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Full Waveform Inversion Using Nonlinearly Smoothed Wavefields

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Summary

The lack of low frequency information in the acquired data makes full waveform inversion (FWI) conditionally converge to the accurate solution. An initial velocity model that results in data with events within a half cycle of their location in the observed data was required to converge. The multiplication of wavefields with slightly different frequencies generates artificial low frequency components. This can be effectively utilized by multiplying the wavefield with itself, which is nonlinear operation, followed by a smoothing operator to extract the artificially produced low frequency information. We construct the objective function using the nonlinearly smoothed wavefields with a global-correlation norm to properly handle the energy imbalance in the nonlinearly smoothed wavefield. Similar to the multi-scale strategy, we progressively reduce the smoothing width applied to the multiplied wavefield to welcome higher resolution. We calculate the gradient of the objective function using the adjoint-state technique, which is similar to the conventional FWI except for the adjoint source. Examples on the Marmousi 2 model demonstrate the feasibility of the proposed FWI method to mitigate the cycle-skipping problem in the case of a lack of low frequency information.

Introduction

Full waveform inversion (FWI) updates the subsurface parameters by minimizing the difference between the predicted and observed data. However, FWI still suffers from a cycle-skipping problem, and thus, may converge to a solution corresponding to a local minimum, specifically when the traveltimes shift between the predicted and observed data is larger than a half of cycle. Data with enough low frequency components, which has a long length of cycle, helps FWI avoid the cycle-skipping problem, but an available frequency band of real seismic data is usually not low enough. On the other hand, an initial model close enough to the true model can mitigate the cycle-skipping problem in FWI, but obtaining a good initial model is not trivial task and requires elaborate techniques, such as traveltimes tomography and migration velocity analysis.

A lot of researches in FWI have been devoted recently to solving the cycle-skipping problem without low frequency. Ma and Hale (2013) estimated the traveltimes shift between the predicted and observed data using dynamic warping method, and tried to minimize the time shift to update the subsurface parameters. Warner and Guasch (2014) calculated the deconvolution filter between the predicted and observed data and penalized the energy away from the optimal Dirac delta function to update velocity model. Wu and Alkhalifah (2016) constructed the objective function based on the data extension and data selective approach. Wu et al. (2014) estimated the envelope of wavefield and inverted the artificial low frequency components included in the envelope of wavefield to update long wavelength components of the model.

On the other hand, Hu (2014) proposed the beat tone inversion, where subtraction between slightly different frequency wavefields carries apparent low-frequency information. However, since the apparent frequency information is not real, it required an elaborate technique, such as the phase-frequency differential and Hilbert transformation, to extract the apparent low frequency information. The motivation of this abstract is that multiplication, instead of subtraction, between slightly different frequency wavefields generates artificial low frequency components, which tends to represent the kinematic components of the wavefield more accurately, thus we can easily extract it by applying a low-pass filter. In order to generalize its process for the whole frequency-band of data, we use a nonlinear operation of multiplying the wavefield with itself. We construct the objective function based on the global-correlation norm (Choi and Alkhalifah, 2012) to deal with the energy (amplitude) imbalance as a function of offset in the nonlinearly smoothed wavefield. We derive the gradient expression of the objective function using the adjoint-state technique, which is similar to the conventional FWI except for the adjoint source. Numerical examples show that the proposed FWI method can mitigate the cycle skipping problem and generate a good convergent result in the case of lack low frequency information.

Theory

A source of low frequency by nonlinearly smoothing wavefield

Hu (2014) proposed the beat tone inversion, where subtraction between slightly different frequency wavefields results in apparent low-frequency information. However, this apparent frequency information is not physical frequency, thus he needed an elaborate technique, such as the phase-frequency differential, to extract low frequency information. The motivation of this abstract is to multiply slightly different frequency wavefields instead of subtraction:

$$\cos(2\pi f t) \cos(2\pi(f + \delta f)t) = \frac{1}{2} \{ \cos(2\pi(2f + \delta f)t) + \cos(2\pi\delta f t) \} \quad (1)$$

where f and t are the frequency and time variables, respectively. We note that the real artificial low frequency ($2\pi\delta f$) is generated through the multiplication. We can easily extract the artificial low frequency component $[\cos(2\pi\delta f t)]$ by applying a low-pass filter to the multiplied wavefield. In order to obtain the artificial low frequency components from full frequency-band of data, we multiply

a wavefield with itself (nonlinear operation). We also generalize the case by adjusting the power of the multiplied wavefield:

$$s(x, t) = S\left(\left(\sqrt{u^2(x, t)}\right)^p\right), \quad (2)$$

where $s(x, t)$ is referred to as the nonlinearly smoothed wavefield, S indicates a triangle low-pass filter and superscript p stands for the power of wavefield. In this abstract, p is 1 or 2.

Global-correlation-based objective function and its gradient

We construct the objective function using the nonlinearly smoothed wavefield in equation 2 following the global-correlation norm approach (Choi and Alkhalifah, 2012):

$$J(\mathbf{m}) = - \sum_x \frac{\int_t s_{syn}(x, t) \cdot s_{obs}(x, t) dt}{\sqrt{\int_t s_{syn}(x, t)^2 dt} \cdot \sqrt{\int_t s_{obs}(x, t)^2 dt}} \quad (3)$$

where $s_{syn}(x, t)$ and $s_{obs}(x, t)$ are the nonlinearly smoothed wavefields of the simulated and observed wavefields, respectively. The gradient of the objective function can be derived by taking the partial derivative of equation 3 with respect to the subsurface model parameter ($\mathbf{m}$) as follows:

$$\frac{\partial J(\mathbf{m})}{\partial \mathbf{m}} = \sum_x \frac{\int_t s_{syn}(x, t) s_{obs}(x, t) dt \int_t s_{syn}(x, t) \cdot \frac{\partial s_{syn}(x, t)}{\partial \mathbf{m}} dt}{\sqrt{\int_t s_{syn}(x, t)^2 dt}^3 \cdot \sqrt{\int_t s_{obs}(x, t)^2 dt}} - \frac{\int_t s_{obs}(x, t) \cdot \frac{\partial s_{syn}(x, t)}{\partial \mathbf{m}} dt}{\sqrt{\int_t s_{syn}(x, t)^2 dt} \cdot \sqrt{\int_t s_{obs}(x, t)^2 dt}}, \quad (4)$$

where

$$\frac{\partial s_{syn}(x, t)}{\partial \mathbf{m}} = \frac{\partial u(x, t)}{\partial \mathbf{m}} p \sqrt{u^2(x, t)}^{p-2} u(x, t). \quad (5)$$

The adjoint source $R(x, t)$ is derived from the above equations:

$$p \sum_x \frac{\int_t s_{syn}(x, t) s_{obs}(x, t) dt \int_t s_{syn}(x, t) \cdot \sqrt{u^2(x, t)}^{p-2} u(x, t) dt}{\sqrt{\int_t s_{syn}(x, t)^2 dt}^3 \cdot \sqrt{\int_t s_{obs}(x, t)^2 dt}} - \frac{\int_t s_{obs}(x, t) \cdot \sqrt{u^2(x, t)}^{p-2} u(x, t) dt}{\sqrt{\int_t s_{syn}(x, t)^2 dt} \cdot \sqrt{\int_t s_{obs}(x, t)^2 dt}}. \quad (6)$$

Finally, the gradient is expressed as

$$\frac{\partial J(\mathbf{m})}{\partial \mathbf{m}} = \sum_x \int_t \frac{\partial u(x, t)}{\partial \mathbf{m}} R(x, t) dt. \quad (7)$$

We observe that the gradient has a similar form to the conventional FWI except for the computation of adjoint source. Therefore, the proposed method has almost the same cost as the conventional FWI.

The envelope-based FWI makes the optimization problem less prone to cycle skipping. But when a cycle-skip between the simulated and observed wavefield is larger than the dominant period, the envelope-based FWI still may encounter a local minimum. However, the radius of attraction of our objective function can be enlarged by increasing the triangle smoothing width. The computed gradient is smoothed according to the smoothing radius of the filter as follows:

$$r_s = v \left(\frac{1}{2f} + \sigma \right), \quad (8)$$

where σ refers to the smoothing width, v is the average velocity, and f is the dominant frequency. The larger the smoothing width (σ) is, the larger the basin of attraction is. Based on the multiscale strategy, we progressively reduce the smoothing width as the inversion proceeds.

Examples

We test the proposed method on the Marmousi 2 model. The true and initial velocity models are shown in figure 1(a) and (b), respectively. A ricker wavelet with peak frequency of 7 Hz is used as the

source. We filtered out the data below 3.5 Hz. The nonlinearly smoothed wavefields are computed based on equation 2 with $p=1$. We start with a large smoothing window and progressively reduce it as 0.16, 0.12 and 0.08s. We also apply a Gaussian smoothing operator on the gradient to remove potential artifacts. The inversion results with a smoothing width $\sigma = 0.16, 0.12$ and 0.08 s are shown in Figures 2a ~ 2c, respectively. As the smoothing width becomes smaller, the resolution is getting higher. Figure 2d shows the subsequent FWI result starting from Figure 2c, which is compatible with the true model. For comparison, we display the inverted models for the envelope-based FWI and conventional FWI in Figures 2e ~ 2g, which show worse results than that of the proposed FWI algorithm especially in this case where we lack the low frequencies. The velocity depth profiles also show a good convergence of the proposed FWI results (Figure 3).

The examples demonstrate that the proposed FWI algorithm successfully generates a long wavelength structure model without low frequency information, which can be used as a good starting model for a subsequent conventional FWI.

Figure 1 The (a) true velocity model and (b) initial model.

Figure 2 The inverted velocity models of the proposed FWI with the triangle smoothing width of (a) 0.16, (b) 0.12, and (c) 0.08 s, (d) subsequent FWI starting from Figure 2c, (e) envelope-based FWI, (f) subsequent FWI starting from Figure 2e, and (g) conventional FWI starting from the initial model in Figure 1b.

Figure 3 The comparison of velocity depth profiles at distances of 3 km (a) and 5 km (b); the true model (red), initial model (cyan), final inverted model starting from the proposed FWI result (blue), and final inverted model starting from the envelope-based FWI result (green).

Conclusions

Multiplication between slightly different frequency wavefields generates artificial low frequency components, which is easily extracted by applying a low-pass filter. For generalization in the whole frequency-band of the data, we multiply a wavefield with itself to generate artificial low frequency components. Also, we construct the objective function based on the global-correlation norm to ameliorate the energy imbalance in the nonlinear smoothed wavefield. We calculate the gradient of the objective function using the adjoint-state technique, which is similar to the conventional FWI except for the adjoint source. Numerical examples demonstrate that the proposed FWI method generates a convergent result for subsequent FWI even in the case of lack of low frequency.

Acknowledgements

We thank KAUST for its support and the SWAG for collaborative environment. Author Yuanyuan Li wishes to thank the China Scholarship Council for support to study abroad.

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