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Frequency Domain Multi-parameter Full Waveform Inversion for Acoustic VTI Media

R. Djebbi* (KAUST), T. Alkhalifah (KAUST)

Summary

Multi-parameter full waveform inversion (FWI) for transversely isotropic (TI) media with vertical axis of symmetry (VTI) suffers from the trade-off between the parameters. The trade-off results in the leakage of one parameter's update into the other during the inversion. It affects the accuracy and convergence of the inversion. The sensitivity analyses suggested a parameterisation using the horizontal velocity v_h , epsilon and eta to reduce the trade-off for surface recorded seismic data. We test the (v_h , epsilon, eta) parameterisation for acoustic VTI media using a scattering integral (SI) based inversion. The data is modeled in frequency domain and the model is updated using a preconditioned conjugate gradient method. We applied the method to the VTI Marmousi II model and in the inversion, we keep eta parameter fixed as the background initial model and we invert simultaneously for both v_h and epsilon. The results show the suitability of the parameterisation for multi-parameter VTI acoustic inversion as well as the accuracy of the inversion approach.

Introduction

Full waveform inversion (FWI) aims to invert the full recorded data content to recover an accurate Earth model. However, most of the existing inversion schemes consider only an acoustic isotropic Earth in order to reduce the computational cost. Additionally, when considering more than one parameter, the ambiguity or trade-off between the different parameters (Gholami et al., 2013; Alkhalifah and Plessix, 2014; Alkhalifah, 2016; Djebbi et al., 2016) affects the accuracy of the inverted model. For transversely isotropic (TI) media with vertical axis of symmetry (VTI), Gholami et al. (2013) and Alkhalifah and Plessix (2014) analyzed the trade-off based on the radiation patterns for several parameterizations. Alkhalifah (2016) studied the short and long wavelength influences of perturbations in the parameters. A qualitative study has been conducted also by Djebbi et al. (2016) where they derived the sensitivity kernels for anisotropy parameter perturbation and analyzed the trade off, not only at the scatterer location, but also along the wave-path.

The VTI wave equation parameterization using the normal move-out velocity v_{nmo} , η and δ parameters is adapted for an inversion that includes reflections and diving waves (Plessix and Cao, 2011; Alkhalifah and Plessix, 2014). For this parameterization, the δ parameter is considered as a secondary parameter, which is used to fit the inaccuracy in the amplitude caused by the acoustic assumption. However, the trade-off between v_{nmo} and η parameter for large scattering angles (diving waves) is significant and results in the leakage of η parameter into v_{nmo} during the inversion. Another widely used parameterization is based on the vertical velocity v_v , δ , and ϵ parameters (Vigh et al., 2014). It suffers from the same type of trade-off for large scattering angles between v_v and ϵ parameter. Alkhalifah (2016) suggested using the horizontal velocity v_h , η and ϵ . This parameterization is considered optimal for conventional surface seismic experiments with the least trade-off. Alkhalifah and Guitton (2016) compared the two last mentioned parameterizations for elastic FWI. The authors confirmed that using v_h , η and ϵ parameterization we can achieve a better result. Wu and Alkhalifah (2016) used the same parameterization for reflection waveform inversion (RWI). The authors assumed $\epsilon = \eta$ to reduce the number of variables and used ϵ as the perturbation parameter. In this abstract, we perform an acoustic VTI inversion using v_h , η and ϵ parameterization. The inversion is tested on the Marmousi II model and is performed in frequency domain using a scattering integral approach. We show that we can achieve reasonable inversion results for both v_h and ϵ .

Theory

Conventionally, Full Waveform Inversion (FWI) is implemented using the adjoint state method. We consider a different approach where the scattering integral (SI) method is used to explicitly compute the gradient. The gradient is given by the product of the complex conjugate Fréchet derivatives matrix with the residual vector:

$$p_1 = \frac{\partial E}{\partial \mathbf{m}} = -\mathbf{K}^T \Delta \mathbf{u}(\mathbf{x}_s, \mathbf{x}_r, \omega), \quad (1)$$

where $\Delta \mathbf{u}(\mathbf{x}_s, \mathbf{x}_r, \omega)$ is the data residual at the receivers positions, $\mathbf{K}$ is the sensitivity kernels matrix, and T is the conjugate transpose operator. The difficulty associated with the SI approach is the large memory required to save the Fréchet derivative (sensitivity) matrix. Liu et al. (2015) proposed to save the Green's functions into memory only and use matrix decomposition to compute the gradient.

The Born sensitivity kernels for VTI media

For a VTI medium parameterized using v_h , ϵ and η , the structure of the sensitivity matrix (Liu et al., 2015) is given as:

$$\mathbf{K} = [\mathbf{K}_{v_h} \quad \mathbf{K}_\epsilon \quad \mathbf{K}_\eta]. \quad (2)$$

The Fréchet derivatives matrix has a total size of $3 \times (n \times m)$, where n is the residual vector size and m is the model size. Each row of the three sub-matrices, given in equation (2), is the sensitivity kernel for v_h , ϵ and η , respectively, for a specific source-receiver pair.

We combine the VTI system of equations into a single 4th order equation. We perturb the three Earth

model parameters to obtain the corresponding single frequency sensitivity kernels:

$$\begin{cases} K^{(v_h)}(\mathbf{x}_r, \mathbf{x}_s, \mathbf{x}, \omega) &= \frac{2\omega^2}{v_0^3} u_0(\mathbf{x}, \mathbf{x}_s, \omega) G_0(\mathbf{x}, \mathbf{x}_r, \omega); \\ K^{(\epsilon)}(\mathbf{x}_r, \mathbf{x}_s, \mathbf{x}, \omega) &= -(u_0(\mathbf{x}, \mathbf{x}_s, \omega) \partial_{zz} G_0(\mathbf{x}, \mathbf{x}_r, \omega) + \partial_{zz} u_0(\mathbf{x}, \mathbf{x}_s, \omega) G_0(\mathbf{x}, \mathbf{x}_r, \omega)); \\ K^{(\eta)}(\mathbf{x}_r, \mathbf{x}_s, \mathbf{x}, \omega) &= -\frac{v_0^2}{\omega^2} (\partial_{xx} u_0(\mathbf{x}, \mathbf{x}_s, \omega) \partial_{zz} G_0(\mathbf{x}, \mathbf{x}_r, \omega) + \partial_{zz} u_0(\mathbf{x}, \mathbf{x}_s, \omega) \partial_{xx} G_0(\mathbf{x}, \mathbf{x}_r, \omega)); \end{cases} \quad (3)$$

where $u_0(\mathbf{x}, \mathbf{x}_s, \omega)$ and $G_0(\mathbf{x}, \mathbf{x}_r, \omega)$ are the source wavefield and the receiver Green's function, respectively.

Update direction using the sensitivity kernels

Preconditioned Conjugate Gradient (PCG) method is used to update the model. The preconditioner is the diagonal of the approximate Hessian and the update direction is given as:

$$\mathbf{p}_2 = \frac{1}{\text{diag}(\mathbf{H}_a)} \mathbf{p}_1 \quad (4)$$

The approximate Hessian matrix $\mathbf{H}_a$ is given by the product of the Fréchet derivatives matrix and its conjugate transpose: $\mathbf{H}_a = \mathbf{K}^T \mathbf{K}$. The step size is estimated for each parameter using a second order approximation of the objective function $E_1(\alpha) = E(\mathbf{m} + \alpha^T \mathbf{p}_2)$. For a VTI medium, the vector $\mathbf{m}$ is composed of three vectors corresponding to: v_h , ϵ and η . The vector $\mathbf{p}_2$ is the update vector. Finally, α is the step size vector given as: $\alpha = [\alpha_{v_h} \quad \alpha_\epsilon \quad \alpha_\eta]^T$. The step size vector elements are obtained by solving the 3×3 system:

$$\begin{bmatrix} \mathbf{p}_{2v_h}^T (\mathbf{K}_{v_h}^T \mathbf{K}_{v_h}) \mathbf{p}_{2v_h} & \mathbf{p}_{2v_h}^T (\mathbf{K}_{v_h}^T \mathbf{K}_\epsilon) \mathbf{p}_{2\epsilon} & \mathbf{p}_{2v_h}^T (\mathbf{K}_{v_h}^T \mathbf{K}_\eta) \mathbf{p}_{2\eta} \\ \mathbf{p}_{2\epsilon}^T (\mathbf{K}_\epsilon^T \mathbf{K}_{v_h}) \mathbf{p}_{2v_h} & \mathbf{p}_{2\epsilon}^T (\mathbf{K}_\epsilon^T \mathbf{K}_\epsilon) \mathbf{p}_{2\epsilon} & \mathbf{p}_{2\epsilon}^T (\mathbf{K}_\epsilon^T \mathbf{K}_\eta) \mathbf{p}_{2\eta} \\ \mathbf{p}_{2\eta}^T (\mathbf{K}_\eta^T \mathbf{K}_{v_h}) \mathbf{p}_{2v_h} & \mathbf{p}_{2\eta}^T (\mathbf{K}_\eta^T \mathbf{K}_\epsilon) \mathbf{p}_{2\epsilon} & \mathbf{p}_{2\eta}^T (\mathbf{K}_\eta^T \mathbf{K}_\eta) \mathbf{p}_{2\eta} \end{bmatrix} \cdot \begin{bmatrix} \alpha_{v_h} \\ \alpha_\epsilon \\ \alpha_\eta \end{bmatrix} = \begin{bmatrix} -\mathbf{p}_{1v_h}^T \mathbf{p}_{2v_h} \\ -\mathbf{p}_{1\epsilon}^T \mathbf{p}_{2\epsilon} \\ -\mathbf{p}_{1\eta}^T \mathbf{p}_{2\eta} \end{bmatrix} \quad (5)$$

The resulting 3 step sizes take into account a perturbation in the three parameters. Therefore, the trade-off is considered using this approach. For the adjoint state based FWI, the elements of the matrix in the left hand side require additional modeling steps (needed to approximate the matrix elements by finite difference). For the scattering integral (SI) approach only vector-matrix multiplications are required. As a result, no additional modeling is required and the main computation occurs during the construction of the sensitivity matrix and its decomposition. The computation is minor in the following inversion process.

Numerical results

In Figure 1, the first row shows the VTI Marmousi II model. The original Marmousi II velocity model and the density are used to derive ϵ and δ Thomsen parameters. Then, those parameters are used to compute the horizontal velocity v_h , ϵ and η parameters. The initial model, shown in the second row of Figure 1, is obtained by considering a zero δ . Actually, long-wavelength δ information is usually obtained from well information and can not be constrained using surface seismic records. Then, we smooth the vertical velocity and ϵ using a triangular smoother with a 1500 m radius. The resulting models are comparable to the models used by Alkhalifah and Guitton (2016), where v_h and v_{nmo} are smoothed instead. The modeling is performed in the frequency domain using an acoustic VTI Helmholtz solver. We consider a multi-scale inversion approach where 4 frequency bands are considered. Table 1 shows the inversion parameters for each frequency band.

Frequency Band	f_{min} (Hz)	f_{max} (Hz)	Δf (Hz)	Number of iterations
1	2.0	4.0	0.25	40
2	3.5	6.0	0.5	40
3	5.5	9.0	0.5	40
4	8.5	12.0	0.5	40

Table 1 Inversion parameters.

As the sensitivity of the recorded data to η is minor (Alkhalifah, 2016), it is not updated during the inversion and we keep it fixed as the initial value. We perform a simultaneous inversion of both v_h

Figure 1 Marmousi II VTI model: (a-c) The exact model, (d-f) the initial model.

and ε parameter. v_h is the main parameter that we aim to invert for and ε plays the secondary role of fitting the amplitude. ε absorbs the inaccuracies in the inversion of short wavelength due to the acoustic approximation. Figure 2 and 3 show the inverted model as well as slices at 7 and 12 km. The horizontal velocity is well inverted and the main features are recovered. We conclude that the acoustic frequency domain multi-parameter inversion using a parameterization based on v_h , ε and η can provide highly resolved and accurate horizontal velocity.

Furthermore, the inverted ε parameter is reasonable and the short wavelength features are well recovered for parts of the model deeper than 1500 m. In fact, the ε parameter is only sensitive to small scattering angles, which results in short wavelength update. From the ε slices, we observe that the shallow part of the initial ε model is far from the true model. Such a difference is not recoverable from the short wavelength update. For the deeper parts, the initial ε is closer to the exact model. As a result, the short wavelength ε update recovers the bumps in the model as seen by Figure 3(c-d). To recover a good ε model, the long wavelength features must exist in the initial model. Also, high frequency information should be used to invert for more accurate short wavelength features. Figure 4 shows the data misfit history for the 4 frequency bands. The misfit is the time domain data over the whole spectrum. The misfit is largely reduced, which shows the convergence of the inversion.

Figure 2 Inverted Marmousi II model: (a) v_h and (b) ε .

Conclusions

We applied a frequency domain FWI using the scattering integral approach to invert VTI medium. The VTI wave equation is parameterized using the horizontal velocity v_h , ε and η anisotropy parameters for reduced trade-off between different parameters. We used a preconditioned conjugate gradient method to update the model. The step size is estimated through a second order approximation of the objective function. It takes into account a perturbation in the three parameters and can take care of the trade-off with no extra modeling cost. The η parameter is kept fixed during the inversion and only v_h and ε are updated. We showed that the method results in accurate inverted models. Finally, we confirmed that the parameterization is suitable for surface recorded seismic data.

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Figure 3 Inverted Marmousi II model slices: (a,b) v_h slices at 7 and 12 km, (c,d) ϵ parameter slices at 7 and 12 km.

Figure 4 Data misfit history.

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