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Optimal Full Waveform Inversion Strategy in Azimuthally Rotated Elastic Orthorhombic Media

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Summary

The elastic orthorhombic assumption is one of the most practical Earth models that takes into account the horizontal anisotropic layering and vertical fracture network. In this model, the rotation angle of the vertical planes of symmetry is a crucial parameter needed to increase the convergence of an anisotropic full waveform inversion (FWI) as well as to provide the fracture geometry along azimuthal direction. As an initial step, we investigate the possibility of recovering the azimuth angle via FWI, which may offer high-resolution information. We first utilize our new parameterization with deviation parameters, which provides the opportunity for multi-stage FWI. Based on the radiation patterns and gradient directions of each parameter, we show that the azimuth angle mainly affects the parameters that have azimuth-dependent radiation patterns, so that we can hierarchically build up the subsurface model from isotropic to VTI to azimuthally rotated orthorhombic models with less trade-offs. From the numerical example for a synthetic 3D model, we expect that both a deviation parameter and the azimuth angle can be recovered in the last stage of FWI with minimum trade-offs.

Introduction

Recently, multi-parameter full waveform inversion (FWI) corresponding to more complex physics is becoming a focus of our community (Operto et al., 2013; Alkhalifah and Plessix, 2014; Köhn et al., 2015; Masmoudi and Alkhalifah, 2016). Luckily, the elastic orthorhombic (ORT) medium assumption (Oh and Alkhalifah, 2016) provides enough degrees of freedom necessary to capture phenomena related to reservoir characteristic, like fracturing. However, for orthorhombic media, accurate azimuth angle of symmetric axis is required for FWI to converge properly, and provide high-resolution information of the fracture network. In this study, as an initial step to estimate the azimuth angle of the symmetric axis using FWI, we analyse the gradient directions of each parameter in azimuthally rotated orthorhombic (rORT) media based on the radiation patterns. Finally, we suggest an optimal FWI strategy for rORT media and examine the possibility of inverting for azimuth angle.

Gradient direction in azimuthally rotated orthorhombic media

The elastic wave equation in azimuthally rotated orthorhombic media can be expressed with particle displacements (u_i), stress (σ_{ij}), strain (ϵ_{ij}) and source (f_i) as follows (Ivanov and Stovas, 2016):

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \partial_j \sigma_{ij} + f_i \quad (1), \quad \text{and} \quad \sigma_{ij} = C_{ijkl}^{\text{rORT}} \epsilon_{kl} \quad (2)$$

In this study, the density (ρ) is fixed at 1 g/cm³ and is not considered as a FWI parameter. In Voigt notation ($C_{ijkl} \rightarrow C_{IJ}$), the stiffness matrix can be expressed as the rotation of orthorhombic stiffness matrix (C_{IJ}^{rORT} ; Oh and Alkhalifah, 2016) around the vertical direction (z) as follows:

$$C_{IJ}^{\text{rORT}} = \mathbf{M}_z C_{IJ}^{\text{ORT}} \mathbf{M}_z^T \quad (3)$$

where

$$C_{IJ}^{\text{rORT}} = \begin{pmatrix} \tilde{C}_{11} & \tilde{C}_{12} & \tilde{C}_{13} & 0 & 0 & \tilde{C}_{16} \\ \tilde{C}_{12} & \tilde{C}_{22} & \tilde{C}_{23} & 0 & 0 & \tilde{C}_{26} \\ \tilde{C}_{13} & \tilde{C}_{23} & \tilde{C}_{33} & 0 & 0 & \tilde{C}_{36} \\ 0 & 0 & 0 & \tilde{C}_{44} & \tilde{C}_{45} & 0 \\ 0 & 0 & 0 & \tilde{C}_{45} & \tilde{C}_{55} & 0 \\ \tilde{C}_{16} & \tilde{C}_{26} & \tilde{C}_{36} & 0 & 0 & \tilde{C}_{66} \end{pmatrix} \quad (4) \quad \text{and} \quad \mathbf{M}_z(\phi) = \begin{pmatrix} \cos^2 \phi & \sin^2 \phi & 0 & 0 & 0 & -\sin 2\phi \\ \sin^2 \phi & \cos^2 \phi & 0 & 0 & 0 & \sin 2\phi \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos \phi & \sin \phi & 0 \\ 0 & 0 & 0 & -\sin \phi & \cos \phi & 0 \\ \frac{\sin 2\phi}{2} & -\frac{\sin 2\phi}{2} & 0 & 0 & 0 & \cos 2\phi \end{pmatrix} \quad (5)$$

The azimuth angle, ϕ , is defined as the angle from x direction in a counter-clockwise direction. The gradient from the least-squares FWI for an arbitrary parameter, p , can be expressed as

$$\nabla_p E = u_{\text{PDW}}^p(\mathbf{m}, t) [d(t) - u(\mathbf{m}, t)] \quad (6) \quad \text{with} \quad \rho \frac{\partial^2 u_{\text{PDW},i}^p}{\partial t^2} = \partial_j \sigma_{\text{PDW},ij}^p + \partial_j \left(\frac{\partial C_{ijkl}^{\text{rORT}}}{\partial p} \epsilon_{kl} \right), \quad (7)$$

where u_{PDW} is the partial derivative wavefields. The last term in eq. (7) is the virtual source from the model parameter perturbation. The gradient direction for any parameter can be obtained by the linear combination of 13 C_{IJ} parameters as follows (Oh and Alkhalifah, 2016):

$$\nabla_p E = \sum_{IJ} \frac{\partial C_{IJ}}{\partial p} (\nabla_{C_{IJ}} E) \quad (8)$$

Optimal parameterization in azimuthally rotated orthorhombic media

For elastic orthorhombic media, Oh and Alkhalifah (2016) suggested the new parameterization, which includes wave velocities (v_{p1} and v_{s1}), anisotropic parameters (ϵ_1 , η_1 and γ_1) for a vertically transversely isotropic (VTI) background and their deviation parameters (ϵ_D , η_D , γ_D and δ_3), as below:

$$\left(v_{p1} \quad v_{s1} \quad \epsilon_1 \quad \eta_1 \quad \gamma_1 \quad \delta_3 \quad \epsilon_D \quad \eta_D \quad \gamma_D \quad \phi \right) \quad (9)$$

This parameterization allows us to build up the subsurface models from isotropic to VTI to ORT media (Oh and Alkhalifah, 2016). Here, we add the azimuth angle of the symmetric axis (ϕ). We first examine the influence of the azimuth angle on the gradient directions. We assume a 4-layers model in Figure 1. The L_2 is our target, which is a strongly rORT layer. We use a homogeneous initial model (L_1) below the water. We assume a pressure source in the middle of the surface and hydrophones on all surface nodal points so that PP waves are dominant in the residual wavefields. Based on this model, we calculate the PP reflection patterns (R_p ; Oh and Alkhalifah, 2016) from the horizontal reflector (L_2) when the azimuth angle of the symmetric axis is equal 0 (Figures 2a). We also show the single-shot gradient direction on xy -plane at the depth of L_2 to examine the azimuthal variations of the gradient direction. As reflected in the radiation patterns, the gradients of the deviation parameters and δ_3 are azimuthally rotated by the background azimuth angle while the 5 other parameters are not (Figures 2b and 2c) because they have azimuth independent radiation patterns. This indicates that we can still build up the isotropic (v_{p1} and v_{s1}) and VTI parameters (ϵ_1 , η_1 and γ_1) with minimal influence of azimuth angle. Then, azimuth angle can be inverted with deviation parameters (ϵ_D , η_D , γ_D and δ_3).

Figure 1 The geometry of a 4-layers model: L_1 – very weak ORT, L_2 - strong rORT ($\phi=45^\circ$), L_3 – same as L_1 . The blue line indicates a source-receiver line. The red, green and yellow dots denote the source, receiver and perturbation point used in Figure 2. The red arrows indicates reflection ray path from the top of L_2 , which are assumed to calculate the reflection pattern in Figure 2a.

Figure 2 (a) The PP reflection patterns in terms of the opening angle (θ_o) in Figure 1 and gradient directions on the top of L_2 when (b) $\phi=0^\circ$ and (c) $\phi=45^\circ$. The dashed blue lines indicate the fast and slow horizontal axes of initial ORT layer. Notice that ϕ_{sr} is the azimuth angle of the source-receiver line in Figure 1, which is not related to the rotation angle of symmetric axis (ϕ).

Azimuth angle as a FWI parameter

As demonstrated, the new parameterization still has hierarchical features because the influences of azimuth angle are also well separated. However, for marine data (PP dominant), inverting for all 9 ORT parameters is not practical due to strong trade-offs among parameters as shown in Figure 2a. To reduce the trade-offs, Oh and Alkhalifah (2016) suggested hierarchically inverting for only v_{p1} , v_{s1} , ϵ_1 and ϵ_D , which we will refer to as the ideal parameters throughout the paper. Figure 3a shows the gradient direction for the ideal parameters on xy -plane depending on different background azimuth angles. Regardless of azimuth angle, we expect to build up two seismic velocities (v_{p1} and v_{s1}) and ϵ_1 . From this VTI model, the parameters, ϵ_D and ϕ , can be recovered as shown in Figure 3b. As we see in Figure 3a, the reflection pattern of the azimuth angle resembles ϵ_D (or δ_3) in Figure 2a, which means

that long offset data are also required to invert for the azimuth angle. However, the trade-offs between ε_D and ϕ are not strong as the gradient directions show. This is because the sensitivity kernel of ϕ has azimuthal polarity changes caused by the definition of angle that has positive values in a counter-clockwise direction but has negative values in the clockwise direction (Masmoudi et al., 2016).

(a)

(b)

Figure 3 (a) The gradient direction of the ideal parameters and the reflection patterns of the azimuth angle and (b) a schematic diagram illustrating the hierarchical parameter update strategy.

(a)

(b)

(c)

Figure 4 The true model: (a) ORT parameters (Table 1) and the azimuth angles (b and c).

Figure 5 The final inverted model of the ideal parameters and azimuth angle in Case-I.

To support our parameter update strategy in rORT media, we perform the multi-stage FWI (Figure 3b) for a more complex 3D model (Figure 4). The true model has a high velocity mud-filled channel structure (L_4) and fractured rORT layer (L_6), resembling a reservoir. The true ORT parameters are listed in Table 1. For the azimuth angle in L_6 , we assume two different structures (Figures 4b and 4c) to examine the potential trade-off of the azimuth angle with other parameters. For the initial models, we use smoothed v_{p1} , v_{s1} and ε_1 and the other 7 parameters are set to 0. We use 100 pressure sources and 160,000 hydrophones on the surface. In Figures 5 and 6, we note that the ideal parameters can be recovered regardless of true azimuth angle. The ϕ also looks reasonable thanks to the multi-stage FWI implementation, which mitigates the trade off with good ε_D . From these examples, we conclude that ϕ is invertible if we have good estimates of the background VTI, which can be obtained in the previous stages of our approach.

Figure 6 The final inverted model of the ideal parameters and azimuth angle in Case-II.

Table 1 The orthorhombic parameters for the true model in Figure 4a.

	v_{p1}	v_{s1}	ϵ_1	ϵ_2	δ_1	δ_2	δ_3	γ_1	γ_2	ρ	Ref.
Layer1	1.5	0	0	0	0	0	0	0	0	1	water
layer2	2.0	1.15	0	0	0	0	0	0	0	1	isotropic
layer3	2.2	1.27	0.12	0.12	0.04	0.04	0	0.08	0.08	1	VTI
layer4	3.4	1.63	0.1	0.1	0.03	0.03	0	0.05	0.05	1	VTI
layer5	2.6	1.5	0.06	0.06	0.03	0.03	0	0.02	0.02	1	VTI
layer6	2.8	1.7	0.07	0.15	0.03	0.08	0.02	0.04	0.08	1	ORT
layer7	3.5	2.15	0.05	0.05	0.03	0.03	0	0.02	0.02	1	VTI
layer8	4.5	2.9	0	0	0	0	0	0	0	1	isotropic

Conclusions

We investigate the feasibility of using FWI in azimuthally rotated elastic orthorhombic media. By analysing the radiation patterns and the gradient direction of each parameter, we are convinced that a parameterization in which we use deviation parameters provides us with the best functionality to perform FWI in azimuthally rotated orthorhombic media. The numerical examples show that a multi-stage FWI implementation reduces the trade-offs between parameters so that the azimuth angle can be estimated.

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