We B2 10

On the Resolution of Inversion for Orthorhombic Anisotropy

V.V. Kazei* (King Abdullah University of Science & Technology), T. Alkhalifah (King Abdullah University of Science & Technology)

Summary

We investigate the resolution of elastic anisotropic inversion for orthorhombic media with P-waves by remapping classic radiation patterns into the wavenumber domain. We show analytically that dynamic linearized inversion (linearized reverse-time migration and full-waveform inversion) for orthorhombic anisotropy based on longitudinal waves is fundamentally sensitive to ϵ parameters only and density, in which the perturbing effects can be represented by particular anisotropy configuration. Singular value decomposition of spectral sensitivities allows us to provide estimates of the number of parameters one could invert in specific acquisition settings, and with certain parametrization. In most acquisition scenarios, a hierarchical parameterization based on the P , S , and SS -wave velocities, along with dimensionless parameters that describe the anisotropy

as velocity ratio in the radial and azimuthal directions, minimizes the tradeoff and increases the sensitivity of the data to velocity

compared to the standard (stiffness, density) parametrization. These features yield more robust velocity estimation, by focusing the inversion on a subset of invertible parameters.

Introduction

Reservoir delineation and monitoring are among the major objectives of modern seismic imaging. The resolution capabilities of most common inversion implementations (full-waveform inversion, linear least-squares model optimizations) are defined by the sensitivity of scattered wavefield to changes in the reservoir properties (Figure 1). Reservoir properties can be well approximated by orthorhombic anisotropy at the scales of common exploration seismic wavelengths. Specifically, it describes a system of orthogonal fractures or layering (vertical) at the microscale level within the reservoir. *P*-wave velocity analysis for orthorhombic media (Tsvankin, 1997) defines the resolution limits of most ray-based methods. Recently, the interest in the dynamic components of the wavefield, despite the additional acquisition and computational requirements, has grown as we seek the higher resolution potential of wavefields. In multiparameter inversion the major challenge is to minimize the crosstalk between parameters. Resolution can be investigated visually by using reflection based radiation patterns (Gholami et al., 2013; Alkhalifah and Plessix, 2014; Oh and Alkhalifah, 2016) and quantitatively by using singular value decomposition analysis in the wavenumber domain (Podgornova et al., 2015). We generalize the latter to 3-D inversion in orthorhombic anisotropy.

Theory

The Born approximation for the scattered wavefield $\mathbf{U}$ for a source located at $\mathbf{x}_s$ and a receiver located at $\mathbf{x}_g$ for a perturbation in density $\delta\rho(\mathbf{x})$ and the stiffness tensor $\delta c_{ijkl}(\mathbf{x})$ can be expressed as follows (Shaw and Sen, 2004, e.g.):

$$\delta U_i(\mathbf{x}_s, \mathbf{x}_g, \omega) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (\delta\rho(\mathbf{x}) u_k^0 \omega^2 - \delta c_{kj pq}(\mathbf{x}) \frac{\partial u_p^0}{\partial x_q} \frac{\partial}{\partial x_j}) G_{ik}(\mathbf{x}_g, \mathbf{x}) d\mathbf{x}. \quad (1)$$

If we assume that both scattered G_{ik} and incident u_k^0 waves are longitudinal plane waves in an isotropic background with velocity V_p (weak anisotropy assumption), then the *P*-wave polarized part of the displacement is proportional to a linear combination of Fourier transforms of the perturbed parameters:

$$\delta U_{PP}(\mathbf{s}, \mathbf{g}, \omega) \propto \mathbf{s} \cdot \mathbf{g} \delta \hat{\rho}(\mathbf{K}) + s_i s_j \delta \hat{c}_{ijkl}(\mathbf{K}) g_k g_l, \quad \mathbf{K} = \frac{\omega}{V_p} (\mathbf{s} + \mathbf{g}). \quad (2)$$

Here, the $\mathbf{s}$ and $\mathbf{g}$ vectors point towards the source and receiver, respectively. For the case of a point scatterer $\delta \hat{\rho}(\mathbf{K}) = \text{const}$, $\delta \hat{c}_{ijkl}(\mathbf{K}) = \text{const}$, equation 2 provides the radiation pattern or scattering function of these scatterers (Eaton and Stewart, 1994; de Hoop et al., 1999; Shaw and Sen, 2004). Under the plane-wave and Born applicability assumptions, equation 2 generalizes the point scatterer radiation patterns to arbitrary perturbations. We investigate the potentially inverted scatterer at certain spatial resolution defined by wavenumber $\mathbf{K}$. This study is designed to handle arbitrary 3D acquisition (finite aperture, azimuth coverage), but we restrict our analysis to the vertical wavenumbers only, which leads to the following relations (the vertical axis is labeled 3, while the azimuth, ϕ , is measured from the x_1 axis):

$$s_1 = -g_1 = \sqrt{1 - \frac{k_z^2}{4}} \cos \phi, \quad s_2 = -g_2 = \sqrt{1 - \frac{k_z^2}{4}} \sin \phi, \quad s_3 = g_3 = k_z/2, \quad (3)$$

Substituting equation 3 into equation 2, we obtain reflection-based radiation patterns (Oh and Alkhalifah, 2016) for horizontal reflectors or equivalently the spectral sensitivities of the vertical wavenumbers (Kazei et al., 2013; Podgornova et al., 2015). For vertical wavenumbers/weak horizontal reflectors *P*-waves are insensitive to the following perturbations of elastic parameters:

$$2\mathbf{C}_{12} - \mathbf{C}_{66}; \quad 2\mathbf{C}_{13} + \mathbf{C}_{55}; \quad 2\mathbf{C}_{23} + \mathbf{C}_{44}; \quad \mathbf{C}_{11} + 2\mathbf{C}_{12} + \mathbf{C}_{22} - \mathbf{C}_{33} + \vec{\rho}. \quad (4)$$

Here, $\mathbf{C}_{ij}$ and $\vec{\rho}$ denote unit perturbations of the stiffness matrix coefficient C_{ij} , and the background *P*-wave slowness perturbation density. The total number of parameters needed to describe the elastic orthorhombic medium is ten. Since equation 4 includes four independent linear combinations, only *six* independent parameters can be inverted from *P* – *P* waves in the linearized anisotropic orthorhombic approximation.

Figure 1 a) A diagram of an orthorhombic reservoir (Tsvankin, 1997) in an isotropic background within the context of a potential acquisition scenario. b). The scattering angle range as a function of vertical model wavenumber. For band limited data acquired with limited aperture in which the maximum scattering angle for a given model point, θ_{max} , the area in green shows a consistent range of model wavenumbers achievable to the depth of that point. On the other hand, lower or higher resolution given in blue is provided by specific scattering angles (the effective range of angles of illumination for these wavenumbers is less than the one defined by aperture).

Spectral sensitivities

For arbitrary perturbation of parameters, the scattered wavefield can be expressed in the wavenumber domain:

$$\delta U_{PP}(k_z, \phi, \omega(k_z, K_z)) \propto \mathbf{A}^T(k_z, \phi) (\delta \rho(K_z), \delta \mathbf{c}(K_z))^T, \text{ or } \delta U_{PP}(K_z) \propto \mathbb{A} \delta \mathbf{m}(K_z). \quad (5)$$

Vector $\mathbf{A}(k_z, \phi)$ here is essentially a set of reflection based radiation patterns remapped into the normalized wavenumber domain. Thus, element $A_1(k_z, \phi)$ (Figure 2(a)) is the radiation pattern of density, while the radiation patterns for the remaining 21 elements, corresponding to the elastic constants, are shown in Figure 2(b), which is ordered to correspond to the Voigt notation elastic matrix (Figure 2(c)). Similarities of certain radiation patterns in Figure 2(b) (for example C_{12} and C_{66}) confirm the presence of Null-space (4). First, sticking with the elastic coefficient, we seek to understand the tradeoff and extract the linear combinations that expose such tradeoff. We compare such analysis on the elastic coefficients with a parameterization driven by a hierarchical relation (Oh and Alkhalifah, 2016). This hierarchical parameterization divides the ten required parameters to describe the orthorhombic model into three groups. Those that represent an isotropic simplification of the model given by the three non-zero parameters – V_p, V_s, ρ . An additional three parameters describing the lower symmetry VTI version – $\epsilon_1, \eta_1, \gamma_1$ (setting these parameters to zero renders the medium isotropic). Finally, four more parameters – $\epsilon_d, \eta_d, \gamma_d, \delta_3$ describing the azimuthal variation (setting these parameters to zero renders the medium VTI). Radiation patterns for any other parametrization can be obtained through the chain rule (Oh and Alkhalifah, 2016, e.g.). Thus, the partial derivatives of the new parameters with respect to the stiffness matrix coefficients are shown in graphic format in Figure 3 (a), which leads to the spectral sensitivities presented in Figure 3(b). Similarities of sensitivities for ρ & ϵ_1 , V_s & η_1 , η_d & γ_d and zero sensitivity of γ_1 demonstrate the existence of a *visible* null-space in this parameterization, which is reasonably close to the inherent null-space mentioned earlier. Changes in the spectral sensitivity of density (compare Figure 2(a) and Figure 3(b)) is the result of the reparameterization.

Resolved parameters

To perform a quantitative analysis of the possible coupling of parameters, we compute the singular value decomposition of matrix $\mathbb{A}$. First, we assume full azimuth/full offset - perfect illumination in both parameterizations ($k_z \in [0, 2]$ and $\phi \in [0, \pi]$). The singular values in Figure 4(a) show that in this case of perfect illumination C_{ij}, ρ parametrization result in better resolution and more stability in the inversion of six parameters, which makes it preferable. However, the model resolution matrix (Menke, 2012) for the conventional C_{ij}, ρ parameterization (Figure 4(b)) shows strong crosstalk between the parameters.

Figure 2 3-D spectral sensitivities for density – a) and stiffness matrix elements – b). Elements of stiffness matrix in Voigt notation – c). Orthorhombic media can be represented by elements in red cells, monoclinic with horizontal symmetry plane by red and blue cells. Parameters in white cells do not scatter $P-P$ waves and thus can not be inverted for weak anisotropic reflectors. Orthorhombic parameters show symmetric w.r.t. 90° scattering, while the rest of monoclinic parameters – antisymmetric. Poisson ratio is assumed to be equal to 0.25 throughout the paper.

Figure 3 The partial derivatives of the C_{ij} parameters (same scheme as in Figure 2(c) is used) in the C_{ij}, ρ parameterization with respect to the Oh and Alkhalifah (2016) parameters - a). The spectral sensitivities of the new parameters - b).

Figure 4 Singular values of the matrix $\mathbb{A}$ - a) for the C_{ij}, ρ – red and the hierarchical – blue parameterizations. Resolution matrices (keeping singular values above 0.1 of the maximum) for full offset/full azimuth data for C_{ij}, ρ parameterization - b) and the hierarchical parameterization - c).

Thus, the inversion needs four additional constraints to eliminate the null-space (equation 4) and to estimate six parameters properly. The crosstalk between V_p and other parameters of the Oh and Alkhalifah (2016) parameterization is minimized (Figure 4(c)), which simplifies inversion for velocity. It is also clear that $V_p, V_s, \epsilon_1, \epsilon_d$ show high contribution to their own estimates (high diagonal values) and can be inverted if the other parameters are kept fixed. Utilizing the effective illumination angle concept in Figure 1(b), we provide an estimate of the number of invertible parameters, given the offset and azimuth coverage in conventional (Figure 5(a)) and hierarchical (Figure 5(b)) parameterizations.

Figure 5 The number of parameters that can be resolved based on the maximum effective opening angle at a model point for the C_{ij}, ρ parameterization -a) and for the hierarchical parameterization -b). Only the singular values above 0.1 of the maximum singular value are used in the analysis.

Discussion & Conclusions

We demonstrate that a linearized dynamic inversion of P waves for anisotropic orthorhombic medium is fundamentally sensitive to six parameters only. Vertical variations of density show the same scattering as a certain combination of elastic constants. We provide estimates of the number of invertible anisotropic orthorhombic parameters, given the acquisition setup, by using singular value decomposition of the spectral sensitivities. While the hierarchical parameterization reduces the tradeoffs between parameters, it also reduces the number of parameters recoverable considering realistic situations. If the goal of the inversion is to obtain a velocity model that explains most of the observed data then the hierarchical parameterization (Oh and Alkhalifah, 2016) is optimal. If we need to extract more information about the anisotropy of a medium, considering optimal conditions and utilization of a Hessian approximation capable of treating the tradeoff, and with a good initial model (such as that constructed using the hierarchical parameterization) then the classic C_{ij}, ρ parameterization is likely to help recover more anisotropic features of the reservoir at later stages.

References

- Alkhalifah, T. and Plessix, R.E. [2014] A recipe for practical full-waveform inversion in anisotropic media: An analytical parameter resolution study. *Geophysics*, **79**(3), R91–R101.
- Eaton, D.W.S. and Stewart, R.R. [1994] Migration/inversion for transversely isotropic elastic media. *Geophysical Journal International*, **119**(2), 667–683.
- Gholami, Y., Brossier, R., Operto, S., Ribodetti, A. and Virieux, J. [2013] Which parameterization is suitable for acoustic vertical transverse isotropic full waveform inversion? Part 1: Sensitivity and trade-off analysis. *Geophysics*, **78**(2), R81–R105.
- de Hoop, M.V., Spencer, C. and Burridge, R. [1999] The resolving power of seismic amplitude data: An anisotropic inversion/migration approach. *Geophysics*, **64**(3), 852–873.
- Kazei, V., Troyan, V., Kashtan, B. and Mulder, W. [2013] On the role of reflections, refractions and diving waves in full-waveform inversion. *Geophysical Prospecting*, **61**(6), 1252–1263.
- Köhn, D., Hellwig, O., De Nil, D. and Rabbel, W. [2015] Waveform inversion in triclinic anisotropic media – a resolution study. *Geophysical Journal International*, **201**(3), 1642–1656.
- Menke, W. [2012] *Geophysical data analysis: Discrete inverse theory*, 45. Academic press.
- Oh, J.W. and Alkhalifah, T. [2016] Elastic orthorhombic anisotropic parameter inversion: An analysis of parameterization. *GEOPHYSICS*, **81**(6), C279–C293.
- Podgornova, O., Leaney, S., Liang, L. et al. [2015] Analysis of resolution limits of VTI anisotropy with full waveform inversion. In: *2015 SEG Annual Meeting*. Society of Exploration Geophysicists.
- Shaw, R.K. and Sen, M.K. [2004] Born integral, stationary phase and linearized reflection coefficients in weak anisotropic media. *Geophysical Journal International*, **158**(1), 225–238.
- Tsvankin, I. [1997] Anisotropic parameters and P-wave velocity for orthorhombic media. *Geophysics*, **62**(4), 1292–1309.