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Centered Differential Waveform Inversion with Minimum Support Regularization

V.V. Kazei* (King Abdullah University of Science & Technology), T. Alkhalifah (King Abdullah University of Science & Technology)

Summary

Time-lapse full-waveform inversion has two major challenges. The first one is the reconstruction of a reference model (baseline model for most of approaches). The second is inversion for the time-lapse changes in the parameters. Common model approach is utilizing the information contained in all available data sets to build a better reference model for time lapse inversion. Differential (Double-difference) waveform inversion allows to reduce the artifacts introduced into estimates of time-lapse parameter changes by imperfect inversion for the baseline-reference model. We propose centered differential waveform inversion (CDWI) which combines these two approaches in order to benefit from both of their features. We apply minimum support regularization commonly used with electromagnetic methods of geophysical exploration. We test the CDWI method on synthetic dataset with random noise and show that, with Minimum support regularization, it provides better resolution of velocity changes than with total variation and Tikhonov regularizations in time-lapse full-waveform inversion.

Introduction

Reservoir delineation and monitoring are among the major objectives of seismic exploration. Time-lapse full-waveform inversion is focused on providing a high-resolution solution to the second of these objectives, yet it heavily relies on an estimate of the reference (typically referred to as baseline) model properties. Watanabe et al. (2004) apply differential full-waveform inversion to reduce the dependence on the precision of the inversion results for the baseline model (Figure 1(b)), Denli et al. (2009) recast the approach for elastic media as Double-difference waveform inversion (DDWI). Raknes and Arntsen (2014) and Asnaashari et al. (2015) utilize localized regularization penalty terms to focus time-lapse inversion. Maharramov et al. (2016) propose joint full-waveform inversion (Figure 1) with total variation (TV) regularization applied to the difference of simultaneously inverted baseline and monitor velocity models, and Alemie and Sacchi (2016) reparametrize the joint inversion in model space to focusing the inversion on time-lapse changes.

While the joint inversion for baseline and monitor models is generally more robust as it allows us to tie the two inverted models, it is also computationally more intensive than the sequential inversion or DDWI as the common velocity structure of baseline and monitor models is inverted twice. This leads to a more efficient common model idea (Hicks et al., 2016). We propose the centered double-difference waveform inversion (CDWI), which combines the benefits of all the approaches mentioned above. It takes all the data available into account when inverting for the reference model and then utilizes the DDWI scheme to invert for the time-lapse changes in velocity (Figure 1(c)). Total variation regularization applied to the time-lapse changes sharpens the differences in the joint time-lapse waveform inversion. Alternatively, we apply a minimum support (MS) regularization (Portniaguine and Zhdanov, 1999). This regularization is commonly used in electromagnetic and gravitational exploration methods to improve the resolution of the inversion beyond the limits of TV regularization. We test if the same applies in time-lapse waveform inversion.

Centered differential waveform inversion

Conventionally in double-difference waveform inversion the baseline model is inverted first and then the monitor model is constructed based on the double-difference misfit functional (Denli et al., 2009). Inversion for the baseline model is a complicated nonlinear problem and normally suffers from insufficient signal to noise ratio. We suggest to invert all data sets (baseline and monitor) simultaneously to build the background model, which is a particular and arguably the most efficient way to use common information for all available surveys (Oghenekohwo et al., 2015; Hicks et al., 2016). In case of data being measured at the same locations, the latter can be easily done by reparametrizing the observed data:

$$d_{avg} = \frac{d_{mon} + d_{bas}}{2}, \quad d_{diff} = \frac{d_{mon} - d_{bas}}{2}, \quad (1)$$

d_{mon} and d_{bas} are the monitor and the baseline data. First we invert for the average model $\mathbf{m}_{avg-inv}$ using standard FWI to fit the average data d_{avg} . Note that the model inverted is not the same as the average of inverted models with this approach because of the presence of noise in real data at low frequencies. Then we perform differential waveform inversion to estimate the time-lapse changes. The misfit functional for the CDWI is the following:

$$J_{CDWI}(\mathbf{m}_{avg-inv} + \mathbf{m}) = \|(d(\mathbf{m}_{avg-inv} + \mathbf{m}) - d(\mathbf{m}_{avg-inv})) - d_{diff}\|^2, \quad (2)$$

where $d(\mathbf{m}_0 + \mathbf{m})$ is data for the velocity model $\mathbf{m}_0$ perturbed with velocity $\mathbf{m}$. The CDWI misfit functional (equation 2) is very similar to the misfit functional of DDWI:

$$J_{DDWI}(\mathbf{m}_{bas-inv} + \mathbf{m}) = \|(d(\mathbf{m}_{bas-inv} + \mathbf{m}) - d(\mathbf{m}_{bas-inv})) - 2d_{diff}\|^2, \quad (3)$$

where $\mathbf{m}_{bas-inv}$ is the inverted baseline model. The difference between *CDWI* and *DDWI* approaches is that *CDWI* allows us to improve signal to noise ratio in the reference model (approximately, as the inversion is nonlinear) by $\sqrt{n}$, where n is the number of surveys recorded, as all the data sets recorded are used for the reference model. This is especially rewarding for the monitored regions with several data sets and at low frequencies which normally show lower signal to noise ratios.

Figure 1 Modern time-lapse waveform inversion strategies. Joint time lapse full-waveform inversion – a), Double-difference waveform inversion – b), centered difference waveform inversion – c).

Minimum support & Sobolev space norm regularizations

Regularization can boost the resolution of the time-lapse inversion (Maharramov et al., 2016; Asnaashari et al., 2015) by imposing constraints on the difference in the model parameters. We compare the often used Tikhonov regularization (laplacian is used as the weighting matrix) and TV regularization, with a Minimum Support (MS) regularization (Portniaguine and Zhdanov, 1999), not yet used in full-waveform seismic inversion. All these regularizations can be introduced into FWI in a standard way through stabilizing functionals:

$$J_{REG-FWI} = J_{FWI} + \frac{1}{2} \alpha_{reg} J_{reg}, \quad \frac{\partial J_{REG-FWI}}{\partial \mathbf{m}}(\mathbf{x}) = \frac{\partial J_{FWI}}{\partial \mathbf{m}}(\mathbf{x}) + \frac{1}{2} \alpha_{reg} \frac{\partial J_{reg}}{\partial \mathbf{m}}(\mathbf{x}), \quad (4)$$

where J_{reg} is a stabilizing functional. Tikhonov and total variation stabilizing functionals are essentially norms in the Sobolev spaces (W_2^1 and W_1^1 , respectively), which we naturally generalize to arbitrary W_p^1 :

$$J_{W_p^1} = \|m\|_{W_p^1} = \int_V (\nabla m \cdot \nabla m + \varepsilon)^{p/2} d\mathbf{x}, \quad J_{Tikh} \equiv J_{W_2^1}, \quad J_{TV} \equiv J_{W_1^1}, \quad \text{where } \nabla m(\mathbf{x}) = \frac{\partial m}{\partial \mathbf{x}}(\mathbf{x}), \varepsilon > 0. \quad (5)$$

The volume of support of a spatially distributed parameter $m(\mathbf{x})$ (area of non-zero values of m in 2-D) can be approximated analytically; $J_{MS} = \int_V \frac{m^2}{m^2 + \varepsilon} d\mathbf{x}$, which is converging to the volume of support of m when ε goes to zero. The gradients of the stabilizing functionals are the following:

$$\frac{\partial J_{W_p^1}}{\partial m}(\mathbf{x}) = -p \nabla \cdot \frac{\nabla m}{(\nabla m \cdot \nabla m + \varepsilon)^{p/2}}, \quad \frac{\partial J_{MS}}{\partial m}(\mathbf{x}) = -2\varepsilon \frac{m}{(m^2 + \varepsilon)^2}. \quad (6)$$

Values of ε in the stabilizing functionals (equation 6) are crucial to make the functional $J_{FWI-REG}$ differentiable for the TV and minimum support regularizations, but using W_p^1 for $p > 1$ allows us to relax the dependence on ε as the stabilizing functional becomes differentiable at $\varepsilon = 0$ (equation 6). MS regularization can not be improved as gracefully, but it is expected to produce sharper results provided that the smoothing parameter ε is chosen appropriately.

Example

We test the proposed method with a time-lapse change introduced into the Marmousi II model (Figure 2). A dense survey with 53 sources and 419 receivers spread across the top of the model is used as "observed data" (an absorbing boundary is used instead of a free surface). We restrict the minimum frequency in the data to be 3 Hz and maximum offset to 10 km; random noise level is assumed to be 30% at frequencies below 4 Hz and 20% for frequencies 4 to 5 Hz and 10% for frequencies above 5 Hz to make data more realistic. Single frequencies from 3 to 8 Hz are inverted sequentially with each subsequent frequency 20% higher than the previous. In the first stage, we perform inversion for the background model (baseline for DDWI, average model for CDWI). To mitigate cycle-skipping at the lowest frequency of 3Hz, the

maximum offset used in inversion is gradually increased starting from 4km and the regularization term is gradually decreased starting with 10% of the data misfit and ending with 5%. The $W_{1,2}^1$ norm is used as the stabilizing functional when inverting for the average model and baseline models. This naturally blends features of Tikhonov and TV regularizations to allow for smooth updates prior to the sharp ones, and thus, establish better spatial model wavenumber continuity (Alkhalifah, 2016; Kazei et al., 2016). Figure 3 shows the baseline model estimate after multiscale inversion with 20 iterations per frequency and offset at 3Hz and 20 iterations of L-BFGS per frequency, otherwise. At the low frequencies the noise is strongly suppressed by the very dense acquisition – so the main features are well recovered. Despite the excessive number of total iterations, the model is not completely resolved at depth since there is noise in the data, which leads to complications for time-lapse inversion. The errors in estimate of velocity are of order of the anomaly in depth, which means that the time-lapse inversion results with such a reference model would not be correct. After that we apply an inversion for the average model (Figure 3(b)), which retrieves the features of the Marmousi model better.

Figure 2 The FWI setup: For baseline data the Marmousi II model is used. The monitor model includes low velocity anomalies. A linearly increasing velocity with depth is used as a starting model for full-waveform inversion.

Next, we perform the second stage of CDWI using the average model obtained (Figure 2). Without regularization the time-lapse image is contaminated with noise, some events have similar amplitude to the time-lapse changes Figure 4. When we add Tikhonov stabilizer the locations of the time-lapse anomalies can be retrieved, but the shape and fine resolution are completely lost (Figure 4(b)). Total variation regularization provides a better estimate of the shape of anomalies, yet the resolution is still low and can be increased by using the Minimum Support regularization (Figure 4(d)). With minimum support regularization the shallow anomaly is perfectly resolved, while the very fine parts of the deeper anomaly can not be resolved. This happens probably due to the resolution limitations at frequency of 8 Hz. The resolution of MS regularized time-lapse waveform inversion has gone far beyond the commonly attributed to FWI resolution of quarter of the local wavelength, this is due to the blocky assumption imposed by the regularization term.

Discussion and conclusions

CDWI merges all available datasets to construct the average (common) model for time-lapse waveform inversion. This is computationally more efficient than the joint full-waveform inversion in the case of a collocated (mergeable) surveys. Several regularization types were compared in the framework of time-

Figure 3 Inverted baseline model – a), inversion with the average data – b).

Figure 4 Estimated time-lapse differences without regularization – a); with classic Tikhonov (Sobolev) – b), total variation – c) and minimum support – d) regularizations.

lapse full-waveform inversion. Total variation works well for regularizing the background model, while minimum support can produce higher resolution time-lapse images. We did not use a-priori information to localize the time-lapse anomalies, yet if available this information can be easily incorporated into the inversion process regardless of the regularization type to improve the results even further. We proposed centered difference waveform inversion and compared it with double-difference waveform inversion in enhancing local features. On clean data all methods produce similar results.

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References

- Alemie, W. and Sacchi, M. [2016] Joint reparametrized time-lapse full-waveform inversion. In: *SEG Technical Program Expanded Abstracts 2016*. 1309–1314.
- Alkhalifah, T. [2016] Full-model wavenumber inversion: An emphasis on the appropriate wavenumber continuation. *GEOPHYSICS*, **81**(3), R89–R98.
- Asnaashari, A., Brossier, R., Garambois, S., Audebert, F., Thore, P. and Virieux, J. [2015] Time-lapse seismic imaging using regularized full-waveform inversion with a prior model: which strategy? *Geophysical prospecting*, **63**(1), 78–98.
- Denli, H., Huang, L. et al. [2009] Double-difference elastic waveform tomography in the time domain. In: *2009 SEG Annual Meeting*. Society of Exploration Geophysicists.
- Hicks, E., Hoerber, H., Houbiers, M., Lescoffit, S.P., Ratcliffe, A. and Vinje, V. [2016] Time-lapse full-waveform inversion as a reservoir-monitoring tool – A North Sea case study. *The Leading Edge*, **35**(10), 850–858.
- Kazei, V., Tessmer, E. and Alkhalifah, T. [2016] Scattering angle-based filtering via extension in velocity. In: *SEG Technical Program Expanded Abstracts 2016*, Society of Exploration Geophysicists, 1157–1162.
- Maharramov, M., Biondi, B.L. and Meadows, M.A. [2016] Time-lapse inverse theory with applications. *Geophysics*, **81**(6), R485–R501.
- Oghenekohwo, F., Kumar, R., Esser, E. and Herrmann, F.J. [2015] Using common information in compressive time-lapse full-waveform inversion. In: *77th EAGE Conference and Exhibition 2015*.
- Portniaguine, O. and Zhdanov, M.S. [1999] Focusing geophysical inversion images. *GEOPHYSICS*, **64**(3), 874–887.
- Raknes, E.B. and Arntsen, B. [2014] Time-lapse full-waveform inversion of limited-offset seismic data using a local migration regularization. *Geophysics*, **79**(3), WA117–WA128.
- Watanabe, T., Shimizu, S., Asakawa, E., Matsuoka, T. et al. [2004] Differential waveform tomography for time-lapse crosswell seismic data with application to gas hydrate production monitoring. In: *2004 SEG Annual Meeting*. Society of Exploration Geophysicists.