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Comparative Study of Shale-Gas Production using Single- and Dual-Continuum Approaches

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Abstract

In this paper, we explore the possibility of specifying the ideal hypothetical positions of matrices blocks and fractures in fractured porous media as a single-continuum reservoir model in a way that mimics the dual-porosity dual-permeability (DPDP) configuration. In order to get an ideal mimic, we use the typical configuration and geometrical hypotheses of the DPDP model for the SDFM. Unlike the DPDP model which consists of two equations for the two-continuum coupled by a transfer term, the proposed single-domain fracture model (SDFM) model consists of a single equation for the single-continuum. Each one of the two models includes slippage effect, adsorption, Knudsen diffusion, geomechanics, and thermodynamics deviation factor. For the thermodynamics calculations, the cubic Peng-Robinson equation of state is employed. The diffusion model is verified by calculating the total mass flux through a nanopore by combination of slip flow and Knudsen diffusion and compared with experimental data. A semi-implicit scheme is used for the time discretization while the thermodynamics equations are updated explicitly. The spatial discretization is done using the cell-centered finite difference (CCFD) method. Finally, numerical experiments are performed under variations of the physical parameters. Several results are discussed such as pressure, production rate and cumulative production. We compare the results of the two models using the same dimensions and physical and computational parameters. We found that the DPDP and the SDFM models production rate and cumulative production behave similarly with approximately the same slope but with some differences in values. Moreover, we found that the poroelasticity effect reduces the production rate and consequently the cumulative production rate but in the SDFM model the reservoir takes

more time to achieve depletion than the DPDP model. The normal fracture factor which appears in the transfer term of the DPDP model is adjusted against the SDFM.

Keywords: shale-gas, dual-porosity dual-permeability, fractured reservoir, porous media, reservoir simulation.

Nomenclature:

b_m	Matrix slip coefficient [Pa]
b_f	Fracture slip coefficient [Pa]
c	Constant [-]
D_{kf}	Knudsen diffusion coefficient
k_m	Apparent matrix (bedrock) permeability [md]
k_f	Apparent fracture permeability [md]
k_{m0}	Matrix (bedrock) initial permeability [md]
k_{f0}	Fracture initial permeability [md]
L_x, L_y, L_z	Fracture spacing [m]
M_w	Molecular weight [kg/mol]
r_w	Wellbore radius [m]
r_e	Drainage radius [m]
R	General gas constant [Pa m ³ /mol K]
t	Time [s]
T	Temperature [K]
T_n	Total time steps
T_c	Critical Temperature [K]
P_0	Initial reservoir pressure [Pa]
P_w	Wellbore pressure [Pa]
P_m	Matrix pressure [Pa]
P_f	Fracture pressure [Pa]
P_L	Langmuir pressure [Pa]
q_a	Absorbed gas volume per unit area of shale surface [kg/m ³]
q_{std}	Adsorption volume per unit mass of shale [kg/m ³]
V_{std}	Mole volume under standard conditions [m ³ /mol]
V_L	Langmuir volume [m ³ /kg]
Z	Gas deviation factor [-]
α	Tangential momentum accommodation coefficient
α_m	Biot's effective parameter
α_f	Fraction Biot's effective parameter
μ_0	Initial gas viscosity [Pa s]
μ_g	Gas viscosity [Pa s]

ρ_g	Gas (methane) density [kg/m ³]
ρ_s	Shale rock density [kg/m ³]
σ	The crossflow coefficient
σ_m	Mean total stress [Pa]
σ_f	Mean total stress [Pa]
φ_m	Matrix porosity [-]
φ_f	Fracture porosity [-]
$\varphi_{0,m}$	Initial Matrix porosity [-]
$\varphi_{r,m}$	Matrix porosity reference [-]
$\varphi_{0,f}$	Initial Fracture porosity [-]
$\varphi_{r,f}$	Fracture porosity reference [-]

Introduction

Producing gas from shale strata is progressively becoming crucial in the world's energy supply lately. This is caused, in part, by the growing shortage in energy supply and the continual increase of energy demand [1]. Even though the current drop of oil price, which halted many of the projects related to the development of shale gas reservoirs, it is expected that this trend will not last long and the demand on energy from shale reservoirs will pick up potential again. The productivity of shale reservoirs is determined by the presence of natural fracture networks [2]. The transport of shale gas is influenced by very complex mechanisms such as adsorption, desorption [3], viscosity changes, diffusion and flow slippage [4]. The gas in shale sediments can be stored in three different ways: as free gas compressed in the fractures and pore space [3]; as gas adsorbed on bedrock surfaces and pore walls (estimated to account for 20%~85% of shale gas in place [5]); and as gas dissolved in solid organic materials such as kerogen [6]. Shale gas reservoirs are composed mainly of two, heterogeneously distributed, parts. These are; low permeability matrices, and assemblage of fractures, which are fully saturated with natural gases. Due to this heterogeneity, the physical properties fluctuate at a scale that is not usually considered during numerical modeling. To account for the different scales encountered in fractured porous media, researchers usually adapt dual continua models. Historically, Warren and Root [7] are probably the first to develop an idealized model to study the behavior flow in fractured porous media based on dual-

continuum models [8]. Dual-Porosity model was used when two porosity values existed in the domain. To come up with a two continua model, Warren and Root's [7] considered the matrix parts as idealized uniform cubical blocks distributed uniformly and separated by fractures, Fig. 1. Each part, either fracture or matrix had different porosity values.

Fig. 1: Warren and Root [7] idealization fractured porous medium

Ozkan [9, 10] used the Dual-Porosity model, in which the matrix was considered as uniform radius spherical blocks. He considered both the matrix diffusive Darcy flow and the fractures stress dependent permeability for naturally fractured reservoirs ignoring sorption and desorption processes. Moridis et al. [8] used the standard Dual-continuum model to account for several mechanisms in Kerogen. They reported that while sorption effects may not be noticeable on production data for tight sand reservoirs, they cause a significant deviation in shale gas reservoirs. Bustin [11] used a standard Dual-continuum model to study the permeability effect governed by the shale gas fabric on the production rate due to Darcy flow in the fractured system. He found that higher gas saturations, gas adsorption capacities, and reservoir pressures would result in more gas production due to the gas stored in shale. Wu [12] used a generalized dual-continuum methodology and proposed general reservoir simulators ignoring the adsorption and desorption processes. His goal was to implement a general theoretical fracture model for simulating fluid and heat flow processes in fractured unconventional reservoirs [12, 13]. Recently, El-Amin [35,41] introduced an analytical solution of the apparent-

permeability and fractional derivative gas-transport equation in porous media. Guo et al. [3] developed a mathematical model including all the above mentioned mechanisms to describe the flow behavior in tight shale gas formulation with ignoring the poroelasticity effect.

In the single discrete fracture modeling, each fracture is represented explicitly using high resolution of mesh. So, as fractures have high permeability and big variations in physics, dense mesh is required in fractures than in matrix. For example, Baca et al. [36] presented a two-dimensional model for single-phase flow with heat and solute transfer in fractured porous media. Juanes et al. [37] proposed a finite-element formulation for a single-phase flow in fractured formations. Matthai et al. [38] introduced numerical investigations for two-phase flow in fractured porous media using control-volume finite-element method.

As mentioned above, most of the research work on the simulation of shale gas reservoirs used one of the dual-continuum models. However, these models hide a lot of details that might significantly affect the flow behavior, (e.g., field parameters, matrix and fracture blocks positions, their spatial distribution, etc.). In this paper, therefore, we represent the matrices blocks and fractures in fractured porous media as a single-continuum. We called the suggested model “single-domain fracture model (SDFM)”. The SDFM consists of a single equation unlike the DPDP model which consists of two equations coupled with a transfer term. The aim of the SDFM is to mimic the DPDP model using a single-continuum model. So, in the SDFM we employ the typical configuration of the DPDP model to check its performance. After testing the two models, DPDP and SDFM separately, the SDFM will be used to adjust the transfer term (in particular the shape factor) of the DPDP model. It is noted that the SDFM is slower than the DPDP model. On the other hand, we use fine mesh for computational issues. This paper also studies the effects of geomechanical stresses. Each model including same physics such as slippage effect, adsorption, diffusion, geomechanics, and thermodynamics deviation factor. For the thermodynamics calculations, we employ the cubic Peng-Robinson equation of state. Also, the model is verified against experimental data from literature by calculating the total mass flux through nanopore by combination of slip flow and Knudsen diffusion. For

the spatial discretization we use the cell-centered finite difference (CCFD) method for both DPDP and SDFM model, and the time discretization are treated using a semi-implicit scheme while the thermodynamics equations are treated explicitly. Finally, numerical experiments are performed under variations of physical and computational parameters. Several results are discussed such as pressure and production rate.

Mathematical modeling

We consider a single-phase gas flow within the shale reservoir with assuming that the reservoir flow is isothermal and the gravity effect is negligible. Moreover, we assume that the gas sorption and desorption adhere to Langmuir curve, which means that it can achieve equilibrium quickly.

A fractured shale-gas system can be described by appropriate mass balance equations. The general mass balance equation applies to both DPDP and SDFM. The DPDP model has two mass conservation equations similar to the general mass balance equation, one for the matrix continuum and one for the fracture continuum, and coupled with a transfer term. On the other hand, the SDFM has a single mass balance equation for the whole domain. The general mass balance equation may be written as [19, 30]:

$$\frac{\partial M}{\partial t} + [\nabla \cdot (\rho \mathbf{u})] = Q \quad (1)$$

where M is mass accumulation term; $\mathbf{u}$ is velocity vector; ρ is density; Q is source term; t is the time. In the matrix blocks of shale strata, free gas and absorbed gas coexist with each other. Mass accumulation term for free gas per unit volume is represented by [3]:

$$M_f = \varphi \sum_{\beta} S_{\beta} \rho_{\beta} \quad (2)$$

where β is fluid phase; φ is porosity; S_{β} is fraction of pore volume occupied by phase β ; ρ_{β} is density of phase β . For gas reservoirs, $S_{\beta} = 1$. On the other hand, adsorbed gas is estimated to account for 20%~85% of shale gas in place [4]. Mass accumulation term which accounts for adsorbed gas on the matrix surface is represented by [19]:

$$M_{ads} = \sum (1 - \varphi) q_a \quad (3)$$

where q_a is the adsorbed gas volume per unit area of shale surface. Gas adsorption is caused by pressure differential in solid organic materials. With free gas production, the pores pressure decrease causing a pressure difference between the matrix and pores. This

drop in pressure value causes the gas to desorb from the matrix surface. The most commonly used way to describe this process is Langmuir isotherm model [3, 14, 15, 16, 17], expressed as:

$$q_a = \frac{\rho_s M_w}{V_{std}} \left(\frac{V_L P_m}{P_L + P_m} \right) \quad (4)$$

where q_{std} is the adsorption volume per unit mass of shale; V_{std} is the mole volume under standard condition (0 °C, 1 atm); V_L is the Langmuir volume; P_L is the Langmuir pressure; ρ_s is the density of shale core; P_m is the matrix pressure; M_w is the molecular weight. Manipulating the time derivative of the adsorbed mass, Eq. (4), such that,

$$\frac{\partial q_a}{\partial t} = \frac{\partial q_a}{\partial P_m} \frac{\partial P_m}{\partial t} = \frac{\rho_s M_w V_L}{V_{std}} \frac{P_L}{(P_L + P_m)^2} \frac{\partial P_m}{\partial t} \quad (5)$$

Langmuir isotherm is developed to describe how the surface coverage of an adsorbed gas depends on the pressure of the gas above that surface at a fixed temperature [18]. Langmuir adsorption assumes that the adsorbent to be an ideal solid surface consisting of series of distinct sites capable of binding the adsorbate. Combining all previously discussed mass accumulation terms of both free and absorbed gas, Eqs. (2)-(4), we get [3],

$$M = \Sigma(\phi \rho_\beta + (1 - \phi) q_a)$$

Now, let us call the real gas law, $PV = ZRT \frac{m}{M_w}$, R is the gas constant, T is the temperature, m is the mass, V is the gas volume, and Z is the gas deviation factor. The mass density of the gas is,

$$\rho_g = \frac{\gamma}{Z} P \quad (6)$$

where $\gamma = \frac{M_w}{RT}$.

Now, we need to conduct thermodynamic calculations to compute the thermodynamics factor, Z . So, we employ the cubic Peng-Robinson equation of state, which can be expressed as follow [34]:

$$Z^3 - (1 - B)Z^2 + (A - 3B^2 - 2B)Z - (AB - B^2 - B^3) = 0$$

where

$$A = \frac{a_T P}{R^2 T^2}$$

$$B = \frac{b_T P}{RT}$$

Coefficients a_T and b_T are functions of the critical properties, thus,

$$a_T = 0.45724 \frac{R^2 T_c^2}{P_c}$$

$$b_T = 0.0778 \frac{RT_c}{P_c}$$

where T_c and P_c are the critical temperature and critical pressure, respectively.

The equation of flow in porous media may be expressed as,

$$\mathbf{u} = -\frac{K}{\mu} \nabla P \quad (7)$$

where K is permeability; μ is viscosity. For gas with low density, the gravity effect is ignored. Pores sizes in shale formations are almost on the order of few nanometers. Because of that, the conventional Darcy law as given above may not be used to describe the flow process in the matrix blocks. Bird et al. [19] found that gas transport mechanisms in fine porous media combine diffusion, slip flow, and viscosity variations. Furthermore, Terzhagi [20] reported that rock deformation based on effective stresses has also some effects on the transport of fluids in fractured systems. On the other hand, natural gas molecules in the nanosize pore space of shale exhibit frequent collision with the wall surface than between the molecules themselves. In this case, the diffusive flux of the gas can be described by Knudsen diffusion [21] as,

$$J_k = -M_w D_{km} (\nabla C_m) \quad (8)$$

where J_k is the mass flux of the gas caused by Knudsen diffusion, D_{km} is the diffusion coefficient of the matrix system, and C_m is the gas mole concentration. The collisions of gas molecules with each other give rise to the concept of viscosity for which it influences the gas flux due to pressure gradient that can be expressed as,

$$J_v = -\frac{\rho_m k_{m0}}{\mu_g} \nabla P_m \quad (9)$$

where J_v is mass flow; ρ_m is gas density in the matrix system; k_{m0} is the intrinsic permeability of the matrix; μ_g is gas viscosity; P_m is pore pressure in matrix.

Due to the small scale passages where the gas molecules in the matrix ought to pass upon flow, the traditional no slip condition may not apply and the gas indeed slip along the surface. Such slip-flow contribute an additional flux that can improve the flow. This additional flux can cause apparent gas permeability to become higher in the same porous medium than the traditional permeability. Historically, the Klinkenberg method was used to correct the effective gas permeability to a liquid-equivalent permeability using a “gas-slippage” factor. The Klinkenberg effect [4] becomes significant in modeling gas flow in reservoirs in systems with low pressure or low permeability. We consider slip flow, viscous flow and Knudsen diffusion. Thus, the matrix apparent permeability can be written as [6],

$$k_m = k_{m0} \left(1 + \frac{b_m}{P_m} \right) \quad (10)$$

where $b_m = \sqrt{\frac{8\pi RT}{M_g}} \frac{\mu_g}{r} \left(\frac{2}{\alpha} - \frac{2984}{3000} \right)$, α is the tangential momentum accommodation coefficient, takes values within the range [0,1]. Invoking this into the matrix flow vector term, we get,

$$\nabla \cdot (\rho u)_m = -\nabla \cdot \frac{k_{m0} \gamma}{\mu} \left(1 + \frac{b_m}{P_m} \right) P_m \nabla P_m \quad (11)$$

For the gas flow in fractures, we consider Knudsen diffusion and viscous flow. The mass flux may be represented as,

$$J_f = -\frac{\rho_f k_{f0}}{\mu_g} \left(1 + \frac{b_f}{P_f} \right) \nabla P_f \quad (12)$$

where $b_f = \frac{D_{kf} \mu_g}{k_{f0}}$.

The apparent permeability of fractures is expressed as,

$$k_f = k_{f0} \left(1 + \frac{b_f}{P_f} \right) \quad (13)$$

Knudsen diffusion coefficient, D_{kf} , for the fracture system, is defined as [6],

$$D_{kf} = \sqrt{\frac{\pi R T k_{f0} \varphi_{0,f}}{M_w}},$$

where $\varphi_{0,f}$ is the initial fracture porosity.

Therefore, the fractures flow equation becomes,

$$\nabla \cdot (\rho \mathbf{u})_f = -\nabla \cdot \frac{k_{f0} \gamma}{\mu} \left(1 + \frac{b_f}{p_f}\right) P_f \nabla P_f \quad (14)$$

As we mentioned early, the DPDP model deals with fractures and matrix blocks in different scales simulated with two different continua and linked with a transfer term that specifies the mass transfer between the two continua [7]. The mass exchange between matrix blocks and fractures is represented by a shape factor [23,29]. Warren and Root [7] defined the shape factor for cubic matrix blocks as,

$$\sigma = \frac{2n(n+2)}{l^2},$$

where n is the set of normal fractures.

$$l = \frac{3L_x L_y L_z}{L_x L_y + L_y L_z + L_z L_x},$$

is a characteristic length, L_x, L_y and L_z are lengths of the sides of a cubic matrix block. These equivalence relations are obtained using volume to surface ratios. Some other methods have been considered to handle the matrix-fracture connection such as the boundary conditions method [24]. In this paper, we use the term S to indicate the transfer of gas between the matrix and fracture systems in the DPDP model:

$$S = \frac{k_{m0} \gamma \sigma (P_m + b_m) (P_m - P_f)}{\mu_g} \quad (15)$$

where σ is the crossflow coefficient between the fracture and matrix systems. In the fracture system, fluid flows into the fracture from the matrix represented by a sink term. In order to defined this sink term, we follow the model developed by Aronofsky and Jenkins [25] for gas production from vertical well, namely,

$$q_p = \frac{k_{f0} \theta \gamma}{z \mu_g \left[\ln\left(\frac{r_e}{r_w}\right) + S + D_q \right]} P_f (P_{fe} - P_w) \quad (16)$$

where $\theta = 2\pi$ when the production well is placed in the center, while, $\theta = \pi/2$ when the production well is located in the corner [26]; P_w is the bottom hole pressure; P_{fe} is the average fractures pressure around the well; r_w is well radius; r_e is the drainage radius, which can be calculated by,

$$r_e = \begin{cases} 0.14 \sqrt{2[(\Delta x)^2 + (\Delta y)^2]} & \text{if } k_x = k_y \\ 0.28 \frac{\sqrt{\left(\frac{k_y}{k_x}\right)^{\frac{1}{2}} \Delta x^2 + \left(\frac{k_x}{k_y}\right)^{\frac{1}{2}} \Delta y^2}}{\left(\frac{k_y}{k_x}\right)^{\frac{1}{4}} + \left(\frac{k_x}{k_y}\right)^{\frac{1}{4}}} & \text{if } k_x \neq k_y \end{cases} \quad (17)$$

where Δx , Δy are the grid lengths in x and y directions.

The source/sink term for the fractures in DPDP model is described as,

$$Q_f = S - q_p \quad (18)$$

But for the matrix blocks, it is described as,

$$Q_m = -S \quad (19)$$

The rock deformation model applied in this paper is based on the concept of effective stress [20, 28]. Biot and Willis [28] developed a generalization for the effective stress model such that the permeability and the porosity are both dependent on the stress. The porosities φ_m , φ_f are related to the mean effective stress according to the following set of equations [12],

$$\varphi_\beta = \varphi_{r,\beta} + (\varphi_{0,\beta} - \varphi_{r,\beta}) \exp(-a \sigma'_\beta), \quad \beta = m, f \quad (20)$$

where σ'_β is effective stress which is defined by,

$$\sigma'_\beta = \sigma_\beta - \alpha_\beta P_\beta \quad (21)$$

where σ_m is the mean total stress in matrix blocks; σ_f is the mean total stress in fractures; α_m, α_f are Biot's effective parameters. $(\varphi_{0,\beta} - \varphi_{r,\beta})$ has a positive value when porosity increases and a negative value when porosity decreases. The derivative of the porosity is given as:

$$\varphi'_\beta = \frac{\partial \varphi_\beta}{\partial P_\beta} = a \alpha_\beta (\varphi_{0,\beta} - \varphi_{r,\beta}) e^{-a \sigma'_\beta}, \quad \beta = m, f \quad (22)$$

$\varphi_{0,m}$ is initial porosity of the matrix system; $\varphi_{0,f}$ is initial porosity of the fractures system; The stress evaluation is added to each term of the mass accumulation equation, Eq. (1). This means that the matrix equation becomes:

$$\left(\frac{\partial M}{\partial t}\right)_m = F_1(P_m) \frac{\partial P_m}{\partial t} \quad (24)$$

where

$$F_1(P_m) = \left\{ \frac{\gamma}{Z} [\varphi_m + \varphi'_m P_m] + \frac{(1-\varphi_m) \rho_s M_w V_L P_L}{V_{std} (P_L + P_m)^2} - \frac{\rho_s M_w V_L \varphi'_m}{V_{std} (P_L + P_m)} \right\}$$

Similarly, for the fracture system becomes,

$$\left(\frac{\partial M}{\partial t}\right)_f = F_1(P_f) \frac{\partial P_f}{\partial t} \quad (25)$$

where

$$F_1(P_f) = \left\{ \frac{\gamma}{Z} [\varphi_f + \varphi'_f P_f] \right\}$$

In conclusion, the DPDP model consists of two equations, that can be written as,

$$F_1(P_m) \frac{\partial P_m}{\partial t} - \nabla \cdot \left(\frac{\gamma k_{m0}}{\mu_g} \left(1 + \frac{b_m}{P_m} \right) P_m \nabla P_m \right) = -S \quad (26)$$

$$F_2(P_f) \frac{\partial P_f}{\partial t} - \nabla \cdot \left(\frac{\gamma k_{f0}}{\mu_g} \left(1 + \frac{b_f}{P_f} \right) P_f \nabla P_f \right) = S - q_p \quad (27)$$

However, the SDFM model consists only of one equation, namely,

$$F(P) \frac{\partial P}{\partial t} - \nabla \cdot \left(\frac{\gamma k_{s0}}{\mu_g} \left(1 + \frac{b}{P} \right) P \nabla P \right) = -q_p \quad (28)$$

We assume the whole domain for the matrix blocks and fractures has the same initial pressure of the domain in DPDP model and SDFM model, i.e.,

$$P_m^{DPDP}(x, y, t = 0) = P_f^{DPDP}(x, y, t = 0) = P^{SDFM}(x, y, t = 0) = P_0 \quad (29)$$

Also, we assume Neumann boundary conditions in the DPDP matrix continuum,

$$F_{m,n}^{DPDP} = 0 \quad \left(\frac{\partial P_m}{\partial n} = 0 \right) \quad (30)$$

We also assume Neumann boundary conditions except for the production is treated as Dirichlet boundary condition in the DPDP fracture continuum and the SDFM single-continuum,

$$F_{f,n}^{DPDP} = 0 \quad \left(\frac{\partial P_f}{\partial n} = 0 \right), \quad P_f^{DPDP}(x, y, t) = P_w \quad (31)$$

$$F_n^{SDFM} = 0 \quad \left(\frac{\partial P}{\partial n} = 0 \right), \quad P^{SDFM}(x, y, t) = P_w \quad (32)$$

Model Verification in Nanopore Scale:

Roy et al. [32] reported that Knudsen diffusion in nanopores can be modeled in terms of the pressure gradient. Mass flux by diffusion of the gas in a nanopore with neglecting viscous effect may be expressed as [2, 32],

$$J_D = \frac{MDK}{RT} \nabla P$$

Mass flux J_a through a circular tube with negligible length of entrance effect can be derived from Hagen-Poiseuille's equation [19],

$$J_a = -\frac{\rho r^2}{8\mu} \nabla P$$

Javadpour et al. [6] used ideal gas law to relate density to pressure in order to integrate over the length,

$$J_a = -\frac{r^2}{8\mu} \frac{\rho_{avg}}{L} \Delta P$$

where ρ_{avg} is the algebraic average of the density at inlet and outlet, and ΔP is the pressure drop between them. However, the gas has a significant variation throughout the reservoir. In order to capture the effect of non-ideality, the compressibility factor was introduced in the density to pressure relation, thus,

$$J_a = -\frac{r^2}{8\mu} \frac{\gamma}{Z} P \Delta P$$

The integral of the flux over the length becomes unsolvable analytically, therefore numerical calculations take place. For nanopore, the no-slip flow boundary condition is not valid [2, 33]. Slip effect on the boundary improves the flow. Brown et al. [33] presented the following theoretical coefficient to correct the slip velocity in a tube,

$$F = 1 + \sqrt{8\pi\gamma} \frac{\mu}{rP_{avg}} \left(\frac{2}{\alpha} - 1 \right)$$

where α tangential momentum accommodation coefficient. In order to determine α for specific systems, experimental measurements are needed. Experimental data for homogeneous porous media was reported by Rot et al. [32], where relatively cylindrical and straight nanopores (≈ 200 nm in diameter) in a 60 μ m thick membrane. The porosity

estimate of the sample was from 0.2 to 0.3. Argon gas was used to inject at different pressure gradients, and total mass flux was measured. Javadpour et al. [2] has reported a value for $\alpha = 0.80$ to fit with the model. Our constructed model results give a good match with experimental results with the same value for $\alpha = 0.8$ (Fig. 2).

Fig. 2: Model verification against experiment data in Roy et al. [32].

Numerical Discretization:

In order to solve the above differential systems, (26)-(32), numerically, we employ the cell-centered finite difference (CCFD) method [31]. The CCFD is considered one of the most commonly used methods for flow and transport in porous media because it is locally conservative. In the CCFD method, the velocity is located on edges, while the pressure and permeability on cell centers. Dividing the total time interval $[0, T_n]$ into N_T time steps such that the time step length is $\Delta t^n = t^{n+1} - t^n$. The current time step is represented by the superscript $n + 1$, while the previous one is represented by n . The backward Euler time discretization is used for the time derivatives terms in the two equations of pressure. We use finer mesh for the SDFM. The time discretization is treated using a semi-implicit scheme while the thermodynamics equation is treated explicitly.

Table 1: Physical and computational parameters.

Parameter	Value	Parameter	Value	Parameter	Value
c	22.2	T	373.15 K	μ_0	1.02E-5 Pa s
k_{m0}	0.01 md	N_T	500	μ_g	1.02E-5 Pa s
k_{f0}	1 md	P_0	10.4E6 Pa	ρ_g	0.66 kg/m ³
L_x, L_y, L_z	0.2 m	P_L	2.07E6 Pa	ρ_s	2550 kg/m ³
M_g	0.016 kg/mol	P_w	3.45E6 Pa	$\varphi_{0,m}$	0.05
r_w	0.14 m	V_{std}	0.0224 m ³ /mol	$\varphi_{r,m}$	0.03
r	1E-8	α_m, α_f	0.8	$\varphi_{0,f}$	0.001
R	8.314 Pa m ³ /mol K	σ_m, σ_f	3E7	$\varphi_{r,f}$	0.0009

In this paper, 2D reservoir is considered (Fig. 3) and due to symmetrically in all quarters, we study only one quarter of the domain. Values of the physical and computational parameters used in this study are stated in Table 1, however, some of these values are changed as required by tests.

Fig. 3: Schematic diagram of 2D reservoir domain.

In DPDP model, we use a uniform mesh for each continuum (Fig. 4a). On the other hand, in the SDFM model, we use different mesh sizes for matrix blocks and for fractures. The number of grids in matrix blocks are different from the number of grids in matrix blocks and, they are distributed uniformly in a single-continuum as shown in Fig. 4b.

Fig. 5a illustrates the dependency of the production rate against time on the of number of grids in matrix blocks, while, Fig. 5b shows the dependency of the cumulative production rate against time on the of number of grids in matrix blocks. Similarly, Fig. 6a illustrates the dependency of the production rate against time on the of number of grid in fractures, while, Fig. 6b shows the dependency of the cumulative production rate against time on the of number of grids in fractures. From these figures, it may be noted that we may need bigger numbers of grids in both matrix blocks and fractures as the time of production increases.

Fig. 4: (a) DPDP regular mesh distribution; (b) SDFM mesh distribution.

Fig. 5: (a) Production rate against time with various values of number of grid matrix blocks (NOB); (b) Cumulative production rate against time with various values of number of grid matrix blocks (NOB).

Fig. 5: (a) Production rate against time with various values of number of grid fracture blocks (NOF); (b) Cumulative production rate against time with various values of number of grid fracture blocks (NOF).

Results and Discussions

In the DPDP model, we use a domain of $60 \text{ m} \times 60 \text{ m}$ with 60×60 uniform mesh, however, in the SDFM, which is of $1.2\text{m} \times 1.2\text{m}$. The DPDP has a specific porosity and permeability used for the whole domain in each continuum. On the other hand, in the SDFM model we specify the exact positions of matrices blocks and fractures in the single-continuum mimicking the DPDP configuration as illustrated in Fig. 7. Both the SDFM and the DPDP models are assumed to have the same configuration except for the parameters appear in the transfer term of the DPDP model. For comparison cases we use same dimensions, namely, $1.2\text{m} \times 1.2\text{m}$.

Fig. 7: Matrix blocks and fractures positions are defined explicitly in SDFM, (left) the lowest permeability [md] and (right) the highest porosity; and vice versa for the fractures.

Fig. 8: Flow behavior (streamlines) of SDFM configuration.

This configuration allows us to observe flow behavior in the domain as illustrated in Fig. 8. It can be seen from the figure that the gas flows out of matrix blocks into fractures to the production well. We may note also how the existence of the production well at the north east corner affects the flow direction within the matrix blocks close to the production well.

Fig. 9: The cumulative production of DPDP and SDFM models (top) with no-stress effect, and (bottom) with stress effect.

Fig. 9 shows that both models, DPDP and SDFM, behave in a similar way but with some quantity difference with and without stress effect. It can be seen also that the SDFM fits when the normal fracture factor equals one ($n=1$) for the case of considering geomechanical stress. However, in the case of no geomechanical stress the SDFM fits when the normal fracture factor is less than one ($n<1$), typically $n=0.735$. This indicates that the shape factor σ (which depends on the factors n [7]) is a major parameter affects

the gas transfer between matrix and fractures. In general, we may conclude that when using the DPDP model, the factor n needs to be calibrated. As in the Warren-Root model [7], the shape factor is expressed as, $\sigma = \frac{2n(n+2)}{l^2}$, l depends only on lengths of sides of a cubic matrix block, which are given. So, we need only to adjust the factor n . Fig. 10 shows the cumulative production of the DPDP model for various values of the factor n . This figure indicates that the cumulative production increases as the factor n increases, because as we stated above that the factor n enhances the gas transfer between the matrix and fractures.

Fig. 10: The cumulative production of the DPDP model for various values of the factor n .

Fig. 11: The production rate and the cumulative production of the SDFM for various values of the matrix permeability.

Fig. 11 shows the production rate and the cumulative production of the SDFM for various values of the matrix permeability. This test gives an indication that the matrix blocks permeability distribution can be considered as a key factor in the SDFM simulation. However, there is a difference between the uniform distribution we choose for our models, DPDP or SDFM, and the actual field data that we need to be aware of during the simulation. For example, Fig. 12 illustrates the SDFM pressure distribution. This figure shows how the pressure around the production well decreases and is affected by matrix blocks and fractures positions. Fig. 13 illustrates the effect of boundary pressure values on both models, SDFM and DPDP. According to Fig. 13, as the pressure on that Dirichlet boundary increase, the production rate decreases in both SDFM and DPDP models.

Fig. 12: SDFM pressure distribution.

Fig. 13: Effect of wellbore pressure variation on the cumulative production of (top) DPDP model, with $n=2$ and (bottom) SDFM.

Initial pressure of the reservoir is very important in defining many parameters that affect the modeling and simulation of the system. Fig. 14 shows the results of initial pressure values on both DPDP and SDFM models. From this figure, we may note that as the initial pressure increases the gas cumulative production increases in both systems.

Fig. 14: Effect of the initial pressure on the cumulative production of (top) DPDP model, with $n=2$, and (bottom) SDFM.

Fig. 15: DPDP production rate (top) and cumulative rate (bottom) of both stress and no stress effect, with $n=2$.

Fig. 15 and Fig. 16 illustrate the variations that occurs on the system when the poroelasticity effect is applied on the SDFM model and the DPDP model. Fig. 15 shows that the stress effect reduces the production rate and the cumulative production in the DPDP model. This may be because the geomechanical stress causes deformations for pores and fractures which might resist the gas transport and therefore reduce the production rate. Fig. 16 also shows that the stress reduces both the production rate and

cumulative production of SDFM model. It is worth to note that in the SDFM model the reservoir takes more time to achieve depletion than the DPDP model for both stress and no-stress cases.

Fig. 16: SDFM production rate (top) and cumulative production (bottom) of both stress and no stress effect.

Fig. 17: Effects of $\Delta\varphi_m$ on the cumulative production in (top) DPDP model, with $n=2$ and (bottom) SDFM

The rock deformation depends on the change of porosity ($\Delta\varphi_\alpha = \Delta\varphi_\alpha = \varphi_{0,\alpha} - \varphi_{r,\alpha}$, $\alpha = m, f$). In Fig. 17 and Fig. 18, we compare the effects of $\Delta\varphi_\alpha$ on both DPDP and SDFM models. It is found that as $\Delta\varphi_m$ increases the cumulative rate decreases and becomes less significant with small $\Delta\varphi_m$.

Fig. 18: Effects of $\Delta\varphi_f$ variation on cumulative production in the DPDP model.

In Fig. 18, we compare different values of $\Delta\varphi_f$ for the DPDP model. It can be seen that as $\Delta\varphi_f$ increases, the cumulative production decreases. It is interesting to note that the reduction of the cumulative production with $\Delta\varphi_f$ in the DPDP model is slower than the reduction of the cumulative rate with $\Delta\varphi_m$. This probably because the values of φ_f are much smaller than the φ_m values.

Our simulation of shale gas production rate and cumulative production (Fig. 13 – Fig. 18) are qualitatively comparable to actual field data represented in Fig. 14 – Fig. 16 of Ref. [39], and in Fig. 4a of Ref. [40]. The behavior looks similar, however, the quantitative comparison is not adequate because we consider 2D small scale reservoir in our simulation. Full comparison to actual data field will be consider in our future work.

Conclusion

In this work, a single-domain fracture model (SDFM) was designed to mimic the DPDP model. Both models, DPDP and SDFM, are simulated under the same configuration and the parameters values. Both models include same physics such as slippage effect, adsorption, diffusion, geomechanics, and thermodynamics deviation factor. The cubic

Peng-Robinson equation of state is employed for thermodynamics calculations. We verified the diffusion model against published experimental data by calculating the total mass flux through a nanopore. Although, both DPDP and SDFM are investigated separately, the SDFM is used to calibrate the transfer term of the DPDP model. We found that the cumulative production of two models behave similarly with some differences. Moreover, numerical experiments have been performed for the two models under various values of governing parameters. We used 2D domain and a uniform mesh DPDP continua, however, in the SDFM model, we use non-uniform mesh to distinguish between matrices and fractures using different porosities and permeabilities. The CCFD method is used for the spatial discretization and a semi-implicit scheme is employed for the time discretization, while thermodynamics equations are treated explicitly. It is worth mentioning that the SDFM fits well when the normal fracture factor equals one ($n=1$). Moreover, we found that the matrix and fractures permeability distributions can be considered as a key factor in fractured reservoir simulation. On the other hand, the poroelasticity effect reduces the production rate and the cumulative production but in the SDFM model the reservoir takes more time to achieve depletion than the DPDP model. The dual-continuum model is fast because it contents a transfer term acts as sink/source term which cause the reservoir depletion.

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Highlights:

- We compare the results of both DPDP and SDFM models using the same parameters.
- We found that the DPDP and the SDFM models behave similarly with approximately the same slope but with some quantity differences.
- We performed numerical experiments on the two models such as variations pressures and porosities.