted perturbation to the background. As we demonstrated with marine data, it enables us to obtain high resolution models.

Figure 2: The inverted background P -wave velocity (a) with the inverted perturbation added to it (b), and (c) the P -wave velocity after EFWI.

ACKNOWLEDGMENTS

The authors would like to thank Statoil ASA and the Volve license partners ExxonMobil E&P Norway AS and Bayerngas Norge AS, for the release of the Volve data. The authors would like to thank Marianne Houbiers from Statoil, who gave some very helpful suggestions and corrections. We also thank KAUST for its support and we thank the SWAG group for collaborative environment.

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Figure 3: The inverted background S -wave velocity (a) with the inverted perturbation added to it (b), and (c) the S -wave velocity after EFWI.

Figure 4: (a) The initial P -wave velocity and (b) the initial S -wave velocity.

Figure 5: (a) The inverted P -wave velocity with perturbation added to it; (b) the inverted S -wave velocity with perturbation added to it.

Figure 6: (a) The inverted perturbation of P -wave velocity and (b) The inverted perturbation of S -wave velocity.

EDITED REFERENCES

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