Simulating propagation of decomposed elastic waves using low-rank approximate mixed-domain integral operators for heterogeneous transversely isotropic media

Jiubing Cheng, Tongji University, State Key Laboratory of Marine Geology, Zedong Wu and Tariq Alkhalifah, King Abdullah University of Science and Technology

SUMMARY

In elastic imaging, the extrapolated vector fields are decomposed into pure wave modes, such that the imaging condition produces interpretable images, which characterize reflectivity of different reflection types. Conventionally, wavefield decomposition in anisotropic media is costly as the operators involved is dependent on the velocity, and thus not stationary. In this abstract, we propose an efficient approach to directly extrapolate the decomposed elastic waves using low-rank approximate mixed space/wavenumber domain integral operators for heterogeneous transverse isotropic (TI) media. The low-rank approximation is, thus, applied to the pseudo-spectral extrapolation and decomposition at the same time. The pseudo-spectral implementation also allows for relatively large time steps in which the low-rank approximation is applied. Synthetic examples show that it can yield dispersion-free extrapolation of the decomposed quasi-P (qP) and quasi-SV (qSV) modes, which can be used for imaging, as well as the total elastic wavefields.

INTRODUCTION

Multicomponent seismic data are increasingly acquired on land and at the ocean bottom in an attempt to better understand the geological structure and characterize oil and gas reservoirs. Seismic modeling, reverse-time migration (RTM), and full-waveform inversion (FWI) in areas with complex geology all require high-accuracy numerical algorithms for time extrapolation of waves. Because seismic waves propagate through the earth as a superposition of P- and S-wave modes, an elastic wave equation is usually more accurate for wavefield extrapolation than an acoustic wave equation. To produce physically interpretable images, which characterize reflectivity of different reflection types, mode decoupling or separation before the application of the imaging condition is required to avoid artifacts from crosstalk among the wave modes (Yan and Sava, 2008).

For isotropic media, far-field P- and S-wave modes can be separated by taking the divergence and curl in the extrapolated elastic wavefields (Aki and Richards, 1980; Sun and McMechan, 2001). Alternatively, Ma and Zhu (2003) extrapolated vector P and S modes separately in an elastic wavefield by decomposing the wave equation into P- and S-wave components. In the mean time, decoupling of wave modes yields familiar scalar wave equations for P and S modes (Aki and Richards, 1980; Du et al., 2014). They can be numerically solved using conventional explicit finite-difference (FD) or pseudo-spectral methods for time extrapolation (Carcione et al., 2002). Various alternative approaches have been developed in the geophysi-

cal literature to achieve stable, dispersion-free extrapolation of scalar waves in heterogeneous isotropic media for large time steps, just to list a few (Tal-Ezer, 1986; Soubaras and Zhang, 2008; Etgen and Brandsberg-Dahl, 2009; Pestana and Stoffa, 2010; Fowler et al., 2010; Fomel et al., 2010, 2013; Song and Fomel, 2011; Wu and Alkhalifah, 2014).

In anisotropic media, one cannot, so simply, derive explicit single-mode time-space-domain differential wave equations. Generally, P and S modes do not respectively polarize parallel and perpendicular to the wave vectors. Dellinger and Etgen (1990) generalized the divergence and curl operations to anisotropic media by constructing separators as polarization projection in the wavenumber-domain. To tackle heterogeneity, these mode separators were rewritten by Yan and Sava (2009) as nonstationary spatial filters determined by the local polarization vectors. Zhang and McMechan (2010) proposed a wavefield decomposition method to separate elastic wavefields into vector P- and S-wave fields for vertically transverse isotropic (VTI) media. Accordingly, Cheng and Fomel (2013, 2014) proposed fast algorithms of mixed space/wavenumber-domain mode separation and vector decomposition by applying low-rank approximation to the involved Fourier integral operators (FIOs) of the general form. These fast algorithms have been used in elastic RTM to generate high-quality angle-domain image gathers for TI media (Wang et al., 2014a).

Anisotropic wave propagation can be formally decoupled in the wavenumber-domain to yield single-mode pseudo-differential equations. Several authors have used this scheme to derive mixed-domain time extrapolation methods for P- or S-wave in anisotropic media (Liu et al., 2009; Chu et al., 2011; Fowler and Lapilli, 2012; Fowler and King, 2011; Fowler and Lapilli, 2012; Song and Alkhalifah, 2013). However, all these efforts are restricted to pure-mode scalar waves and do not honor the elastic effects such as mode conversion.

In this abstract, we first present a mixed-domain operator to achieve dispersion-free extrapolation of elastic waves for heterogeneous anisotropic media. Then we review the approaches to decompose a 2D elastic wavefield into its qP and qSV components for TI media. Accordingly, we propose an efficient implementation of simultaneous extrapolation and decomposition of the elastic wavefields using low-rank approximate mixed-domain integral algorithm. At the end, synthetic examples on a two-layer TI model are shown to demonstrate the validity of the proposed method.

Propagation of decomposed elastic waves

MIXED-DOMAIN EXTRAPOLATION OF ELASTIC WAVES homogeneous VTI medium using:

We start from the 2nd-order elastic wave equation in a 2D VTI medium,

$$\begin{aligned}\rho \frac{\partial^2 u_x}{\partial t^2} &= C_{11} \frac{\partial^2 u_x}{\partial x^2} + C_{44} \frac{\partial^2 u_x}{\partial z^2} + (C_{13} + C_{44}) \frac{\partial^2 u_z}{\partial x \partial z}, \\ \rho \frac{\partial^2 u_z}{\partial t^2} &= C_{44} \frac{\partial^2 u_z}{\partial x^2} + C_{33} \frac{\partial^2 u_z}{\partial z^2} + (C_{13} + C_{44}) \frac{\partial^2 u_x}{\partial x \partial z},\end{aligned}\quad (1)$$

where u_x and u_z represent the x - and z -components of the displacement wavefields, and C_{11} , C_{33} , C_{44} , and C_{13} are the stiffness coefficients. For constant density homogeneous media, applying the 2nd-order time-marching scheme leads to the pseudo-spectral formula,

$$\begin{aligned}\tilde{u}_x(\mathbf{k}, t + \Delta t) &= w_x(\mathbf{k}) \tilde{u}_x(\mathbf{k}, t) + w_{xz}(\mathbf{k}) \tilde{u}_z(\mathbf{k}, t) - \tilde{u}_x(\mathbf{k}, t - \Delta t), \\ \tilde{u}_z(\mathbf{k}, t + \Delta t) &= w_z(\mathbf{k}) \tilde{u}_z(\mathbf{k}, t) + w_{xz}(\mathbf{k}) \tilde{u}_x(\mathbf{k}, t) - \tilde{u}_z(\mathbf{k}, t - \Delta t),\end{aligned}\quad (2)$$

in which Δt is the time step, $\mathbf{k} = (k_x, k_z)$ is the wavenumber vector, and

$$\begin{aligned}w_x(\mathbf{k}) &= 2 - \Delta t^2 [C_{11} k_x^2 + C_{44} k_z^2], \\ w_z(\mathbf{k}) &= 2 - \Delta t^2 [C_{44} k_x^2 + C_{33} k_z^2], \\ w_{xz}(\mathbf{k}) &= -\Delta t^2 [C_{13} + C_{44}] k_x k_z.\end{aligned}\quad (3)$$

To tackle heterogeneity, we extend equation 2 as mixed-domain integral operators:

$$\begin{aligned}\tilde{u}_x(\mathbf{x}, t + \Delta t) &= -\tilde{u}_x(\mathbf{x}, t - \Delta t) \\ &+ \int e^{i\mathbf{k}\mathbf{x}} w_x(\mathbf{x}, \mathbf{k}) \tilde{u}_x(\mathbf{k}) d\mathbf{k} + \int e^{i\mathbf{k}\mathbf{x}} w_{xz}(\mathbf{x}, \mathbf{k}) \tilde{u}_z(\mathbf{k}) d\mathbf{k}, \\ \tilde{u}_z(\mathbf{x}, t + \Delta t) &= -\tilde{u}_z(\mathbf{x}, t - \Delta t) \\ &+ \int e^{i\mathbf{k}\mathbf{x}} w_z(\mathbf{x}, \mathbf{k}) \tilde{u}_z(\mathbf{k}) d\mathbf{k} + \int e^{i\mathbf{k}\mathbf{x}} w_{xz}(\mathbf{x}, \mathbf{k}) \tilde{u}_x(\mathbf{k}) d\mathbf{k},\end{aligned}\quad (4)$$

in which $\mathbf{x} = (x, z)$, and the mixed-domain matrices satisfy,

$$\begin{aligned}w_x(\mathbf{x}, \mathbf{k}) &= 2 - \Delta t^2 [C_{11}(\mathbf{x}) k_x^2 + C_{44}(\mathbf{x}) k_z^2], \\ w_z(\mathbf{x}, \mathbf{k}) &= 2 - \Delta t^2 [C_{44}(\mathbf{x}) k_x^2 + C_{33}(\mathbf{x}) k_z^2], \\ w_{xz}(\mathbf{x}, \mathbf{k}) &= -\Delta t^2 [C_{13}(\mathbf{x}) + C_{44}(\mathbf{x}) k_x k_z].\end{aligned}\quad (5)$$

Note that the 2D elastic wavefields are coupled with qP- and qSV-wave modes.

This method can be easily extended to 3D and other anisotropic media. In the case of TI media with a tilted symmetry axis, the elastic tensor loses its simple form. We may derive the corresponding mixed-domain integral operators by locally rotating the coordinate system so that its z -axis coincides with the symmetry axis of the media. More sophisticated but complicated strategy should be used to enhance stability in variable-tilt TI media (Zhang et al., 2011; Bube et al., 2012) and orthorhombic media (Lapilli and Fowler, 2013).

In addition, a k -space adjustment to the spatial derivatives may convert the time-stepping pseudo-spectral model into an exact model for homogeneous media, and stable for larger time steps (for a given level of accuracy) in heterogeneous media (Liu, 1995; Firouzi et al., 2012; Fang et al., 2013; Wang et al., 2014b).

SIMULTANEOUSLY EXTRAPOLATE AND DECOMPOSE ELASTIC WAVEFIELDS

According to Zhang and McMechan (2010), one can decompose qP and qSV modes from the 2D elastic wavefields in a

$$\begin{aligned}u_x^{(m)}(\mathbf{k}) &= b_x^{(m)}(\mathbf{k}) \tilde{u}_x(\mathbf{k}) + b_{xz}^{(m)}(\mathbf{k}) \tilde{u}_z(\mathbf{k}), \\ u_z^{(m)}(\mathbf{k}) &= b_{xz}^{(m)}(\mathbf{k}) \tilde{u}_x(\mathbf{k}) + b_z^{(m)}(\mathbf{k}) \tilde{u}_z(\mathbf{k}),\end{aligned}\quad (6)$$

where $m = \{qP, qSV\}$, and

$$\begin{aligned}b_x^{(m)}(\mathbf{k}) &= a_x^{(m)}(\mathbf{k}) a_x^{(m)}(\mathbf{k}), \\ b_z^{(m)}(\mathbf{k}) &= a_z^{(m)}(\mathbf{k}) a_z^{(m)}(\mathbf{k}), \\ b_{xz}^{(m)}(\mathbf{k}) &= a_x^{(m)}(\mathbf{k}) a_z^{(m)}(\mathbf{k}),\end{aligned}\quad (7)$$

in which $a_x^{(m)}(\mathbf{k})$ and $a_z^{(m)}(\mathbf{k})$ represent the x - and z -components of the normalized polarization vector of a given mode.

Following Cheng and Fomel (2013, 2014), one can decompose the wave modes in a heterogeneous TI medium using the following mixed-domain integral operations:

$$\begin{aligned}u_x^{(m)}(\mathbf{x}) &= \int e^{i\mathbf{k}\mathbf{x}} b_x^{(m)}(\mathbf{x}, \mathbf{k}) \tilde{u}_x(\mathbf{k}) d\mathbf{k} + \int e^{i\mathbf{k}\mathbf{x}} b_{xz}^{(m)}(\mathbf{x}, \mathbf{k}) \tilde{u}_z(\mathbf{k}) d\mathbf{k}, \\ u_z^{(m)}(\mathbf{x}) &= \int e^{i\mathbf{k}\mathbf{x}} b_{xz}^{(m)}(\mathbf{x}, \mathbf{k}) \tilde{u}_x(\mathbf{k}) d\mathbf{k} + \int e^{i\mathbf{k}\mathbf{x}} b_z^{(m)}(\mathbf{x}, \mathbf{k}) \tilde{u}_z(\mathbf{k}) d\mathbf{k},\end{aligned}\quad (8)$$

with

$$\begin{aligned}b_x^{(m)}(\mathbf{x}, \mathbf{k}) &= a_x^{(m)}(\mathbf{x}, \mathbf{k}) a_x^{(m)}(\mathbf{x}, \mathbf{k}), \\ b_z^{(m)}(\mathbf{x}, \mathbf{k}) &= a_z^{(m)}(\mathbf{x}, \mathbf{k}) a_z^{(m)}(\mathbf{x}, \mathbf{k}), \\ b_{xz}^{(m)}(\mathbf{x}, \mathbf{k}) &= a_x^{(m)}(\mathbf{x}, \mathbf{k}) a_z^{(m)}(\mathbf{x}, \mathbf{k}),\end{aligned}\quad (9)$$

where $a_x^{(m)}(\mathbf{x}, \mathbf{k})$ and $a_z^{(m)}(\mathbf{x}, \mathbf{k})$, respectively, represent the x - and z -components of the normalized polarization vector of the given mode at the location of $\mathbf{x}$.

Inserting equation 2 into equation 8, we obtain mixed-domain integral operators to simultaneously extrapolate and decompose the vector wavefields for qP-wave mode,

$$\begin{aligned}u_x^{(qP)}(\mathbf{x}, t + \Delta t) &= -u_x^{(qP)}(\mathbf{x}, t - \Delta t) \\ &+ \int e^{i\mathbf{k}\mathbf{x}} o_{x1}^{(qP)}(\mathbf{x}, \mathbf{k}) \tilde{u}_x(\mathbf{k}, t) d\mathbf{k} + \int e^{i\mathbf{k}\mathbf{x}} o_{x2}^{(qP)}(\mathbf{x}, \mathbf{k}) \tilde{u}_z(\mathbf{k}, t) d\mathbf{k}, \\ u_z^{(qP)}(\mathbf{x}, t + \Delta t) &= -u_z^{(qP)}(\mathbf{x}, t - \Delta t) \\ &+ \int e^{i\mathbf{k}\mathbf{x}} o_{z1}^{(qP)}(\mathbf{x}, \mathbf{k}) \tilde{u}_x(\mathbf{k}, t) d\mathbf{k} + \int e^{i\mathbf{k}\mathbf{x}} o_{z2}^{(qP)}(\mathbf{x}, \mathbf{k}) \tilde{u}_z(\mathbf{k}, t) d\mathbf{k},\end{aligned}\quad (10)$$

and for qSV-wave mode,

$$\begin{aligned}u_x^{(qSV)}(\mathbf{x}, t + \Delta t) &= -u_x^{(qSV)}(\mathbf{x}, t - \Delta t) \\ &+ \int e^{i\mathbf{k}\mathbf{x}} o_{x1}^{(qSV)}(\mathbf{x}, \mathbf{k}) \tilde{u}_x(\mathbf{k}, t) d\mathbf{k} + \int e^{i\mathbf{k}\mathbf{x}} o_{x2}^{(qSV)}(\mathbf{x}, \mathbf{k}) \tilde{u}_z(\mathbf{k}, t) d\mathbf{k}, \\ u_z^{(qSV)}(\mathbf{x}, t + \Delta t) &= -u_z^{(qSV)}(\mathbf{x}, t - \Delta t) \\ &+ \int e^{i\mathbf{k}\mathbf{x}} o_{z1}^{(qSV)}(\mathbf{x}, \mathbf{k}) \tilde{u}_x(\mathbf{k}, t) d\mathbf{k} + \int e^{i\mathbf{k}\mathbf{x}} o_{z2}^{(qSV)}(\mathbf{x}, \mathbf{k}) \tilde{u}_z(\mathbf{k}, t) d\mathbf{k},\end{aligned}\quad (11)$$

with,

$$\begin{aligned}o_{x1}^{(m)}(\mathbf{x}, \mathbf{k}) &= b_x^{(m)}(\mathbf{x}, \mathbf{k}) w_x(\mathbf{x}, \mathbf{k}) + b_{xz}^{(m)}(\mathbf{x}, \mathbf{k}) w_{xz}(\mathbf{x}, \mathbf{k}), \\ o_{x2}^{(m)}(\mathbf{x}, \mathbf{k}) &= b_x^{(m)}(\mathbf{x}, \mathbf{k}) w_{xz}(\mathbf{x}, \mathbf{k}) + b_{xz}^{(m)}(\mathbf{x}, \mathbf{k}) w_z(\mathbf{x}, \mathbf{k}), \\ o_{z1}^{(m)}(\mathbf{x}, \mathbf{k}) &= b_{xz}^{(m)}(\mathbf{x}, \mathbf{k}) w_x(\mathbf{x}, \mathbf{k}) + b_z^{(m)}(\mathbf{x}, \mathbf{k}) w_{xz}(\mathbf{x}, \mathbf{k}), \\ o_{z2}^{(m)}(\mathbf{x}, \mathbf{k}) &= b_{xz}^{(m)}(\mathbf{x}, \mathbf{k}) w_{xz}(\mathbf{x}, \mathbf{k}) + b_z^{(m)}(\mathbf{x}, \mathbf{k}) w_z(\mathbf{x}, \mathbf{k}).\end{aligned}\quad (12)$$

To derive the time extrapolation of the decomposed vector fields using equations 10 and 11, we update the total elastic wavefields by superposing qP- and qSV-waves, namely

$$\begin{aligned}u_x(\mathbf{x}) &= u_x^{(qP)}(\mathbf{x}) + u_x^{(qSV)}(\mathbf{x}), \\ u_z(\mathbf{x}) &= u_z^{(qP)}(\mathbf{x}) + u_z^{(qSV)}(\mathbf{x}).\end{aligned}\quad (13)$$

Propagation of decomposed elastic waves

Therefore, equations 10 to 13 compose the mixed-domain integral operators to simultaneously extrapolate and decompose the elastic wavefields for 2D VTI media. For tilted TI (TTI) media, the mixed-domain matrices in equation 12 should be constructed with the help of locally rotating the coordinate system.

LOW-RANK APPROXIMATE SOLUTIONS

The discrete implementation of each integral operator in equations 4, 8, 10 and 11 naturally arises as a numerical approximation of a continuous FIO of the general form. Note that phase terms in the FIOs can be absorbed into inverse Fourier transforms. Following Cheng and Fomel (2013, 2014), we may achieve efficient solutions to these FIOs using low-rank approximation of the mixed-domain matrices in the kernels.

For example, any mixed-domain matrix (denoted as $W(\mathbf{x}, \mathbf{k})$) in the FIOs can be approximated by the following separated representation (Fomel et al., 2013):

$$W(\mathbf{x}, \mathbf{k}) \approx \sum_{m=1}^M \sum_{n=1}^N B(\mathbf{x}, \mathbf{k}_m) A_{mn} C(\mathbf{x}_n, \mathbf{k}), \quad (14)$$

in which $B(\mathbf{x}, \mathbf{k}_m)$ is a mixed-domain matrix with reduced wavenumber dimension, $C(\mathbf{x}_n, \mathbf{k})$ is a mixed-domain matrix with reduced spatial dimension, A_{mn} is a $M \times N$ matrix. As we observe, the ranks M and N are generally very small for our applications.

Therefore, the above low-rank approximation speeds up computation of the FIOs since

$$\begin{aligned} & \int e^{i\mathbf{k}\mathbf{x}} W(\mathbf{x}, \mathbf{k}) \tilde{u}_j(\mathbf{k}) d\mathbf{k} \\ & \approx \sum_{m=1}^M B(\mathbf{x}, \mathbf{k}_m) \left(\sum_{n=1}^N A_{mn} \left(\int e^{i\mathbf{k}\mathbf{x}} C(\mathbf{x}_n, \mathbf{k}) \tilde{u}_j(\mathbf{k}) d\mathbf{k} \right) \right), \end{aligned} \quad (15)$$

with $j = x, z$. Evaluation of the last formula is effectively equivalent to applying N inverse fast Fourier transforms each time step. Construction of the separated representation follows the method of Engquist and Ying (2009). In multiple-core implementations, the matrix operations in equation 15 are easy to parallelize.

EXAMPLES

We test our approaches on a two-layer model, in which the first layer is a VTI medium with $v_{p0} = 2500\text{m/s}$, $v_{s0} = 1200\text{m/s}$, $\epsilon = 0.2$, and $\delta = -0.2$, and the second layer is a VTI medium with $v_{p0} = 3600\text{m/s}$, $v_{s0} = 1800\text{m/s}$, $\epsilon = 0.2$, and $\delta = 0.1$. A point source is placed at the center of this model. In the following experiments, the ranks are automatically calculated based on the complexity of the model and the estimate of the approximation accuracy to the mixed-domain matrices (Here we aim for the relative single-precision accuracy of 10^{-6}).

First, we simulate elastic wave propagation by solving the original elastic wave equation using pseudo-spectral method (equation 4) with the help of low-rank approximation. Figure 1 shows the two wavefield components at the time of 0.3 s. In this case, ranks $M = N = 2$ for the first two matrices and $M =$

$N = 1$ for the third matrix in equation 5 are required for the low-rank approximation. It takes CPU time of 0.23 second to finish the wavefield extrapolation of one time step.

Then we simulate propagation of the decomposed elastic waves using the mixed-domain integral operators, which consist of equations 10 to 13. As shown in Figure 2, this approach simulate decomposed and coupled elastic wave propagation very well using the low-rank approximate algorithm with ranks $M = N = 2$. We can observe all transmitted and reflected waves including mode conversions. For a single time step, it takes CPU time of 0.55 second to extrapolate the decomposed elastic waves. To obtain the same accuracy, more time is required if we extrapolate the coupled elastic wavefields using a 10th-order finite-difference algorithm and then decompose them into qP- and qSV-wave modes using the fast algorithm proposed by Cheng and Fomel (2013, 2014). Comprehensive accuracy and efficiency analysis, as well as applications to 3D or more complex models and elastic RTM, will be shown at the conference.

CONCLUSIONS

We have proposed a recursive integral method to simultaneously extrapolate and decompose the elastic wavefields for heterogeneous TI media. The computational efficiency is guaranteed by merging the operations of wavefield extrapolation and vector decomposition into a unified Fourier integral framework and speeding up the solutions using the low-rank approximation. The synthetic example shows that our method can produce dispersion-free decomposed and coupled elastic wavefields efficiently. This implies that the proposed approach has great potential for applications such as elastic RTM and FWI in multicomponent seismic exploration.

ACKNOWLEDGMENTS

The first author would like to thank Sergey Fomel for sharing his experience in designing low-rank approximate algorithm, and Chenlong Wang and Tengfei Wang for helpful discussion. We thank KAUST for its support. We thank the authors of *Madagascar* for providing this software platform for reproducible computational experiments.

Propagation of decomposed elastic waves

Figure 1: Elastic wavefields synthesized by solving the original elastic wave equation using low-rank approximate pseudo-spectral method: (a) x- and (b) z-components.

Figure 2: Elastic wavefields synthesized by using low-rank approximate pseudo-spectral operators that can extrapolate and decompose the elastic waves simultaneously: (a) x- and (b) z-components of the qP-wave displacement wavefields; (c) x- and (d) z-components of the qSV-wave displacement wavefields; (e) x- and (f) z-components of the elastic wavefields updated with their qP- and qSV-components.

EDITED REFERENCES

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2014 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

REFERENCES

- Aki, K., and P. Richards, 1980, Quantitative seismology, 2nd ed.: University Science Books.
- Bube, K. P., T. Nemeth, J. P. Stefani, R. Ergas, W. Liu, K. T. Nihei, and L. Zhang, 2012, On the instability in second-order systems for acoustic VTI and TTI media : *Geophysics*, **77**, no. 5, T171–T186, <http://dx.doi.org/10.1190/geo2011-0250.1>.
- Carcione, J. M., G. C. Herman, and A. P. E. ten Kroode, 2002, Seismic modeling: *Geophysics*, **67**, 1304–1325, <http://dx.doi.org/10.1190/1.1500393>.
- Cheng, J., and S. Fomel, 2013, Fast algorithms for elastic wave mode separation and vector decomposition using low-rank approximation for anisotropic media: 83rd Annual International Meeting, SEG, Expanded Abstracts, doi: 10.1190/segam2013-0049.1.
- Cheng, J., and S. Fomel, 2014, Fast algorithms of elastic wave mode separation and vector decomposition using low-rank approximation for anisotropic media : *Geophysics*, **79**, no. 4, C97–C110, <http://dx.doi.org/10.1190/geo2014-0032.1>.
- Chu, C., B. Macy, and P. Anno, 2011, An accurate and stable wave equation for pure acoustic TTI modeling: 81st Annual International Meeting, SEG, Expanded Abstracts, 179–184.
- Dellinger, J., and J. Etgen, 1990, Wavefield separation in two-dimensional anisotropic media : *Geophysics*, **55**, 914–919, <http://dx.doi.org/10.1190/1.1442906>.
- Du, X., P. J. Fowler, and R. P. Fletcher, 2013, Recursive integral time-extrapolation methods for waves: A comparative review: *Geophysics*, **79**, no. 1, T9–T26, <http://dx.doi.org/10.1190/geo2013-0115.1>.
- Engquist, B., and L. Ying, 2009, A fast directional algorithm for high frequency acoustic scattering in two dimensions : *Communications Mathematical Sciences*, **7**, no. 2, 327–345, <http://dx.doi.org/10.4310/CMS.2009.v7.n2.a3>.
- Etgen, J., and S. Brandsberg-Dahl, 2009, The pseudo-analytical method: Application of pseudoLaplacians to acoustic and acoustic anisotropic wave propagation: 79th Annual International Meeting, SEG, Expanded Abstracts, 2552–2556.
- Fang, G., S. Fomel, and Q. Du, 2013, Seismic wave extrapolation on a staggered grid using low-rank decomposition and low-rank finite-differences: 83rd Annual International Meeting, SEG, Expanded Abstracts, doi: 10.1190/segam2013-0274.1.
- Firouzi, K., B. T. Cox, B. E. Treeby, and N. Saffari, 2012, A first-order k-space model for elastic wave propagation in heterogeneous media : *The Journal of the Acoustical Society of America*, **132**, no. 3, 1271–1283, <http://dx.doi.org/10.1121/1.4730897>. [PubMed](#)
- Fomel, S., L. Ying, and X. Song, 2010, Seismic wave extrapolation using a low-rank symbol approximation: 80th Annual International Meeting, SEG, Expanded Abstracts, 3092–3096.
- Fomel, S., L. Ying, and X. Song, 2013, Seismic wave extrapolation using low-rank symbol approximation: *Geophysical Prospecting*, **61**, 1–11.

- Fowler, P., and R. King, 2011, Modeling and reverse time migration of orthorhombic pseudo-acoustic P-waves: 81st Annual International Meeting, SEG, Expanded Abstracts, 190–195.
- Fowler, P., and C. Lapilli, 2012, Generalized pseudospectral methods for orthorhombic modeling and reverse-time migration: 82nd Annual International Meeting, SEG, Expanded Abstracts, doi: 10.1190/segam2012-0708.1.
- Fowler, P. J., X. Du, and R. P. Fletcher, 2010, Recursive integral time extrapolation methods for scalar waves: 80th Annual International Meeting, SEG, Expanded Abstracts, 3210–3215.
- Lapilli, C., and P. J. Fowler, 2013, Rotation parameters for model building and stable parameter inversion in orthorhombic media: 83rd Annual International Meeting, SEG, Expanded Abstracts, doi: 10.1190/segam2013-0366.1.
- Liu, F., S. Morton, S. Jiang, L. Ni, and J. Leveille, 2009, Decoupled wave equations for P- and SV-waves in an acoustic VTI media: 79th Annual International Meeting, SEG, Expanded Abstracts, 2844–2848.
- Liu, Q., 1995, Generalization of the k-space formulation to elastodynamic scattering problems: The Journal of the Acoustical Society of America, **97**, no. 3, 1373–1379, <http://dx.doi.org/10.1121/1.412079>.
- Ma, D. T., and G. M. Zhu, 2003, P- and S-wave separated elastic wave equation numerical modeling (in Chinese): Oil Geophysical Prospecting, **38**, 482–486.
- Pestana, R., and P. L. Stoffa, 2010, Time evolution of the wave equation using rapid expansion method: Geophysics, **75**, no. 4, T121–T131, <http://dx.doi.org/10.1190/1.3449091>.
- Song, X., and T. Alkhalifah, 2013, Modeling of pseudoacoustic P-waves in orthorhombic media with a low-rank approximation: Geophysics, **78**, no. 4, C33–C40, <http://dx.doi.org/10.1190/geo2012-0144.1>.
- Song, X., and S. Fomel, 2011, Fourier nite-difference wave propagation: Geophysics, **76**, no. 5, T123–T129, <http://dx.doi.org/10.1190/geo2010-0287.1>.
- Soubaras, R., and Y. Zhang, 2008, Two-step explicit marching method for reverse-time migration: 70th Annual International Conference and Exhibition, EAGE, Extended Abstracts, F041.
- Sun, R., and G. A. McMechan, 2001, Scalar reverse-time depth migration of prestack elastic seismic data: Geophysics, **66**, 1519–1527, <http://dx.doi.org/10.1190/1.1487098>.
- Tal-Ezer, H., 1986, Spectral methods in time for hyperbolic problems: SIAM Journal on Numerical Analysis, **23**, no. 1, 11–26, <http://dx.doi.org/10.1137/0723002>.
- Wang, C., J. Cheng, and T. Wang, 2014a, Local angle domain elastic reverse-time migration in TI media: Presented at the 76th Annual International Conference and Exhibition, EAGE.
- Wang, C., J. Cheng, and T. Wang, 2014b, Propagating decomposed elastic waves by first- and second-order wave equations using low-rank approximate k-space method for isotropic media: Presented at the 84th Annual International Meeting, SEG.
- Wu, Z., and T. Alkhalifah, 2014, The optimized expansion based low-rank method for wavefield extrapolation: Geophysics, **79**, no. 2, T51–T60, <http://dx.doi.org/10.1190/geo2013-0174.1>.
- Yan, J., and P. Sava, 2008, Isotropic angle domain elastic reverse-time migration: Geophysics, **73**, no. 6, S229–S239, <http://dx.doi.org/10.1190/1.2981241>.

- Yan, J., and P. Sava, 2009, Elastic wave-mode separation for VTI media : Geophysics, **74**, no. 5, WB19–WB32, <http://dx.doi.org/10.1190/1.3184014>.
- Zhang, Q., and G. A. McMechan, 2010, 2D and 3D elastic wavefield vector decomposition in the wavenumber domain for VTI media : Geophysics, **75**, no. 3, D13–D26, <http://dx.doi.org/10.1190/1.3431045>.
- Zhang, Y., H. Zhang, and G. Zhang, 2011, A stable TTI reverse-time migration and its implementation : Geophysics, **76**, no. 3, WA3–WA11, <http://dx.doi.org/10.1190/1.3554411>.