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Wave-equation Migration Velocity Analysis Using
Plane-wave Common Image Gathers

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ABSTRACT

Wave-equation migration velocity analysis (WEMVA) based on subsurface-offset, angle domain or time-lag common image gathers (CIGs) requires significant computational and memory resources because it computes higher dimensional migration images in the extended image domain. To mitigate this problem, a WEMVA method using plane-wave CIGs is presented. Plane-wave CIGs reduce the computational cost and memory storage because they are directly calculated from prestack plane-wave migration, and the number of plane waves is often much smaller than the number of shots. In the case of an inaccurate migration velocity, the moveout of plane-wave CIGs is automatically picked by a semblance analysis method, which is then linked to the migration velocity update by a connective function. Numerical tests on two synthetic datasets and a field dataset validate the efficiency and effectiveness of this method.

INTRODUCTION

An accurate estimate of the velocity model is important to obtain focused images with migration. Two different categories of methods have been discussed extensively in the literature: data-domain methods such as full waveform inversion (FWI) (Tarantola, 1984; Pratt, 1999; Virieux and Operto, 2009), and image-domain methods such as migration velocity analysis (MVA) (Stork, 1992; Biondi et al., 1999; Yilmaz, 2001; Mulder and Ten Kroode, 2002; Sava and Biondi, 2004; Shen and Symes, 2008). FWI inverts for the velocity model with high resolution, yet it suffers from the problem of local minima (Bunks et al., 1995) and requires a good initial velocity model to converge to the correct velocity. MVA inverts for the migration velocity which maximizes the similarities of a group of images, which is referred to as common image gathers (CIGs). The inverted velocity has moderate resolution and can then serve as the initial model for FWI. The assumption employed by MVA is that using an accurate background velocity, the reflectors in the subsurface must be imaged at the same location for all the images, so that the CIGs are flat or focused. On the contrary, an inaccurate velocity will lead to shifts or defocusing between different images, and MVA back-projects the displacement or defocusing information to give the velocity update. The back-projection can be realized with either a ray-based operator (Stork, 1992; Yilmaz, 2001), or a wave-equation based operator (Biondi et al., 1999; Mulder and Ten Kroode, 2002; Sava and Biondi, 2004; Shen and Symes, 2008). In general wave-equation migration velocity analysis (WEMVA) is intrinsically more robust than the ray-based MVA because it avoids the high-frequency assumption in the ray-tracing method.

WEMVA can be implemented using CIGs in different domains such as the shot (Perrone et al., 2014), angle (Xu et al., 2001; Sava and Fomel, 2003; Biondi and Symes, 2004),

subsurface-offset (Rickett and Sava, 2002) and time-lag domains (Sava and Fomel, 2006). Subsurface-offset and time-lag CIGs require an extended imaging condition in the space-lag and time-lag domains for each shot. Besides the physical dimensions in x-y-z, an extended image for a single shot has extra dimensions for space or time lags, which requires a significant increase in memory-storage space. Angle-domain CIGs are computed by a slant stack of the subsurface-offset CIGs (Sava and Fomel, 2003), or by calculating wavefield propagation directions during migration (Xu et al., 2011; Dickens and Winbow, 2011; Zhang, 2014). The final CIGs result from stacking all the shot gathers together. Computing these CIGs for WEMVA has a high computational cost and memory-storage requirement for large 3D datasets.

To mitigate this problem, plane-wave migration (Whitmore, 1995; Duquet et al., 2001; Zhang et al., 2005; Liu et al., 2006) can be applied to form plane-wave CIGs. This method combines multiple shot gathers into a composite plane-wave gather and migrates the plane-wave gathers with different ray parameters p to obtain plane-wave CIGs. The benefit is that there is no need for the extra dimensions in the extended image domain. Moreover, the number of plane waves is often much smaller than the number of shots in the survey. Because of these two reasons, plane-wave CIGs save computational cost and memory space compared to CIGs in other domains.

Plane-wave technology has been used in the exploration geophysics community to save computational cost in full waveform inversion (Vigh and Starr, 2008) and least-squares migration (Dai and Schuster, 2013; Wang et al., 2014). Jiao et al. (2002) applied residual migration velocity analysis in the plane-wave domain, where the migration velocities were scanned and picked to flatten plane-wave CIGs by a moveout correction based on an analytical moveout formula. However, this formula is only valid for a 1D velocity model or a

2D model with small dip angles.

To overcome this limitation, we present a WEMVA method using plane-wave CIGs, denoted as PWEMVA. PWEMVA inverts for the migration velocity by minimizing the objective function which is the squared summation of the local shifts between a plane-wave migration image and a reference image. The idea of defining the objective function as the local displacements between different images or data has been proposed to invert for the subsurface velocity model (Ma and Hale, 2013; Perrone et al., 2014, 2015), or the velocity variations during a 4D time-lapse seismic survey (Perrone and Sava, 2013). However these methods still require a high computational cost because they compute images or data for every shot profile.

PWEMVA computes the local shift at a given image point by choosing a parabola which best fits the moveout of the plane-wave CIG. Similar approaches have been used by Zhang and Biondi (2013) and Zhang et al. (2015), except that they estimate the moveout of the shot and angle-domain CIGs. To compute the gradient of the objective function, we need to derive the Fréchet derivative of the local image shifts with respect to the migration velocity. This derivative is derived by employing a connective function and the implicit function theorem (Luo and Schuster, 1991).

After the introduction, the theory section summarizes the theory of plane-wave migration, introduces the PWEMVA objective function and derives its gradient. The formula for updating the velocity model is given and a work flow is presented for implementing the PWEMVA method. This is followed by the numerical results section which presents the results of applying PWEMVA to synthetic data and a marine dataset recorded in the Gulf of Mexico. The last two sections include the discussions and conclusions.

THEORY

Plane-wave migration

For a 2D medium, let $S_k(\omega, \mathbf{x})$ represent the source-side wavefield of a single shot located at $(x_k, z = 0)$ at the surface. The plane-wave source-side wavefield $\bar{S}(\omega, \mathbf{x})$ is the summation of all the delayed shot wavefields from single shots:

$$\bar{S}(\omega, \mathbf{x}) = \sum_{k=1}^{n_s} e^{i\omega p(x_k - x_0)} S_k(\omega, \mathbf{x}), \quad (1)$$

where n_s represents the number of shots, i is the imaginary number, ω denotes the angular frequency and $(x_0, 0)$ is the location at the surface where the plane wave is initiated at $t = 0$. Here, $p = \sin \theta / v$ is the ray parameter of the plane wave, where θ is referred to as the shooting angle and v is the velocity at the surface.

For the k^{th} shot, the receiver-side wavefield $R_k(\omega, \mathbf{x})$ is calculated by extrapolating the recorded data backwards. Similar to equation 1, the plane-wave receiver-side wavefield $\bar{R}(\omega, \mathbf{x})$ is the summation of all the delayed $R_k(\omega, \mathbf{x})$:

$$\bar{R}(\omega, \mathbf{x}) = \sum_{k=1}^{n_s} e^{i\omega p(x_k - x_0)} R_k(\omega, \mathbf{x}). \quad (2)$$

A plane-wave gather can be seen as the plane-wave receiver wavefield at the receiver position $\mathbf{x}_g$. $\bar{S}(\omega, \mathbf{x})$ and $\bar{R}(\omega, \mathbf{x})$ are functions of p , and for simplicity this dependence is omitted in the notation.

The prestack plane-wave migration image is obtained by multiplying the plane-wave source-side wavefield with the complex conjugate of the receiver-side wavefield in the fre-

quency domain and summing over all frequencies:

$$m(\mathbf{x}) = \Re \left\{ \sum_{\omega} \bar{S}(\omega, \mathbf{x}) \bar{R}(\omega, \mathbf{x})^* \right\}, \quad (3)$$

where $\Re\{\}$ represents the real part.

Objective function and gradient

The objective function J for PWEMVA is defined as the squared summation of the vertical local shift $\Delta w_j(\mathbf{x}_o)$ between two patches $B(\mathbf{x}_o)$ (with the width and height of a wavelength) centered at $\mathbf{x}_o$ of a plane-wave migration image m_j and the reference image m_0 :

$$J = \frac{1}{2} \sum_{j=1}^{n_p} \sum_{\mathbf{x}_o \in B} \Delta w_j(\mathbf{x}_o)^2, \quad (4)$$

where j denotes the plane-wave index, n_p represents the number of plane waves and B stands for the set of all patches in the migration image. The local vertical shift $\Delta w_j(\mathbf{x}_o)$ aligns $m_0(x, z + \Delta w_j(\mathbf{x}_o))$ with $m_j(x, z)$ for $\mathbf{x} \in B(\mathbf{x}_o)$. In practice the plane-wave image with the ray direction perpendicular to the subsurface structure is chosen to be the reference image. The details are further discussed in the work-flow section.

The gradient of the objective function with respect to the slowness $c(\mathbf{x}')$ (reciprocal of the migration velocity) is

$$\frac{\partial J}{\partial c(\mathbf{x}')} = \sum_{j=1}^{n_p} \sum_{\mathbf{x}_o \in B} \frac{\partial \Delta w_j(\mathbf{x}_o)}{\partial c(\mathbf{x}')} \Delta w_j(\mathbf{x}_o). \quad (5)$$

In order to calculate the Fréchet derivative of the local shift of the window centered at

$\mathbf{x}_o$ with respect to the slowness perturbation at $\mathbf{x}'$, a connective function is defined as the local cross-correlation between $m_0(B(\mathbf{x}_o))$ and $m_j(B(\mathbf{x}_o))$:

$$f_j(c(\mathbf{x}'), w_j(\mathbf{x}_o)) = \sum_{\mathbf{x} \in B(\mathbf{x}_o)} m_0(x, z + w_j(\mathbf{x}_o)) m_j(x, z), \quad (6)$$

where $\mathbf{x} = (x, z)$ and $w_j(\mathbf{x}_o)$ is an arbitrary local shift. The correct image shift $\Delta w_j(\mathbf{x}_o)$ aligns $m_0(x, z + \Delta w_j(\mathbf{x}_o))$ with $m_j(x, z)$ within $B(\mathbf{x}_o)$, so that the connective function in equation 6 is maximized. This means that the derivative of f_j with respect to $w_j(\mathbf{x}_o)$ should be zero at $\Delta w_j(\mathbf{x}_o)$:

$$\begin{aligned} \bar{f}_j(c(\mathbf{x}'), \Delta w_j(\mathbf{x}_o)) &= \sum_{\mathbf{x} \in B(\mathbf{x}_o)} \frac{\partial f_j(c(\mathbf{x}'), w_j(\mathbf{x}_o))}{\partial w_j(\mathbf{x}_o)} \Big|_{w_j(\mathbf{x}_o) = \Delta w_j(\mathbf{x}_o)} \\ &= \sum_{\mathbf{x} \in B(\mathbf{x}_o)} m_j(x, z) \dot{m}_0(x, z + \Delta w_j(\mathbf{x}_o)) \\ &= 0, \end{aligned}$$

where the dot represents the derivative with respect to z .

The implicit function theorem gives

$$\frac{\partial \Delta w_j(\mathbf{x}_o)}{\partial c(\mathbf{x}')} = - \frac{\partial \bar{f}_j / \partial c(\mathbf{x}')}{\partial \bar{f}_j / \partial \Delta w_j(\mathbf{x}_o)}, \quad (7)$$

where the denominator is given by

$$\frac{\partial \bar{f}_j}{\partial \Delta w_j(\mathbf{x}_o)} = \sum_{\mathbf{x} \in B(\mathbf{x}_o)} \ddot{m}_0(x, z + \Delta w_j(\mathbf{x}_o)) m_j(x, z). \quad (8)$$

Here the double dots represent the second-order derivative with respect to z .

Assuming only m_j is a function of the migration slowness (this assumption is further

discussed in the work-flow section), the numerator of equation 7 is

$$\frac{\partial \bar{f}_j}{\partial c(\mathbf{x}')} = \sum_{\mathbf{x} \in B(\mathbf{x}_o)} \dot{m}_0(x, z + \Delta w_j(\mathbf{x}_o)) \frac{\partial m_j(\mathbf{x})}{\partial c(\mathbf{x}')}.$$
 (9)

Inserting equations 7, 8 and 9 into equation 5 yields

$$\frac{\partial J}{\partial c(\mathbf{x}')} = \sum_{j=1}^{n_p} \sum_{\mathbf{x}_o \in B} \frac{- \sum_{\mathbf{x} \in B(\mathbf{x}_o)} \Delta w_j(\mathbf{x}_o) \dot{m}_0(x, z + \Delta w_j(\mathbf{x}_o)) \frac{\partial m_j(\mathbf{x})}{\partial c(\mathbf{x}')}}{\sum_{\mathbf{x} \in B(\mathbf{x}_o)} \ddot{m}_0(x, z + \Delta w_j(\mathbf{x}_o)) m_j(\mathbf{x})}.$$
 (10)

Substituting the Fréchet derivative $\partial m_j(\mathbf{x})/\partial c(\mathbf{x}')$ derived in Appendix A into equation 10, the gradient of the objective function is

$$\frac{\partial J}{\partial c(\mathbf{x}')} = \sum_{j=1}^{n_p} \sum_{\mathbf{x}_o \in B} \frac{\sum_{\mathbf{x} \in B(\mathbf{x}_o)} g_1 + g_2}{\sum_{\mathbf{x} \in B(\mathbf{x}_o)} \ddot{m}_0(x, z + \Delta w_j(\mathbf{x}_o)) m_j(\mathbf{x})},$$
 (11)

$$\text{where } g_1 = \Re \left\{ \sum_{\omega} 2\omega^2 c(\mathbf{x}') \bar{S}(\omega, \mathbf{x}') \left[\overbrace{G(\mathbf{x}'|\mathbf{x})^* \mathcal{M}(\mathbf{x}) \bar{R}(\omega, \mathbf{x})}^{\text{upward-propagated receiver wavefield}} \right]^* \right\},$$

$$\text{and } g_2 = \Re \left\{ \sum_{\omega} 2\omega^2 c(\mathbf{x}') \overbrace{G(\mathbf{x}'|\mathbf{x}) \mathcal{M}(\mathbf{x}) \bar{S}(\omega, \mathbf{x})}^{\text{upward-propagated source wavefield}} \bar{R}(\omega, \mathbf{x}')^* \right\},$$

$$\text{in which } \mathcal{M}(\mathbf{x}) = -\Delta w_j(\mathbf{x}_o) \dot{m}_0(x, z + \Delta w_j(\mathbf{x}_o)).$$
 (12)

Here, $G(\mathbf{x}'|\mathbf{x})$ represents the Green's function recorded at $\mathbf{x}'$ due to a harmonic point source at $\mathbf{x}$ oscillating at a specific angular frequency ω . The gradient in equation 11 has two terms. The first term g_1 corresponds to the source-side wavepath, which is the dot product at $\mathbf{x}'$ between the downward-propagated source-side wavefield $\bar{S}(\omega, \mathbf{x}')$ and the upward-propagated receiver-side wavefield. The upward-propagated receiver-side wavefield is generated by a virtual source at the image point $\mathbf{x}$, which is redatumed from the

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6 receivers at the surface as shown in Figure 1a. Similarly, the second term g_2 can be inter-
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interpreted as the receiver-side wavepath, which is the dot product at $\mathbf{x}'$ between the downward-
propagated receiver-side wavefield $\bar{R}(\omega, \mathbf{x}')$ and the upward-propagated source-side wave-
field. The upward-propagated source-side wavefield is excited by a virtual source at the
image point $\mathbf{x}$, which is redatumed from the sources at the surface as shown in Figure 1b.
The migration slowness is updated by smearing the energy of the local-image shifts at $\mathbf{x}$
along its plane-wave paths associated with the sources and receivers as shown in Figure 1c.

Equations 4 to 12 are for the vertical shifts between 2D images. To be more general,
the local shift between 3D images is a three-component vector, and the objective function
in equation 4 can be generalized as the squared summation of the length of the shift vector
(Huang et al., 2014; Perrone et al., 2014, 2015; Perrone and Sava, 2015). In this case, the
corresponding gradient with respect to the migration slowness is derived in Appendix B.

Given the gradient, the steepest-descent method (Nocedal and Wright, 2006) can be
used to iteratively update the migration slowness until the shifts in plane-wave CIGs are
sufficiently small. An alternative is the conjugate gradient method (Nocedal and Wright,
2006).

WORK FLOW

This section describes the work flow of the PWEMVA method, which is summarized as 3
steps: calculate the objective function, the gradient, and the step length. The implementa-
tion of the first two steps is now described in detail.

1. Calculate the objective function.

First, the shot profile data are transformed into plane-wave gathers based on equation

2. Then plane-wave prestack reverse time migration (RTM) is applied to each plane-wave gather to obtain plane-wave CIGs. The poorly illuminated boundary areas of the migration images are masked.

The objective function in equation 4 requires calculating the local shifts between the same event in a plane-wave migration image and a reference image. In theory the reference image is assumed to be invariant to migration velocity changes, and this assumption is only valid for the image migrated with an accurate velocity model. This is hardly feasible since the true velocity model is never known. In practice, the reference image is assumed to be the plane-wave migration image with the ray direction perpendicular to the subsurface structure because, for a moderate complex velocity, it usually suffers the least from velocity errors.

We use semblance analysis to calculate the shifts in the plane-wave CIGs. The semblance spectrum (Taner and Koehler, 1969; Neidell and Taner, 1971) is calculated by scanning over different curvatures α of a parabola to fit the plane-wave CIGs:

$$\Delta\tilde{w}_j(\mathbf{x}) = \alpha p^2, \quad (13)$$

where $\Delta\tilde{w}_j(x, z)$ is the local shift which aligns $m_0(x, z)$ with $m_j(x, z + \Delta\tilde{w}_j(x, z))$.

$\Delta\tilde{w}_j(x, z)$ can then be transformed into $\Delta w_j(x, z)$ by

$$\Delta w_j(x, z) = -\Delta\tilde{w}_j(x, z - \Delta\tilde{w}_j(x, z)). \quad (14)$$

The reason for fitting the plane-wave CIG with a parabola is explained in Appendix C.

However, equation 13 implicitly assumes the apex of the parabola at the image with

$p = 0$, and this is not accurate for a large dip-angle interface. In this case, equation

13 is replaced with

$$\Delta \tilde{w}_j(\mathbf{x}) = \alpha(p - p_0)^2, \quad (15)$$

where $p_0 = \sin \beta / v(\mathbf{x})$.

Here p_0 is the ray parameter of the plane-wave reflecting perpendicularly off the interface dipping at an angle β and $v(\mathbf{x})$ is the migration velocity at $\mathbf{x}$. β can be computed from the migration image.

After calculating the semblance spectrum, the curvature corresponding to the maximum energy is automatically picked using the method proposed by Fomel (2009). As an example, Figure 2a shows a semblance spectrum computed from a plane-wave CIG shown in Figure 2b and the picked curvature parameters are then transformed into the shift based on equations 13 or 15 as shown in Figure 2c.

2. Calculate the gradient.

When calculating the gradient, equation 12 is simplified as

$$\mathcal{M}(\mathbf{x}) = -\Delta w_j(\mathbf{x}_o) \dot{m}_j(\mathbf{x}), \quad (16)$$

by assuming $\dot{m}_0(x, z + \Delta w_j(\mathbf{x}_o)) \approx \dot{m}_j(x, z)$. The denominator of equation 11 is also omitted in implementation because dividing an image can be unstable.

The Green's functions in equation 11 are computed by solving the two-way acoustic wave equation in the time domain while the background slowness model needs to be smoothed to avoid reflection events in the Green's functions. Given the gradient, a numerical line search method is used to calculate the step length and update the slowness model by the steepest-descent or the conjugate gradient methods (Nocedal

and Wright, 2006).

NUMERICAL RESULTS

The PWEMVA method is applied to two synthetic datasets and a marine dataset recorded in the Gulf of Mexico. These tests are designed to demonstrate the strengths and limitations of this velocity analysis method.

Synthetic test 1

The PWEMVA method is first tested on the data generated for a simple 2D model, which is 2 km wide and 0.5 km deep. Synthetic shot gathers are computed by finite-difference solutions to the 2D acoustic wave equation for the velocity model shown in Figure 3a. The source wavelet is a Ricker wavelet with a 40-Hz peak frequency, and there are 201 shots with 402 active receivers per shot. The sources and receivers are evenly distributed on the surface with intervals of 10 m and 5 m, respectively. The shot profile data are transformed into 41 plane-wave gathers with $-0.471 \text{ s/km} \leq p \leq 0.471 \text{ s/km}$ and shooting angles changing from -45 to 45 degrees. The plane-wave gather with $p = 0$ is shown in Figure 3b, and the initial velocity model is homogeneous with $v = 1.5 \text{ km/s}$. These CIGs using the initial velocity model are shown in Figure 4a. The semblance spectra and the moveout residuals computed from the CIGs at two locations are shown in Figures 4b and 4c. The inverted velocity models after 10 and 20 iterations are shown in Figures 5a and 5b, respectively, where the final tomogram accurately resembles the true velocity model. Figure 5c depicts the values of the objective function at each iteration, and the plane-wave CIGs shown in Figure 6 are mostly flattened using the inverted velocity model after 20 iterations. We also compare the

stacked plane-wave images using the homogeneous and inverted velocity models in Figure 7. It is evident that the inverted velocity model produces more focused and flatter image. For the deepest reflector, the minor distortions shown in Figure 7b and the slight curvatures in Figure 6 are due to the circular bottom and the vertical artifacts at $x = 0.5$ km and $x = 1.5$ km of the inverted velocity model in Figure 5b. These errors are the results of uneven illumination and the velocity/depth ambiguity (Bickel, 1990; Ross, 1994).

Synthetic test 2

The second test inverts the synthetic data generated by a staggered-grid acoustic modeling algorithm. The velocity and density models are shown in Figure 8 with the width of 21 km and the depth of 7.9 km taken from a portion of the BP2004 model. The source wavelet is a Ricker wavelet with a 15-Hz peak frequency. There are 690 shots with 2070 receivers per shot. The shots and receivers are evenly distributed on the surface at 30 m and 10 m intervals, respectively. These data are transformed into 81 plane-wave gathers with the ray parameters ranging from -0.33 s/km to 0.33 s/km and the shooting angles varying from -30 to 30 degrees. The laterally homogeneous velocity model shown in Figure 9a is the initial velocity, which produces the plane-wave CIGs with strong residual moveouts between 2 km to 8 km and 12.5 km to 15.5 km along the horizontal distance as shown in Figure 9b. Figure 10 depicts the semblance spectra and the picked curvatures at different horizontal locations. The velocity of the water layer is not updated in the inversion.

The inverted velocity models after 5 and 10 iterations are shown in Figures 11a and 11b, respectively. After 10 iterations, the tomogram recovers most of the low-wavenumber components of the true-velocity model in Figure 8a. Figure 11c shows the values of the

objective function at each iteration, and the plane-wave CIGs associated with the inverted velocity after 10 iterations are mostly flattened as shown in Figure 12. We now compare the stacked plane-wave images in Figure 13 using the initial and inverted velocities with the density model in Figure 8b, since density perturbations generate most of the reflection data. It is clear that the migration image computed with the inverted velocity model closely resembles the high-wavenumber features of the density model with more focused structures in the deep section.

Field data test

The third example is for a 2D marine dataset recorded in the Gulf of Mexico. The streamer data consist of 496 shots with a shot interval of 37.5 m. Each shot has 480 hydrophones with a receiver interval of 12.5 m. The maximum source-receiver offset is approximately 6 km, the nearest offset is 198 m, and the recording time is 5 seconds. Figure 14a shows a recorded common shot gather (CSG), where the 496 shot gathers are transformed into common midpoint profiles (CMPs) and a 2D spline interpolation is used to fill in the near-offset trace gap after normal moveout correction (Yilmaz, 2001). The interpolated CMPs are then transformed into common receiver gathers (CRGs) with a split-spread acquisition geometry using reciprocity (Liu et al., 2004). In the CRGs, each trace is multiplied by $\sqrt{i/\omega}$ in the frequency domain and then scaled by $\sqrt{t}$ in the time domain to correct for the 3D geometrical spreading (Dai, 2012). A tau-p transform is applied to each CRG to generate 51 plane-wave gathers with $-0.33 \text{ s/km} \leq p \leq 0.33 \text{ s/km}$. The shooting angles vary from -30 to 30 degrees, and a plane-wave gather is shown in Figure 14b. The plane-wave gathers are filtered with a Wiener filter to transform the original wavelet to a Ricker wavelet with a 25-Hz peak frequency.

The initial-velocity model shown in Figure 15a is obtained by inverting the traveltimes of the first arrivals. The plane-wave CIGs calculated by plane-wave RTM are shown in Figure 15b, with significant residual moveouts below 2 km and between 1 km to 9 km along the horizontal axis. The semblance analysis of the plane-wave CIGs illustrated in Figure 16 is used to calculate the spatial shifts for the migration image. After 10 iterations, the inverted velocity model is shown in Figure 17a, which produces the plane-wave CIGs shown in Figure 17b, with more flattened events especially below 2 km and between 1 to 9 km along the horizontal distance.

We now compare the shot profile RTM images from the initial and inverted velocities, as shown in Figures 18a and 18b, respectively. The magnified views of the two images are compared in Figure 19. The comparison clearly shows that the RTM image using the inverted velocity model is better focused than that using the traveltime tomogram.

DISCUSSIONS

In this section we discuss three issues which require attention for implementing the PWEMVA method. The first issue is the range of shooting angles and the number of plane waves. The second issue is the PWEMVA performance with a time-domain two-way wave equation compared to that with a one-way equation in the frequency domain. The third issue is the comparison between semblance analysis and dynamic image warping (DIW) (Hale, 2013), which also computes the relative displacement between two images. At last the limitations of the PWEMVA method are discussed.

The maximum shooting angle α_2 for a plane wave usually ranges between 30 to 45 degree. The shooting angle does not need to be too large, since the velocity of the real earth typically

increases with depth, so that a large shooting angle will reach a critical angle at certain shallow depths and fail to penetrate below. On the other hand, if the maximum shooting angle is too small, plane-wave CIGs will not have sufficient moveout information for the inversion. The minimum shooting angle α_1 is the negative of the maximum shooting angle. After determining the maximum and minimum shooting angles, the number of plane waves will determine the efficiency of the PWEMVA method. Zhang et al. (2005) state that the number of plane waves N_p must satisfy the constraint

$$N_p \geq \frac{L_s f (\sin \alpha_2 - \sin \alpha_1)}{v_s}, \quad (17)$$

where L_s is the length of a CRG and f is the frequency. This criterion ensures that the stacked plane-wave image is similar to a common-shot migration image. Since the PWEMVA method does not involve stacking, the required number of plane waves is fewer than N_p and can be decided by a trial-and-error procedure. An oversampled number of plane waves is inefficient while an undersampled number will generate significant aliasing artifacts in the CIGs.

In this paper, we migrate the plane-wave gathers and calculate the Green's functions by solving the two-way wave equation in the time domain. The alternative implementation is to solve the one-way wave equation in the frequency domain. The time-domain method is more accurate, yet requires more computational resources. This problem is more severe for surveys with a large source-receiver offset because, when computing a plane-wave gather, the delay time of a shot gather with a source at $(x_s, 0)$ is equal to $p * (x_s - x_o)$. This means that the longer source-receiver offset in a CRG, the longer the delay time, so that more computation time is required for the time-domain method. On the other hand, a

frequency-domain implementation is immune to this problem, hence it is much faster than the time-domain method. However, solving the one-way wave equation in the frequency domain is an approximation to the two-way wave equation so that it is less accurate.

Besides semblance analysis, DIW can also be used to calculate relative depth shifts between two seismic images. The advantage of DIW is that unlike the semblance analysis method, it does not assume the moveout of CIGs to be parabolic. However, the success of DIW primarily depends on the assumption that the vertical traces in two different images are the warped versions of each other (Hale, 2013). In practice this assumption is not always satisfied because (1) each plane-wave image does not necessarily have the same number of events: structures with large dip angles, for example, can only be imaged by a certain range of plane waves, and (2) in field data, sometimes the image is contaminated with a certain level of noise. On the contrary, semblance analysis can handle images with moderate signal-to-noise ratios because it computes semblance spectra by stacking all the plane-wave images. To sum up, if the assumption of DIW is satisfied, then it is a more accurate method to calculate image shifts; otherwise semblance analysis is more robust.

One disadvantage of the PWEMVA method is the artifacts in the plane-wave CIGs. Each trace in a CIG is computed by prestack plane-wave migration, thus it suffers from less SNR and cross-term artifacts compared to traces in the angle domain, subsurface-offset or time-lag CIGs, which is the result of stacking across all the shots. These artifacts in the plane-wave CIGs might bias the extraction of the moveout information. Moreover, for a velocity with strong lateral variations, the plane-wave CIGs will possibly suffer from the multi-pathing or kinematic artifacts, and such problems also exist for surface-offset or shot-profile CIGs (Stolk and Symes, 2004; Perrone et al., 2014). However, the angle domain is immune to this problem (Brandsberg-Dahl et al., 1999; Xu et al., 2001; Sava and Fomel,

2003). Another disadvantage is that the plane-wave gathers calculated from the CSGs with sparse-source distributions are usually aliased. This problem is more severe in the cross-line direction when extending the PWEMVA method to 3D velocity inversion. To mitigate this problem, CSGs need to be interpolated first and then transformed into plane-wave gathers, as was done to our marine dataset from the Gulf of Mexico.

CONCLUSIONS

A WEMVA method using plane-wave CIGs is presented which reduces the computational cost and memory-storage space, compared to the MVA analysis of CIGs in angle, subsurface-offset and time-lag domains. These benefits result from the following two properties. First, plane-wave CIGs are the direct output of prestack plane-wave migration, which does not require an imaging condition in the extended dimension or extra processing of the extrapolated wavefields during migration. Second, the number of plane waves is significantly fewer than the number of shots. In the second numerical test and the field data example, the number of plane waves is only approximately 10 percent of the number of shots, so that in these cases PWEMVA saves at least 90 percent of the computational time compared to other types of WEMVA method. The performance of the PWEMVA method largely depends on the quality of the plane-wave CIGs, thus the disadvantage of this method is that the artifacts in the plane-wave CIGs might bias the computation of the moveout information. One future work direction is to extend the PWEMVA method to 3D data.

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APPENDIX A

DERIVATION OF THE MIGRATION IMAGE CHANGE WITH RESPECT TO THE MIGRATION SLOWNESS PERTURBATION

Based on the plane-wave migration imaging condition in equation 3, the Fréchet derivative of the image at $\mathbf{x}$ with respect to the slowness perturbation at $\mathbf{x}'$ consists of two terms:

$$\frac{\partial m(\mathbf{x})}{\partial c(\mathbf{x}')} = \overbrace{\Re\left\{\sum_{\omega} \frac{\partial \bar{S}(\omega, \mathbf{x})}{\partial c(\mathbf{x}')} \bar{R}(\omega, \mathbf{x})^*\right\}}^{\gamma_1} + \overbrace{\Re\left\{\sum_{\omega} \frac{\partial \bar{R}(\omega, \mathbf{x})^*}{\partial c(\mathbf{x}')} \bar{S}(\omega, \mathbf{x})\right\}}^{\gamma_2}, \quad (\text{A-1})$$

where the subscript j representing the plane-wave index is silent.

To derive the formula for γ_1 , we note that equation 1 leads to

$$\frac{\partial \bar{S}(\omega, \mathbf{x})}{\partial c(\mathbf{x}')} = \sum_{k=1}^{n_s} e^{i\omega p(x_k - x_o)} \frac{\partial S_k(\omega, \mathbf{x})}{\partial c(\mathbf{x}')}.$$
(A-2)

The wavefield $S_k(\omega, \mathbf{x})$ is initiated by a point source located at $\mathbf{x}_k = (x_k, 0)$ at the surface

and is defined as

$$S_k(\omega, \mathbf{x}) = G(\mathbf{x}|\mathbf{x}_k)W(\omega), \quad (\text{A-3})$$

where $G(\mathbf{x}|\mathbf{x}_k)$ represents the Green's function recorded at $\mathbf{x}$ due to a harmonic point source at $\mathbf{x}_k$, and $W(\omega)$ represents the source spectrum. Here, we assume the source is zero phase.

Inserting equation A-3 into equation A-2 leads to

$$\frac{\partial \bar{S}(\omega, \mathbf{x})}{\partial c(\mathbf{x}')} = \sum_{k=1}^{n_s} e^{i\omega p(x_k - x_o)} \frac{\partial G(\mathbf{x}|\mathbf{x}_k)}{\partial c(\mathbf{x}')} W(\omega). \quad (\text{A-4})$$

The perturbed Green's function $\Delta G(\mathbf{x}|\mathbf{x}_k)$ can be expressed under the Born approximation as

$$\Delta G(\mathbf{x}|\mathbf{x}_k) = \int 2\omega^2 c(\mathbf{x}'') G(\mathbf{x}|\mathbf{x}'') G(\mathbf{x}''|\mathbf{x}_k) \Delta c(\mathbf{x}'') d\mathbf{x}'', \quad (\text{A-5})$$

where $\Delta c(\mathbf{x}'')$ is the slowness perturbation. Assuming

$$\Delta c(\mathbf{x}'') = \Delta c \delta(\mathbf{x}'' - \mathbf{x}'), \quad (\text{A-6})$$

we obtain

$$\frac{\partial G(\mathbf{x}, \mathbf{x}_k)}{\partial c(\mathbf{x}')} = 2\omega^2 c(\mathbf{x}') G(\mathbf{x}|\mathbf{x}') G(\mathbf{x}'|\mathbf{x}_k). \quad (\text{A-7})$$

Substituting the combination of equations A-4 and A-7 into the expression of γ_1 in equation

A-1 yields

$$\gamma_1 = \Re \left\{ \sum_{\omega} 2\omega^2 c(\mathbf{x}') W(\omega) \sum_{k=1}^{n_s} e^{i\omega p(x_k - x_o)} G(\mathbf{x}'|\mathbf{x}_k) G(\mathbf{x}|\mathbf{x}') \bar{R}(\omega, \mathbf{x})^* \right\}. \quad (\text{A-8})$$

Inserting equations 1 and A-3 into equation A-8 and using the reciprocity property of the

Green's function $G(\mathbf{x}|\mathbf{x}') = G(\mathbf{x}'|\mathbf{x})$, equation A-8 is simplified as

$$\gamma_1 = \Re \left\{ \sum_{\omega} 2\omega^2 c(\mathbf{x}') \bar{S}(\omega, \mathbf{x}') [G(\mathbf{x}'|\mathbf{x})^* \bar{R}(\omega, \mathbf{x})]^* \right\}. \quad (\text{A-9})$$

The formula for γ_2 is derived in a similar way. Based on equation 2

$$\frac{\partial \bar{R}(\omega, \mathbf{x})}{\partial c(\mathbf{x}')} = \sum_{k=1}^{n_s} e^{i\omega p(x_k - x_o)} \frac{\partial R_k(\omega, \mathbf{x})}{\partial c(\mathbf{x}')}, \quad (\text{A-10})$$

$$\text{and } R_k(\omega, \mathbf{x}) = \sum_{\mathbf{x}_g} G(\mathbf{x}|\mathbf{x}_g)^* d(\omega, \mathbf{x}_g, \mathbf{x}_k), \quad (\text{A-11})$$

where $R_k(\omega, \mathbf{x})$ is the backward extrapolated wavefield computed by the time-reversed propagation of the data $d(\omega, \mathbf{x}_g, \mathbf{x}_k)$ recorded at $\mathbf{x}_g$ excited by the source at $\mathbf{x}_k$.

Similar to equation A-7, Born modeling gives

$$\frac{\partial G(\mathbf{x}, \mathbf{x}_g)}{\partial c(\mathbf{x}')} = 2\omega^2 c(\mathbf{x}') G(\mathbf{x}|\mathbf{x}') G(\mathbf{x}'|\mathbf{x}_g). \quad (\text{A-12})$$

Substituting equations A-11 and A-12 into equation A-10 gives

$$\begin{aligned} \frac{\partial \bar{R}(\omega, \mathbf{x})}{\partial c(\mathbf{x}')} &= \sum_{k=1}^{n_s} e^{i\omega p(x_k - x_o)} 2\omega^2 c(\mathbf{x}') G(\mathbf{x}|\mathbf{x}')^* \sum_{\mathbf{x}_g} G(\mathbf{x}'|\mathbf{x}_g)^* d(\omega, \mathbf{x}_g, \mathbf{x}_k), \\ &= 2\omega^2 c(\mathbf{x}') G(\mathbf{x}|\mathbf{x}')^* \bar{R}(\omega, \mathbf{x}'). \end{aligned} \quad (\text{A-13})$$

Inserting equation A-13 into the expression of γ_2 in equation A-1 and using the reciprocity property $G(\mathbf{x}|\mathbf{x}') = G(\mathbf{x}'|\mathbf{x})$ yields

$$\gamma_2 = \Re \left\{ \sum_{\omega} 2\omega^2 c(\mathbf{x}') [\bar{S}(\omega, \mathbf{x}) G(\mathbf{x}'|\mathbf{x})] \bar{R}(\omega, \mathbf{x}')^* \right\}. \quad (\text{A-14})$$

Summarizing the previous derivations, we have

$$\begin{aligned} \frac{\partial m(\mathbf{x})}{\partial c(\mathbf{x}')} &= \gamma_1 + \gamma_2, \\ \text{where } \gamma_1 &= \Re \left\{ \sum_{\omega} 2\omega^2 c(\mathbf{x}') \bar{S}(\omega, \mathbf{x}') [G(\mathbf{x}'|\mathbf{x})^* \bar{R}(\omega, \mathbf{x})]^* \right\}, \\ \text{and } \gamma_2 &= \Re \left\{ \sum_{\omega} 2\omega^2 c(\mathbf{x}') [\bar{S}(\omega, \mathbf{x}) G(\mathbf{x}'|\mathbf{x})] \bar{R}(\omega, \mathbf{x}')^* \right\}. \end{aligned} \quad (\text{A-15})$$

APPENDIX B

EXTENSION OF THE OBJECTIVE FUNCTION AND GRADIENT TO 3D CASES

The local shift between two small cubes $B(\mathbf{x}_o)$ centered at $\mathbf{x}_o$ of a 3D plane-wave migration image $m_j(\mathbf{x})$ and a reference image $m_0(\mathbf{x})$ is a three-component vector $\Delta \mathbf{u}_j(\mathbf{x}_o) = (\Delta u_j(\mathbf{x}_o), \Delta v_j(\mathbf{x}_o), \Delta w_j(\mathbf{x}_o))$, where Δu_j , Δv_j and Δw_j are the components in the x , y and z directions, respectively. Similar to equation 4, the objective function is defined as

$$J = \frac{1}{2} \sum_{j=1}^{n_p} \sum_{\mathbf{x}_o \in B} \|\Delta \mathbf{u}_j(\mathbf{x}_o)\|^2, \quad (\text{B-1})$$

where $\|\cdot\|$ denotes the length of the vector. The local shift vector $\Delta \mathbf{u}_j(\mathbf{x}_o)$ aligns $m_0(\mathbf{x} + \Delta \mathbf{u}_j(\mathbf{x}_o))$ with $m_j(\mathbf{x})$ for $\mathbf{x} \in B(\mathbf{x}_o)$, where the size of $B(\mathbf{x}_o)$ is a wavelength. The gradient of the objective function with respect to the migration slowness $c(\mathbf{x}')$ is

$$\frac{\partial J}{\partial c(\mathbf{x}')} = \sum_{j=1}^{n_p} \sum_{\mathbf{x}_o \in B} \left\{ \frac{\partial \Delta u_j(\mathbf{x}_o)}{\partial c(\mathbf{x}')} \Delta u_j(\mathbf{x}_o) + \frac{\partial \Delta v_j(\mathbf{x}_o)}{\partial c(\mathbf{x}')} \Delta v_j(\mathbf{x}_o) + \frac{\partial \Delta w_j(\mathbf{x}_o)}{\partial c(\mathbf{x}')} \Delta w_j(\mathbf{x}_o) \right\}. \quad (\text{B-2})$$

The connective function is defined as

$$f_j(c(\mathbf{x}'), \mathbf{u}_j(\mathbf{x}_o)) = \sum_{\mathbf{x} \in B(\mathbf{x}_o)} m_0(\mathbf{x} + \mathbf{u}_j(\mathbf{x}_o)) m_j(\mathbf{x}), \quad (\text{B-3})$$

where $\mathbf{u}_j(\mathbf{x}_o)$ is an arbitrary local shift vector. The correct image shift $\Delta \mathbf{u}_j(\mathbf{x}_o)$ aligns $m_0(\mathbf{x} + \Delta \mathbf{u}_j(\mathbf{x}_o))$ with $m_j(\mathbf{x})$. This means that the gradient of f_j with respect to $\mathbf{u}_j(\mathbf{x}_o)$ should be zero at $\Delta \mathbf{u}_j(\mathbf{x}_o)$:

$$\begin{aligned} \bar{\mathbf{f}}_j(c(\mathbf{x}'), \Delta \mathbf{u}_j(\mathbf{x}_o)) &= \nabla f_j(c(\mathbf{x}'), \mathbf{u}_j(\mathbf{x}_o)) \Big|_{\mathbf{u}_j(\mathbf{x}_o) = \Delta \mathbf{u}_j(\mathbf{x}_o)} \\ &= (\bar{f}x_j, \bar{f}y_j, \bar{f}z_j) \\ &= \sum_{\mathbf{x} \in B(\mathbf{x}_o)} m_j(\mathbf{x}) \left(\frac{\partial m_0(\mathbf{x} + \Delta \mathbf{u}_j(\mathbf{x}_o))}{\partial x}, \frac{\partial m_0(\mathbf{x} + \Delta \mathbf{u}_j(\mathbf{x}_o))}{\partial y}, \frac{\partial m_0(\mathbf{x} + \Delta \mathbf{u}_j(\mathbf{x}_o))}{\partial z} \right) \\ &= (0, 0, 0). \end{aligned} \quad (\text{B-4})$$

From the implicit function theorem we have

$$\begin{pmatrix} \frac{\partial \Delta u_j(\mathbf{x}_o)}{\partial c(\mathbf{x}')} \\ \frac{\partial \Delta v_j(\mathbf{x}_o)}{\partial c(\mathbf{x}')} \\ \frac{\partial \Delta w_j(\mathbf{x}_o)}{\partial c(\mathbf{x}')} \end{pmatrix} = - \begin{pmatrix} \frac{\partial \bar{f}x_j}{\partial \Delta u_j(\mathbf{x}_o)} & \frac{\partial \bar{f}x_j}{\partial \Delta v_j(\mathbf{x}_o)} & \frac{\partial \bar{f}x_j}{\partial \Delta w_j(\mathbf{x}_o)} \\ \frac{\partial \bar{f}y_j}{\partial \Delta u_j(\mathbf{x}_o)} & \frac{\partial \bar{f}y_j}{\partial \Delta v_j(\mathbf{x}_o)} & \frac{\partial \bar{f}y_j}{\partial \Delta w_j(\mathbf{x}_o)} \\ \frac{\partial \bar{f}z_j}{\partial \Delta u_j(\mathbf{x}_o)} & \frac{\partial \bar{f}z_j}{\partial \Delta v_j(\mathbf{x}_o)} & \frac{\partial \bar{f}z_j}{\partial \Delta w_j(\mathbf{x}_o)} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\partial \bar{f}x_j}{\partial c(\mathbf{x}')} \\ \frac{\partial \bar{f}y_j}{\partial c(\mathbf{x}')} \\ \frac{\partial \bar{f}z_j}{\partial c(\mathbf{x}')} \end{pmatrix}. \quad (\text{B-5})$$

Inserting the expressions of $\overline{fx_j}$, $\overline{fy_j}$ and $\overline{fz_j}$ in equation B-4 into equation B-5 yields

$$\begin{pmatrix} \frac{\partial \Delta u_j(\mathbf{x}_o)}{\partial c(\mathbf{x}')} \\ \frac{\partial \Delta v_j(\mathbf{x}_o)}{\partial c(\mathbf{x}')} \\ \frac{\partial \Delta w_j(\mathbf{x}_o)}{\partial c(\mathbf{x}')} \end{pmatrix} = -A^{-1} \begin{pmatrix} \sum_{\mathbf{x} \in B(\mathbf{x}_o)} \frac{\partial m_j(\mathbf{x})}{\partial c(\mathbf{x}')} \frac{\partial m_0(\mathbf{x} + \Delta \mathbf{u}_j(\mathbf{x}_o))}{\partial x} \\ \sum_{\mathbf{x} \in B(\mathbf{x}_o)} \frac{\partial m_j(\mathbf{x})}{\partial c(\mathbf{x}')} \frac{\partial m_0(\mathbf{x} + \Delta \mathbf{u}_j(\mathbf{x}_o))}{\partial y} \\ \sum_{\mathbf{x} \in B(\mathbf{x}_o)} \frac{\partial m_j(\mathbf{x})}{\partial c(\mathbf{x}')} \frac{\partial m_0(\mathbf{x} + \Delta \mathbf{u}_j(\mathbf{x}_o))}{\partial z} \end{pmatrix}, \quad (\text{B-6})$$

$$\text{where } A = \sum_{\mathbf{x} \in B(\mathbf{x}_o)} m_j(\mathbf{x}) \begin{pmatrix} \frac{\partial^2 m_0(\mathbf{x} + \Delta \mathbf{u}_j(\mathbf{x}_o))}{\partial x^2} & \frac{\partial^2 m_0(\mathbf{x} + \Delta \mathbf{u}_j(\mathbf{x}_o))}{\partial x \partial y} & \frac{\partial^2 m_0(\mathbf{x} + \Delta \mathbf{u}_j(\mathbf{x}_o))}{\partial x \partial z} \\ \frac{\partial^2 m_0(\mathbf{x} + \Delta \mathbf{u}_j(\mathbf{x}_o))}{\partial y \partial x} & \frac{\partial^2 m_0(\mathbf{x} + \Delta \mathbf{u}_j(\mathbf{x}_o))}{\partial y^2} & \frac{\partial^2 m_0(\mathbf{x} + \Delta \mathbf{u}_j(\mathbf{x}_o))}{\partial y \partial z} \\ \frac{\partial^2 m_0(\mathbf{x} + \Delta \mathbf{u}_j(\mathbf{x}_o))}{\partial z \partial x} & \frac{\partial^2 m_0(\mathbf{x} + \Delta \mathbf{u}_j(\mathbf{x}_o))}{\partial z \partial y} & \frac{\partial^2 m_0(\mathbf{x} + \Delta \mathbf{u}_j(\mathbf{x}_o))}{\partial z^2} \end{pmatrix}.$$

Substituting equation B-6 into equation B-2 gives the gradient of the objective function in equation B-1.

APPENDIX C

MOVEOUT ANALYSIS OF PLANE-WAVE CIGS

For a stack of N horizontal layers, Jiao et al. (2002) show that the migration image depth

$z_N^m(p)$ of the plane-wave gather with the ray parameter p is

$$z_N^m(p) = \sum_{i=1}^N \Delta z_i^t \frac{\sqrt{(c_i^t)^2 - p^2}}{\sqrt{(c_i^m)^2 - p^2}}, \quad (\text{C-1})$$

where c_i^t (c_i^m) represents the correct (incorrect) migration slowness at the i -th layer.

Using the Taylor expansion and truncating after the second term gives

$$\frac{\sqrt{(c_i^t)^2 - p^2}}{\sqrt{(c_i^m)^2 - p^2}} \approx \alpha_i^{(0)} + \alpha_i^{(2)} p^2, \quad (\text{C-2})$$

where

$$\begin{aligned} \alpha_i^{(0)} &= c_i^t / c_i^m, \\ \alpha_i^{(2)} &= \frac{1}{2(c_i^m)^2} \left(\frac{c_i^t}{c_i^m} - \frac{c_i^m}{c_i^t} \right). \end{aligned} \quad (\text{C-3})$$

Substituting equation C-2 into equation C-1 yields

$$z_N^m(p) \approx \sum_{i=1}^N \Delta z_i^t \left(\alpha_i^{(0)} + \alpha_i^{(2)} p^2 \right). \quad (\text{C-4})$$

Based on equation C-4, the depth shift between the plane-wave migration images with ray parameters of p and 0 is

$$z_N^m(p) - z_N^m(p=0) = \left(\sum_{i=1}^N \alpha_i^{(2)} \right) p^2, \quad (\text{C-5})$$

which suggests that the moveout of the plane-wave CIG can be approximated by a parabola.

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LIST OF FIGURES

1 Wavepath diagrams illustrate (a) g_1 , (b) g_2 and (c) $g_1 + g_2$, where $\mathcal{M}(\mathbf{x})$ is an
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20 (a) True velocity model and (b) true density model.

21 (a) Initial velocity model and (b) the associated plane-wave CIGs.

22 Semblance spectra at (a) $x = 3.5$ km, (b) $x = 6.5$ km, (c) $x = 9.5$ km, (d) $x = 12.5$

23 km and (e) $x = 15.5$ km. The red lines represent the picked curvatures of the maximum

energy.

11 Inverted velocity model after (a) 5 iterations and (b) 10 iterations. Panel (c) shows
the objective function at each iteration.

12 Plane-wave CIGs using the inverted velocity model shown in Figure 11b.

13 Stacked plane-wave RTM images (a) using the initial velocity model in Figure 9a
and (b) using the inverted velocity model in Figure 11b.

14 (a) A CSG recorded in the Gulf of Mexico data and (b) a plane-wave gather with
 $p = -0.04$ s/km.

15 (a) The initial velocity model computed by inverting the traveltimes of the first
arrivals and (b) the associated plane-wave CIGs.

16 Plane-wave CIGs are used to calculate the semblance spectra and the shifts of
CIGs.

17 (a) The inverted velocity model after 10 iterations and (b) the associated plane-
wave CIGs.

18 Shot profile RTM images based on the (a) traveltime tomogram in Figure 15a and
(b) inverted velocity model shown in Figure 17a.

19 (a), (c), and (e) are the zoom-in views of the red, green and the yellow squares in
Figure 18a, respectively. (b), (d), and (f) are the zoom-in views of the red, green and the
yellow squares in Figure 18b, respectively.

Figure 1: Wavepath diagrams illustrate (a) g_1 , (b) g_2 and (c) $g_1 + g_2$, where $M(\mathbf{x})$ is an interface of a positive value as shown by the black horizontal line in panels (a) and (b).

458x476mm (300 x 300 DPI)

Figure 2: (a) A semblance spectrum calculated from (b) a plane-wave CIG. (c) The vertical shift of the CIG calculated from the picked curvature parameters in panel (a). The data are generated for a laterally homogeneous 6-layer velocity model, and then migrated using a homogeneous velocity slower than the true velocity. The curvature parameters corresponding to the maximum energy of the spectrum are picked automatically as shown by the red line in panel a, and then transformed into the shift values in panel c. The dashed red lines in panel b represent the depth calculated from the shift values, which match well with the depths of the migrated events.

171x74mm (300 x 300 DPI)

Figure 3: (a) True velocity model and (b) the plane-wave gather with $p = 0$.

58x18mm (300 x 300 DPI)

Figure 4: (a) Plane-wave CIGs migrated using a homogeneous velocity model. The calculated semblance spectra and the picked curvatures at 0.75 km and 1 km are shown in (b) and (c).

237x282mm (300 x 300 DPI)

Figure 5: (a) Inverted velocity after 10 iterations and (b) 20 iterations. (c) The objective function at each iteration.

200x118mm (300 x 300 DPI)

Figure 6: Plane-wave CIGs using the inverted velocity model shown in Figure 5b.

115x66mm (300 x 300 DPI)

(a) Plane-wave RTM Image using Initial Velocity

(b) Plane-wave RTM Image using Inverted Velocity

Figure 7: Stacked plane-wave RTM images (a) using the initial homogeneous velocity model and (b) using the inverted velocity model in Figure 5b.

159x214mm (300 x 300 DPI)

Figure 8: (a) True velocity model and (b) true density model.

191x198mm (300 x 300 DPI)

Figure 9: (a) Initial velocity model and (b) the associated plane-wave CIGs.

212x222mm (300 x 300 DPI)

Figure 10: Semblance spectra at (a) $x = 3.5$ km, (b) $x = 6.5$ km, (c) $x = 9.5$ km, (d) $x = 12.5$ km and (e) $x = 15.5$ km. The red lines represent the picked curvatures of the maximum energy.

361x438mm (300 x 300 DPI)

Figure 11: Inverted velocity model after (a) 5 iterations and (b) 10 iterations. Panel (c) shows the objective function at each iteration.

260x341mm (300 x 300 DPI)

Figure 12: Plane-wave CIGs using the inverted velocity model shown in Figure 11b.

127x79mm (300 x 300 DPI)

Figure 13: Stacked plane-wave RTM images (a) using the initial velocity model in Figure 9a and (b) using the inverted velocity model in Figure 11b.

148x175mm (300 x 300 DPI)

Figure 14: (a) A CSG recorded in the Gulf of Mexico data and (b) a plane-wave gather with $p = -0.04$ s/km.

256x344mm (300 x 300 DPI)

Figure 15: (a) The initial velocity model computed by inverting the traveltimes of the first arrivals and (b) the associated plane-wave CIGs.

226x230mm (300 x 300 DPI)

Figure 16: Plane-wave CIGs are used to calculate the semblance spectra and the shifts of CIGs.

378x499mm (300 x 300 DPI)

Figure 17: (a) The inverted velocity model after 10 iterations and (b) the associated plane-wave CIGs.

224x229mm (300 x 300 DPI)

Figure 18: Shot profile RTM images based on the (a) traveltime tomogram in Figure 15a and (b) inverted velocity model shown in Figure 17a.

269x308mm (300 x 300 DPI)

Figure 19: (a), (c), and (e) are the zoom-in views of the red, green and the yellow squares in Figure 18a, respectively. (b), (d), and (f) are the zoom-in views of the red, green and the yellow squares in Figure 18b, respectively.

293x263mm (300 x 300 DPI)