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Salt-Body Inversion with Minimum Gradient Support and Sobolev Space Norm Regularizations

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Summary

Full-waveform inversion (FWI) is a technique which solves the ill-posed seismic inversion problem of fitting our model data to the measured ones from the field. FWI is capable of providing high-resolution estimates of the model, and of handling wave propagation of arbitrary complexity (visco-elastic, anisotropic); yet, it often fails to retrieve high-contrast geological structures, such as salt. One of the reasons for the FWI failure is that the updates at earlier iterations are too smooth to capture the sharp edges of the salt boundary. We compare several regularization approaches, which promote sharpness of the edges. Minimum gradient support (MGS) regularization focuses the inversion on blocky models, even more than the total variation (TV) does. However, both approaches try to invert undesirable high wavenumbers in the model too early for a model of complex structure. Therefore, we apply the Sobolev space norm as a regularizing term in order to maintain a balance between sharp and smooth updates in FWI. We demonstrate the application of these regularizations on a Marmousi model, enriched by a chunk of salt. The model turns out to be too complex in some parts to retrieve its full velocity distribution, yet the salt shape and contrast are retrieved.

Introduction

Full-waveform inversion (FWI) is one of the most efficient techniques to tackle the non-linear ill-posed inverse problem of seismic imaging. FWI provides higher-resolution results in most cases compared to ray-based methods, but suffers from convergence problems in the presence of complex geological bodies, such as salt domes in the region of interest. The most commonly used solution for this problem is (semi-)manual salt flooding at the early stages of the inversion. Several approaches have been suggested to make the inversion procedure automated. All of them are based on a-priori information (assumptions) favoring the geologically plausible solution with blocky structure of the salt and high contrasting physical properties to that of the neighborhood. Kadu et al. (2016) imposed the blocky structure with a level set method, which is not very easily translated to complicated background models. Esser et al. (2016); Qiu et al. (2016) utilize variation-based regularization constraints, which seems to be a more versatile approach. This study primarily focuses on non-linear regularization methods in the prospect of retrieving the salt body through FWI. Successful inversion of the salt requires a blocky update even at low frequencies in order to help FWI mitigate local minima. First, we apply minimum gradient support (MGS), a non-linear regularizer that is often used in geophysical imaging with electromagnetic and gravity fields. Historically, Portniaguine and Zhdanov (1999) introduced MGS to produce sharper inversion results compared to the most commonly used in seismic exploration regularizations such as total-variation (TV) and Tikhonov. However, updating higher wavenumbers of the model at early iterations might cause FWI failure. Hence, we introduce the W_p^1 norm-based regularization. Being a natural blend of Tikhonov and TV regularizations, the latter is supposed to maintain a balance between lower and higher wavenumbers of the model at early stages.

Theory

Figure 1 MGS – a) and TV – b) stabilizing functionals at different gradient $\mathbf{x}$ and parameter ε .

The roles of Tikhonov and TV regularizations in an inversion process are well-known. Tikhonov stabilizing functional favors smoother models, which is good to promote continuity of the wavenumber coverage (Sirgue and Pratt, 2004; Alkhalifah, 2016) throughout the inversion. On the other hand, TV stabilizing functional does not depend on local smoothness of the model, and thus, "preserves the edge" of the model. MGS regularization penalizes the area where strong model parameter variations and discontinuity occur. Mathematically, MGS can be formulated as an integral stabilizing functional:

$$J_{mgs} = \int_{\mathbf{v}} j_{mgs}(\mathbf{x}) d\mathbf{x}, \quad j_{mgs} = \frac{\nabla m(\mathbf{x}) \cdot \nabla m(\mathbf{x})}{\nabla m(\mathbf{x}) \cdot \nabla m(\mathbf{x}) + \varepsilon}, \quad \varepsilon > 0, \quad (1)$$

where $m(\mathbf{x})$ is the squared slowness at location $\mathbf{x}$; the smoothing parameter ε defines the smoothness/differentiability of the regularizing term. Equation 1 suggests that the area with a gradient nearing zero has no contribution, whereas the area with high enough gradient (no matter how large) has contributions equal to one. As a result, it promotes sharp boundaries rather than smooth variations in the velocity model.

Now, the regularized FWI objective function with a stabilizing functional J_{reg} can be written as,

$$J_{FWI} = J_d + \beta J_{reg}, \quad (2)$$

where J_d represents the data fidelity term; β is a regularization coefficient, which measures the confidence level between the two terms. At the early iterations where the modeled data does not match the observed data well, we set β higher.

Gradient computation of J_{mgs}

According to the conventional approach by Portniaguine and Zhdanov (1999), the functional J_{mgs} can be approximated by simple L_2 weighting of the model m for the gradient evaluation. Although, this approximation turns the problem into conventional Tikhonov regularization, it pays the price with a loss in sensitivity of the functional. However, the gradient evaluation can be derived without any approximations using variational calculus. The formula for the gradient of MGS functional is given by,

$$\frac{\delta J_{mgs}}{\delta m}(\mathbf{x}) = -2\nabla \cdot \frac{\nabla m(\mathbf{x})\varepsilon}{(\nabla m(\mathbf{x}) \cdot \nabla m(\mathbf{x}) + \varepsilon)^2}. \quad (3)$$

Role of ε

It is crucial to start the inversion process with a suitable ε in order to accelerate its convergence. The value of ε decides the smoothness in the solution. For instance, higher values tend to over smooth the solution, whereas lower values do the opposite.

Figure 1 shows the behavior of ε in MGS compared to TV regularizer. When ε is large then both regularizations have parabolic shapes in $|\nabla m| = x$, which is equivalent to Tikhonov. In Figure 1(b) at higher gradient values, the functional becomes steeper, which means that a smooth model is favored over a blocky one in the classic total variation approach. In Figure 1(a) at higher gradient values the functional becomes less steep, which means that the blocky models including only very high and very low values of model gradient will be favorable with this stabilizing functional.

W_p^1 regularization

This is a straightforward generalization of the TV and minimum support regularizations, which penalizes the W_p^1 norm of the model parameter gradient. Mathematically, it is given by

$$J_{wp} = \int_{\mathbf{v}} j_{wp}(\mathbf{x}) d\mathbf{x}, \quad j_{wp} = (\nabla m(\mathbf{x}) \cdot \nabla m(\mathbf{x}) + \varepsilon)^{p/2}, \quad \varepsilon > 0. \quad (4)$$

The gradient of J_{wp} can be evaluated as follows,

$$\frac{\delta J_{wp}}{\delta m}(\mathbf{x}) = -p\nabla \cdot \frac{\nabla m}{(\nabla m \cdot \nabla m + \varepsilon)^{1-p/2}}. \quad (5)$$

Figure 2 The W_p^1 stabilizing functional for various $|\nabla m| = x$ and p values, no regularizing ε is added.

Figure 2 displays the Sobolev space W_p^1 norm dependence on the local parameter gradient value. The norm W_p^1 is equivalent to the L_p norm of the gradient. At $p = 1$ it gives the total variation norm, and at $p = 2$ it becomes Tikhonov. If we go further down towards $p = 0.5$ it has the features similar to the Minimum gradient support stabilizer (Figure 1).

Example

We demonstrate the application of MGS and W_p^1 on a synthetic data set of 3 Hz frequency, simulated from the Marmousi model with a modification by introducing a moderate-sized salt body in the center. Figure 3 displays the modified Marmousi model and the initial velocity, which is linearly increasing with depth. The Marmousi model in this form includes: complicated fine structures – primarily in the right side of the model; horizontal layering – left side; homogeneous part – salt in the middle – the most suitable for blocky regularization approaches. First, we try to perform standard full-waveform inversion with 50 sources and 100 receivers equidistantly spread across the top of the model (an absorbing boundary is used instead of a free surface). For all the regularization approaches, we perform a cascading inversion w.r.t. regularization parameter β . We perform 30 iterations of L-BFGS for a high regularization coefficient β value and then decrease β by 20% and so on, for 5 values of the regularization coefficient β . The swipe through the regularization coefficients β (similar to the usage of different TV regularization constraints in (Esser et al., 2016)) is repeated 4 times leading to 600 iterations of L-BFGS in total for each picture. Thus, we can assume that the results are as good as they can be if no other tools are used inside in the inversion. We choose ε empirically so that it is about 5% of the mean square of the gradient. Initial β is chosen so that the regularizing term is 10% of the data misfit functional. We compare the results of the approaches described above in Figure 4. As expected, the updates of MGS and TV regularizer are too blocky to invert even for the moderate size salt in a complex background. Also, conventional FWI with and without Tikhonov are not successful. The $W_{1,2}^1$ approaches shows better retrieval of the salt body in the inversion process, yet even for this regularization approach the right part of the Marmousi, which has the most geological complexity, is not inverted very well. This defines the moderate success in application of full-waveform (Figure 4(f)) without regularization using the results of the inversion with W_p^1 norm. However, the salt body shape and contrast are retrieved by using the new approach for the single frequency (3 Hz) data.

Figure 3 (a) A chunk of a salt body is placed inside the Marmousi model; (b) initial velocity model.

Discussion & Conclusions

The reconstruction of a salt body from the seismic data through FWI is a major challenge. We introduced MGS and W_p^1 as regularizers in order to ameliorate the ill-posed salt body inversion problem. We attribute the failure of the regularization methods to provide a decent estimate of the model to the major complexities (fine structures of the original Marmousi) in the model itself and lack of low frequencies (actually we used a single frequency of 3 Hz). The complexities do not fall easily into a regularized update, an additional tool taking into account local model features such as a scattering angle-based filter (Kazei et al., 2016) could help the convergence of the inversion.

None of the Tikhonov, MGS or TV regularizations succeeded in the reconstruction of the salt body because of the model complexity and the lack of low frequencies. The Sobolev space norm $W_{1,2}^1$, which naturally blends the essential properties of the TV and Tikhonov stabilizers, served the purpose better. However, just like TV and MGS regularizations, it requires user intervention in selecting the appropriate

Figure 4 Full-waveform inversion results without any regularization – a), with standard Tikhonov regularization – b), the Sobolev space norm $W_{1,2}^1$ – c), total variation – d), minimum gradient support – e). In all approaches the regularization coefficient was decreased in 5 steps, which lead to sharp but wrong results for Tikhonov regularization e.g. FWI without any regularization but using c) as the initial velocity model – f).

regularization and smoothing parameters. In future work, we intend to investigate the possibility of an automated selection of these parameters.

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