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Key Points:

- First study to simulate the generation and evolution of internal solitary waves in the Red Sea
- Tide background has been analyzed and properties of ISWs have been estimated
- The main generation mechanism is tide topography interaction, tidal beam affects little

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On the generation and evolution of internal solitary waves in the southern Red Sea

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Abstract Satellite observations recently revealed trains of internal solitary waves (ISWs) in the off-shelf region between 16.0°N and 16.5°N in the southern Red Sea. The generation mechanism of these waves is not entirely clear, though, as the observed generation sites are far away (50 km) from the shelf break and tidal currents are considered relatively weak in the Red Sea. Upon closer examination of the tide properties in the Red Sea and the unique geometry of the basin, it is argued that the steep bathymetry and a relatively strong tidal current in the southern Red Sea provide favorable conditions for the generation of ISWs. To test this hypothesis and further explore the evolution of ISWs in the basin, 2-D numerical simulations with the nonhydrostatic MIT general circulation model (MITgcm) were conducted. The results are consistent with the satellite observations in regard to the generation sites, peak amplitudes and the speeds of first-mode ISWs. Moreover, our simulations suggest that the generation process of ISWs in the southern Red Sea is similar to the tide-topography interaction mechanism seen in the South China Sea. Specifically, instead of ISWs arising in the immediate vicinity of the shelf break via a hydraulic lee wave mechanism, a broad, energetic internal tide is first generated, which subsequently travels away from the shelf break and eventually breaks down into ISWs. Sensitivity runs suggest that ISW generation may also be possible under summer stratification conditions, characterized by an intermediate water intrusion from the strait of Bab el Mandeb.

1. Introduction

Internal waves have been widely observed over continental margins, including shelves and slopes, and coastal oceans such as the South China [Liu and Hsu, 2004], the Sulu Sea [Apel et al., 1985], and the Strait of Gibraltar [Alpers et al., 2008]. Satellite imagery, *in situ* observations and numerical simulations provide valuable information on how large-amplitude disturbances, particularly internal solitary waves (ISWs), are generated and evolve. Previous studies suggest that the majority of the observed internal waves arise from the interaction of the barotropic tide with local topographic features. However, how specific mechanisms manifest depends on the particular stratifications, topographies and the strength of tidal currents. Vlasenko et al. [2005] summarized the generation of baroclinic tides, due to flow-topography interactions, using different parameters. In particular, generation mechanisms are classified into linear and nonlinear processes depending on the steepness of bottom topography and the intensity of tidal currents. The different generation processes of lee waves, soli-bores, ISWs, and multimodal baroclinic tides have all been included in this classification. In addition, there is the so-called local generation mechanism of ISWs proposed by New and Pingree [1992] in connection with field observations in the Bay of Biscay. To explain the fact that ISWs were found at large distances from the shelf break, New and Pingree [1992] argued that these ISWs, rather than being generated by direct interaction of the barotropic tide with the shelf topography, arose from a remotely generated internal tidal beam impinging on the pycnocline locally. This scenario has since been explored by several studies [Gerkema, 2001; New and Da Silva, 2002; Akylas et al., 2007]. Other factors that affect the generation of internal waves include background currents [Du et al., 2008], eddies [Dunphy, 2009], and seasonality of water stratification [Kartadikaria et al., 2011].

Recently, satellite observations revealed trains of ISWs in the off-shelf region between 16.0°N and 16.5°N of the southern Red Sea [Da Silva et al., 2012]. These ISWs are characterized by coherent crest lengths exceeding 80 km and crest-to-crest distances of more than 2 km, compatible with signatures of large-amplitude solitary waves. What makes these observations intriguing, however, is that (i) the ISWs are generated near

the center of the basin, far away (50 km) from the shelf break; (ii) the tidal current strength in the Red Sea is significantly smaller than in other locations where ISWs have been found. Thus, the appropriate generation mechanism is not obvious.

In response to these puzzling observations, *Da Silva et al.* [2012] suggested three possible mechanisms for the generation of ISWs in the Red Sea: (1) resonance and disintegration of interfacial tides; (2) local generation by impinging internal tidal beams that are remotely generated; and (3) same as (2), but for geometrically focused and amplified internal tidal beams. Among these mechanisms, (1) and (3) are closely connected with the unique bathymetry of the southern Red Sea.

The Red Sea is an elongated, narrow-shaped sea, which extends from north to south over a distance of approximately 2,000 km. The average width is about 280 km, with a maximum breadth of 306 km in the southern basin [Bower and Furey, 2012]. Unlike other seas where internal waves are observed, instead of a seamount or ridge, the Red Sea features a long but narrow trench in its center.

To shed some light on the generation and evolution of ISWs in the southern Red Sea, we conduct numerical simulations using the nonhydrostatic MITgcm model, applied to a two-dimensional (2-D) realistic bathymetry and driven by barotropic tidal currents.

Based on analyses of the tide properties in the Red Sea and the unique geometry of the basin, we argue that the steep bathymetry and a relatively strong tide current in the southern part of the Red Sea actually provide favorable conditions for the generation of ISWs. This claim is supported by our simulations, which furthermore suggest that the generation process of ISWs is, in fact, similar to the tide-topography interaction mechanism seen in the South China Sea [Li et al., 2011]: the induced disturbance initially is in the form of a broad internal tide, which is subject to long-wave dispersion. It subsequently travels away from the shelf and eventually evolves into ISWs as nonlinear steepening overcomes long-wave dispersion. This provides an explanation for the observation of ISWs being generated near the center of the basin. Moreover, the simulation results are consistent with the observed ISW amplitudes and speeds.

The remainder of the paper is organized as follows. In section 2, we analyze tide data collected at five stations along the Red Sea and examine the local bathymetry characteristics, in order to assess the possibility of ISW generation via tide-topography interaction in the southern Red Sea. In addition, to enable comparison of the model predictions with the satellite observations, we estimate the ISW amplitudes and speeds from the satellite images. The numerical model setup and related sensitivity experiments are described in section 3. The simulation results are discussed and compared to the satellite observations in section 4. Finally, section 5 provides a summary of the main conclusions.

2. Properties of ISWs in the Southern Red Sea

2.1. Analysis of Synthetic Aperture Radar (SAR) Images

We have analyzed SAR images from Envisat-1 in the southern Red Sea to examine the properties of the observed ISWs. Five of these images showing ISWs are included in Figure 1 on top of depth contours in the southern Red Sea. Dashed rectangles indicate the locations of the five images. Images (a) and (d) show two groups of ISWs propagating to the south-west and north-east, respectively. Image (b) shows a group of ISWs propagating to the south-west while images (c) and (e) show ISWs propagating to the north-east. All of the observed ISWs propagate in the direction perpendicular to the axis of the Red Sea, which is consistent with what was reported by *Da Silva et al.* [2012]. Based on this, the numerical experiments discussed in section 3 use a 2-D topography representing a cross section of the southern Red Sea. According to the SAR images in Figure 1, the maximum length of crests of the observed ISWs is around 60 km, which is comparable with the signatures of large-amplitude solitary waves found in other sites. The distorted fronts of ISWs in image (e) are probably due to the effects of refraction by bottom topography or eddies [Xie et al., 2015].

The estimation of ISW amplitudes from SAR images has been discussed in numerous studies [Zheng et al., 2001; Pan et al., 2007; Xue et al., 2013; Romeiser and Graber, 2015]. Here, we followed the peak-to-peak method in Zheng et al. [2001] and Xue et al. [2013]. Specifically, the peak-to-peak distance is equal to the distance between the maximum current convergence and divergence and can be determined from the ISW signatures on SAR images. For a given stratification, the solution of the Korteweg-de Vries (KdV) equation can relate the distance between the current convergence and divergence to the ISW amplitudes, which is then

Figure 1. Five SAR images are shown overlaying a contour map of the bathymetry in the southern Red Sea. Dash rectangles show the locations of the images. (a) Taken on 21 March 2012 at 07:10:48 UTC in wide-swath mode with HH polarization; (b) taken on 5 May 2004 at 07:16:26 UTC in wide-swath mode with VV polarization; (c) taken on 16 May 2004 at 19:13:54 UTC in wide-swath mode with VV polarization; (d) taken on 20 April 2008 at 19:13:21 UTC in wide-swath mode with HH polarization; (e) taken on 20 February 2012 at 07:10:54 UTC in wide-swath mode with HH polarization.

used to infer the latter from the peak-to-peak distance as estimated from the SAR images (the reader is referred to *Xue et al.* [2013, equations (1)–(8)] for full details). We followed this method to estimate the amplitudes of the first two ISWs in Figure 1d. Figure 2 plots the intensity along the transect AB in Figure 1d, from which the peak-to-peak distances of the first two ISWs are estimated at $S_1 = 0.84$ km and $S_2 = 0.73$ km, and the corresponding amplitudes estimated are 72.48 m and 63.03 m, respectively. These amplitudes are comparable to those of large-amplitude ISWs found in the South China Sea [*Liu and Hsu*, 2004] and the Sulu Sea [*Apel et al.*, 1985].

2.2. Bathymetry and Stratification in the Southern Red Sea

The sea floor topography of the Red Sea includes a deep, narrow trough in its center, the characteristics of which vary from north to south. Figure 3 plots the bathymetry of the Red Sea as provided by the ETOP01 Global Relief Model (2009) with a spatial resolution of 1 arc-minute. A review of 200 m isobaths characterizing the shape of the long trough indicates that the north is considerably wider than the south. Topographical cross sections in Figures 3b–3d show that in the northern and central Red Sea the width of the trough is approximately 200 km at its basin; however, in the south, where relatively large shelves occupy most of the basin, the trough width is restricted to 70–80 km. With a depth in the same range, the slope gradient in the southern Red Sea is much larger than that in the northern or central Red Sea.

The topography profile in Figure 3d was extracted from a southern area in the Red Sea where a hot spot for ISWs has been reported. All of the observed ISWs propagated from the center to the shore, perpendicular to isobaths; so, the cross section (d) represents the propagation direction of the generated ISWs. Accordingly, in order to simplify the model setup, the 2-D topography in our standard simulation case R0 (section 3) is chosen so that the shallowest depth is at 50 m west and 30 m east, and the trench bottom is set to 1300 m (see Figure 10, bottom). Along the propagation direction the trench has a width of 75 km and includes two sills, A and B, located at longitudes 41.1°E and 41.7°E and depths 750 m and 600 m, respectively.

According to Oregon State University Tidal Inversion Software (OTIS) data [*Egbert and Erofeeva*, 2002], the most important barotropic tidal components between 16.0°N and 16.5°N in the southern Red Sea are M_2 , K_1 , N_2 , S_2 and O_1 , among which, the tidal velocity of M_2 is the most dominant (Table 1). Based on this, the

Figure 2. The image intensity along the transect AB shown in Figure 1d; S1 and S2 are marked for the first two solitary waves.

M_2 tidal frequency $\omega = 1.4053 \times 10^{-4}$ is implemented in the standard numerical experiment (R0, described in section 3). Figure 4 plots the homogeneous domain-wide profile of initial temperature $T(z)$ and salinity $S(z)$ extracted from the World Ocean Atlas 2005 (WOA2005). The buoyancy frequency $N(z)$, calculated in

Figure 3. (a) Sea floor topography of the Red Sea; contours of isobaths of the Red Sea; green lines indicate areas with a 200 m depth. Three red lines a-b, c-d and e-f are cross lines selected perpendicular to the isobaths to represent the topography in northern, central, and southern Red Sea, respectively; five tidal stations are labeled with black squares. (b-d) Topography profiles refer to northern, central, and southern Red Sea, respectively. Two sills A and B are labeled accordingly.

Table 1. Comparison of the Elevation Amplitudes for the Five Main Tidal Components in the Red Sea

Tide Stations	Data From	M ₂ (cm)	S ₂ (cm)	O ₁ (cm)	N ₂ (cm)	K ₁ (cm)
Duba	Station	25.14	8.05	1.43	6.31	2.03
	TPXO	23.48	6.96	0.73	6.69	1.49
Jeddah	Station	6.82	1.83	1.70	2.80	2.32
	TPXO	4.71	1.31	1.42	1.39	2.18
Jizan	Station	31.61	11.61	1.42	11.67	0.89
	TPXO	28.43	10.76	1.39	8.37	1.71
Rabigh	Station	11.57	3.62	1.41	4.41	3.17
	TPXO	9.70	2.90	1.28	2.77	2.09
Yanbu	Station	15.21	4.79	1.47	5.76	2.38
	TPXO	11.22	3.27	1.24	3.22	2.06

terms of the density $\rho(z)$ using the formula $N^2 = -\frac{g}{\rho} \left(\frac{\partial \rho}{\partial z} \right)$, shows that the pycnocline depth is about 100 m and the maximum buoyancy frequency $N_{\max}$ is 0.02 rad s^{-1} . The boundary value problem (BVP) for the internal wave modes taken the form

$$\frac{d^2 \psi_i(z)}{dz^2} + k_i^2 \frac{N^2(z) - \omega^2}{\omega^2 - f^2} \psi_i(z) = 0;$$

$$\psi_i(z)(0) = \psi_i(z)(-H) = 0.$$

Here $\psi_i(z)$ is the vertical modal structure function, k_i is horizontal wave number, H

is the water depth and f is the Coriolis parameter. The results shown in Figure 4 suggest that the second baroclinic mode has a reverse of polarity at depth around 100 m. At the middle of the transect in Figure 3d, where the average latitude ϕ is 16.37°N , the Coriolis parameter $f = 2\Omega \sin \phi = 4.097 \times 10^{-5}$; since $f < \omega < N_{\max}$, internal waves generated at tidal frequencies are not trapped.

2.3. Tidal Current Strength in the Southern Red Sea

Da Silva et al. [2012] argued that the tidal current in the Red Sea is relatively weak compared with other sites where large amplitude ISWs have been observed. To assess this claim, the spatial distribution of tidal currents in the Red Sea was analyzed using data from both tidal gauge stations and TPXO7.2. Figure 5 shows the sea surface elevation at five locations along the Saudi coast, where the red and green curves represent observations from tidal gauges and TPXO7.2, respectively. TPXO7.2 is a global model of ocean tides, which best-fits, in a least-squares sense, the Laplace Tidal Equations and the along track averaged data from

Figure 4. Temperature, salinity, buoyancy frequency profiles, and the first two modes of baroclinic tide in the southern Red Sea.

TOPEX/Poseidon and Jason (on TOPEX/POSEIDON tracks since 2002) obtained from OTIS. In Figure 5, from top to bottom, (a), (b), (c), (d), and (e) refer to the tidal gauge stations located at the major Saudi shelves along the Red Sea coast, Duba, Yanbu, Rabigh, Jeddah and Jizan, respectively (see in Figure 3). The red lines are the original tidal elevation data including the mean sea level signal, and the green lines are the barotropic tide model outputs from TPXO7.2. Despite some differences, the plot shows a good match between observational and TPXO 7.2 values. Note that the amplitudes of elevation vary considerably with latitude: the northern ((a) Duba) and southern ((e) Jizan) basins exhibit relatively large elevation amplitudes compared to the central basin ((d), Jeddah).

Table 1 outlines the results of a harmonic analysis of elevation using tidal observations and TPXO7.2 outputs. The elevations of the five main tidal components, M_2 , S_2 , N_2 , O_1 , and K_1 , are listed in Table 1. The M_2 elevation is much larger than that of the other components, and varies markedly at different locations, consistent with our discussion above. For example, the M_2 component has an amplitude of only 6.82 cm near Jeddah, but increases 5-fold near Jizan to reach 31.61 cm. Comparison of TPXO7.2 outputs and tide gauge data suggests that for all five tidal components, elevations calculated by TPXO7.2 are relatively small compared with those collected from tide gauge measurements. According to Zhang *et al.* [2011], this discrepancy could be due to some unresolved complex topography and assumptions of the barotropic tidal currents model.

Outputs from the TPXO model provide a complete spatial picture that agrees well with the observed values, supporting its use for the analysis of tidal current spatial variability in the Red Sea. The spatial distributions of the magnitude and phase of the TPXO M_2 tidal component in the Red Sea are depicted in Figure 6. The spatial distribution of the M_2 phase (Figure 6a) is clearly symmetrical, with the amphidromic point (the point of zero amplitude) at the center of the Red Sea located at 20.3°N, 38.9°E. The amplitude of elevation of the M_2 tidal component is smaller than 0.1 m in the central part of the basin and gradually increases toward the

Figure 5. Sea surface elevations at five tide stations in the Red Sea; the red lines refer to the tide station data; the green lines refer to the TPXO data. From top to the bottom, (a-e) the tide stations located at Duba, Yanbu, Rabigh, Jeddah and Jizan, respectively. Locations of tide stations refer to Figure 3.

Figure 6. (left) The spatial distribution of phase and (right) elevation of the M2 tidal component in the Red Sea.

northern and southern basins (Figure 6b). In the south, where ISWs are observed, amplitudes of elevations vary between 0.25 m and 0.3 m.

A tidal ellipse map of the M_2 component (Figure 7) focused on the southern Red Sea shows a rather irregular distribution because of complex topography. Although the one-layer setup of the barotropic tidal model TPXO 7.2 may not be suitable to analyze the details especially for areas near the islands, it could still provide us the basic characteristics: First, the tidal current in shallow water is much higher than in deep water. Second, the major axis of the tidal ellipse, which stands for the main direction of the tidal current, varies with location along the deep trench bounded by the two islands. In the northwest, the major tidal ellipse axis has a south-to-north orientation at 45° to the trench; meanwhile in the northeast, the major ellipse axis has an east-to-west orientation, also at 45° to the trench. In both the southwestern and southeastern parts, the major tidal ellipse axis has a southwest-to-northeast orientation, which is perpendicular to the trench. By comparing the tidal ellipse map with the observed locations of ISWs, we identified the area with southwest-to-northeast tidal currents as the location where ISWs are observed. The amplitude of the tidal current velocity in the direction perpendicular to the trench on the shelves varies between 10 cm s^{-1} and 30 cm s^{-1} and could reach up to 50 cm s^{-1} in some areas. Comparing tidal currents between the data from a mooring site and TPXO outputs, Zhang *et al.* [2011] argued that TPXO data underestimate the tidal velocity due to under-resolved bathymetry. To some extent this means that one could expect a stronger tidal current than what the TPXO model estimates in the southern Red Sea. This calls for long-term moorings deployment in the southern Red Sea to measure the strength of tidal currents.

As reported by Da Silva *et al.* [2012], all the observed ISWs appear in the region between 15.5°N and 16.5°N , which is slightly south of the Jizan tide station (16.86°N , 42.55°E). To date, no observations of ISWs have been reported in other areas of the Red Sea. From the above analysis of tidal strength and our earlier discussion of topographic variations in the Red Sea, we conclude that the southern Red Sea features a relatively strong tidal current and a steep topography; both these conditions are favorable for the generation of

Figure 7. The tidal ellipse map of the M2 tidal component in the southern Red Sea; green refers to clockwise ellipses and red refers to anti-clockwise ellipses. The tidal gauge JIZAN is indicated with a black square.

ISWs. By contrast, in the central Red Sea, despite its steep topography, relatively weak tides are not conducive to the formation of internal waves. Although the tide strengthens in the northern basin, there is an insufficient topography gradient to produce ISWs. The numerical simulations below support our claim that the combination of the steep bathymetry with the relatively strong tidal current enables the generation of ISWs in the southern Red Sea.

3. Model Setup

Guided by the properties of observed ISWs and environmental conditions discussed above, we performed numerical simulations of the generation of internal waves using the MITgcm, which includes fully nonlinear and nonhydrostatic capabilities [Marshall *et al.*, 1997]. The model configuration is a 2-D x-z domain with a length of 223.8 km and maximum depth of 1300 m. Grid resolution was set to 100 m in the x-direction and 10 m in the z-direction, which was adequate for capturing the details of the generated internal wave structures. A sponge layer was employed for the open boundaries on each side, set such that the grid size dx increases exponentially from 100 m to 10^5 m to avoid wave reflections. To satisfy the Courant-Friedrichs-Lewy condition, the time step was set to 10 s. In the model, the polynomial equation of state (EOS) suggested by McDougall *et al.* [2003], known as the MDJWF equation, was selected because it provides better results than the linear EOS equation [Dorostkar *et al.*, 2010]. We used the PP81 scheme [Pacanowski and Philander, 1981] to set the viscosity and diffusivity: $\nu = \frac{\nu_0}{(1 + \alpha Ri)^n} + \nu_b$ and $k = \frac{\nu}{1 + \alpha Ri} + k_b$, where $Ri = N^2(z) / (u_z^2 + v_z^2)$ is the Richardson number, $N(z)$ is the buoyancy frequency, u_z and v_z are the horizontal and vertical gradients, respectively. The background viscosity $\nu_b = 1.0 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$, background diffusivity $k_b = 1.0 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$, $\alpha = 5$, $n = 1$ and $\nu_0 = 1.5 \times 10^{-2} \text{ m}^2 \text{ s}^{-1}$. The model was implemented with a free surface. The Coriolis parameter is set as $f = 2 \Omega \sin \Psi = 4.097 \times 10^{-5}$, where $\Psi = 16.37^\circ \text{ N}$ is the average latitude of the study area.

Following Vlasenko *et al.* [2010], the forcing of the tidal current was implemented with a periodic barotropic tidal flow of frequency ω and amplitude of horizontal velocity U_0 . The ellipse map of the M₂ tidal current in Figure 7 shows that most of the major axes of ellipses are in the direction of the cross line perpendicular to the crests of observed internal waves, which makes this simplification reasonable. Specifically, the term $F_x = U_0 H_0 / H(x, y) \omega \cos(\omega t)$ is added to the zonal momentum balance equation as external forcing, where H_0 is the depth where the tide has velocity amplitude U_0 . The values in the standard experiment R0, $H_0 = 300$ m and $U_0 = 0.05 \text{ m s}^{-1}$ were estimated from the TPXO 7.2 model [Egbert and Erofeeva, 2002]. The tidal frequency is that of the M₂ component with period of 12.42 h.

We conducted a number of different numerical experiments to gain a thorough understanding of the generation process of ISWs in the southern Red Sea. As summarized in Table 2, the standard simulation R0 using the real topography was set up with the amplitude of the M₂ barotropic tidal velocity U_0 equal to

Table 2. Configurations of Numerical Experiments

Cases	Changes Respect to R0	U_0 (m s^{-1})
R0	Standard case	0.05
R1	$U_0 \times 2$	0.10
R2	$U_0 \times 0.5$	0.025
R3	Topo 2 (east removed)	0.05
R4	Topo 3 (west removed)	0.05
R5	Topo 2 without sill A	0.05
R6	Topo 3 without sill B	0.05
R7	R3 with 10 tidal periods running	0.05
R8	R3 with summer stratification	0.05

0.05 m s^{-1} at a depth of 300 m. To explore the influence of tidal velocity on the generation process, simulations R1 and R2 were configured with double U_0 and half U_0 , respectively. To observe the generation process more clearly and remove any influence of wave interactions in different directions, simulations R3 and R4 were designed without the eastern and western parts of the topography, respectively. This setup also allowed us to investigate the resonance scenario suggested by *Da Silva et al.*

[2012]: with one part of the topography removed, the structure for the possible reflection of the internal tide is broken, so the resonance effect can be eliminated. On the basis of the results obtained with Topo2 (used in R3), simulations R5 and R6 had sill A and B removed, respectively, enabling us to explore the effect of these sills on the generation of high-mode internal waves. To examine the local generation mechanism suggested by *Da Silva et al.* [2012], in R7, we extended the running time to 10 tidal periods, this allowed possible tidal beams that could hit the pycnocline to be fully generated. In R8, we performed a sensitivity experiment with summer stratification.

4. Results and Discussion

4.1. Generation and Propagation Mechanisms of ISWs

Simulation R0 is the standard case, which uses the M_2 tidal component with amplitude of tidal velocity 0.05 m s^{-1} at a depth of 300 m. Figure 8 plots four snapshots of temperature contours at different times during the second tidal period. Due to the superposition of wave disturbances generated by the western and eastern parts of the topography, it is difficult to analyze the details of the generation mechanism from these snapshots. Instead, it is more convenient to illustrate the generation and evolution process in simulation R3 shown in Figure 9, where the eastern part of the topography has been removed; thus, we solely focus on the west-to-east propagating internal waves.

Figure 9a shows a snapshot at the end of the first quarter of the second tidal period, during which the eastward tidal flow increased from zero up to maximum. The increasing ebb tidal current appears above the western slope and depresses the isotherm throughout the first period. This depression is asymmetrical and can be divided into: i) a component with a steep slant in the front, at $x = 20 \text{ km}$ from the edge; and ii) a second component with two layers, an upper layer (50 m to 100 m) with a structure very different from that below 100 m, at $x = 10 \text{ km}$ from the edge. In this lower layer, isotherms are strongly depressed and have a “U” shape that is opposite from that of those in the upper layer. In the next phase of the generation process, these two components of the depression behave differently. Although the direction of the tidal current remains west-to-east in the second quarter period, its strength decreases to zero. Under a “softened” depression, isotherms appear to be restored and the two components of the depression propagate eastward at a slower speed. At the end of the second quarter of the second period ($t = 1.5 T$) in Figure 9b, the first component of the depression appears 35 km from the edge, and is steeper than it was in the first quarter period, with isotherms displaced by more than 30 m. The second component of the depression is located 18 km from the edge and differences between the upper and lower layers persist. After the midtidal period, the ebb tide switches into a flood tide, meaning that the tidal current switches to an east-to-west orientation. During the third quarter period, the two components of the depression propagate slowly from west to east, traveling at a slower pace due to the advection of the increasing flood tidal current. Figure 9c shows that at the end of the third quarter of the second tidal period ($t = 1.75 T$), the first part of the depression breaks into a series of solitary waves, where the leading wave has an amplitude of about 40 m; the second component of the depression also steepens but does not break. Notice that the location of the first component of the depression is at 52 km while the second is located at around 25 km, which means that the second component of the depression is propagating at a lower speed than the first component. In the last quarter period, the flood tidal current still flows in the opposite direction to the propagation of solitary waves, however, with less strength. The solitary waves in the front keep propagating eastward and two new waves are generated at the tail; the second component also propagates slowly and breaks into two waves,

Figure 8. Four snapshots show the generation process of ISWs in the southern Red Sea for the standard case R0. Current velocity shown refers to the depth of 300 m. Red dot indicates the time of each snapshot.

Figure 9. Four snapshots show the generation process of ISWs in the southern Red Sea for case R3 in which the eastern part topography was removed. Current velocity shown refers to the depth of 300 m. Red dot indicates the time of each snapshot.

Figure 10. (top) The diagram for case R3 showing temperature $T(x,t)$ in $^{\circ}\text{C}$ at $Z = -200$ m. The red line refers to the barotropic tidal current; the black dashed line indicates the speed of first-mode waves. (bottom) The topography implemented in case R3.

still with opposite wave shapes in the upper and lower layers. The second component of the depression is a mode-2 internal wave, whose vertical structure shows good agreement with the theoretical mode shape (figure 4). It is indicated that some high-mode internal waves could be generated by small sills in the topography, as in the South China Sea [Li *et al.*, 2011]. The relationship between high-mode waves and these sills in the southern Red Sea will be discussed in part 4.3. As a twin case of R3, simulation R4 leads to a similar result, but with an east-to-west generation and propagation process.

According to the simulation results, the ISWs are first generated more than 40 km from the edge of the shelf, beyond the middle of the trough. This is consistent with previous analysis of satellite images in Da Silva *et al.* [2012], where the observed ISWs are all originated from the middle of the southern Red Sea and propagate to the shore. The half wavelength of the leading wave in the group has been estimated by the backscatter profile in the satellite image to be around 2 km (Figure 2), which also agrees with our simulations. The leading wave of four generated ISWs has an amplitude of more than 50 m and a half wavelength of around 2 km (Figure 9d). Despite the limitations of 2-D numerical modeling and the simplification of the topography and tidal forcing, the simulations are in good agreement with the observations.

The distribution of the temperature field at a depth of 200 m for case R3 plotted in Figure 10 provides further insight into the generation process of solitary waves. The red sinusoid indicates the barotropic tidal current, and the black dashed line indicates the speed of the first-mode internal waves. Along this line, the yellow portion is wide and continuous at the beginning, which indicates a long depression, but later becomes narrow and “noncontinuous,” suggesting that the long depression steepens and breaks into a series of solitary waves. The timing is consistent with the generation process discussed above. The speed of first-mode ISWs is about 1.5 m s^{-1} , somewhat faster than the estimate of about 1.3 m s^{-1} by Da Silva *et al.* [2012] based on a sequence of SAR images.

It is also useful to compute the Froude number $\text{Fr} = kU_{\text{max}}/\omega$, where $k = 2\pi/\lambda$ is the horizontal wave-number of the generated waves, $\omega = 2\pi/T$ is the tidal frequency, and U_{max} is the maximum amplitude of the barotropic tidal current. Using $U_{\text{max}} = 0.30 \text{ m s}^{-1}$ (approximately estimated from TPXO, see Figure 7), $\lambda = 68.5 \text{ km}$ (approximately estimated solving BVP of internal waves), and $T = 12.42 \text{ h}$ (the M2 tidal

component period), the Froude number turns out to be about 0.20. According to the classification of *Vlasenko et al.* [2005, chap. 6, pp. 305–306], for Froude number of 0.2 and a steep bottom, the weakly nonlinear theory for flow-topography internal wave generation should apply. In this case, first and second mode baroclinic tides are expected to be generated, which is consistent with the generation process in our simulations.

4.2. The Effect of Tidal Current Intensity

The intensity of the tidal current is one of the most important factors in the generation process of internal waves. For this reason the amplitude of the tidal velocity was doubled and halved in simulations R1 and R2, respectively, and results are shown in Figure 11. At the end of the second tidal period, the isotherms were considerably different in these two cases. In R1, a group of five solitary waves has propagated eastward to around 60 km. The leading wave had an amplitude of approximately 70 m, a height that decreases only slightly for the following three waves. However, the tail wave had an amplitude of only 10 m. A comparison against the standard case R0 presented in Figure 8d, shows that in R1 the amplitudes of the leading solitary waves were 20 to 30 m larger, and the decrease in amplitude from the head to the tail wave was slower. Furthermore, in case R1 an additional group of solitary waves is generated 45 km from the edge and propagated to the west, while in case R0 there is only one long depression (Figure 9d). The second-mode wave breaks into three solitary waves 30 km from the edge in R1, while it was limited to one wave in R0. Note that all ISWs observed in the southern Red Sea are generated close to the center of the basin and the pattern of internal waves we found in R1 has never been seen in satellite images. Hence, doubling the intensity of the tidal velocity leads to a response that is not consistent with how the ISWs are generated in situ in the southern Red Sea. As might be expected, no ISWs were generated in simulation R2; instead, long depressions were found over the whole domain. This result might explain why ISWs are not observed in the central Red Sea (18°N–22°N) where the tidal velocity is relatively weak.

4.3. Sill Effects and Local Generation Mechanism

As suggested by *Da Silva et al.* [2012], the local mechanism due to remotely generated tidal beams could be relevant to the ISWs in the southern Red Sea. Simulations R5–R7 were designed to explore the influence of this factor as well as the effects of the sills A and B.

Figure 11. Comparison of the temperature field as simulated by cases R1 and R2; (top) the temperature field for case R1, with the amplitude of tidal velocity $U_1 = 2 \cdot U_0$; (bottom) the temperature field for case R2 with $U_2 = 0.5 \cdot U_0$. The snapshots are all at time $t = 2.0 T$.

Figure 12. Comparison between cases R3 and R5 for studying the effect of sills (a) horizontal velocity field in R3; (b) isotherm field in the upper layer (0–250 m) in R3; (c) horizontal velocity field in R5; (d) isotherm field in the upper layer (0–250 m) in R5. Removing sill A has little effect on the generation of ISWs.

Specifically, in simulations R5 and R6, we modified the topography used in R3 and R4, as shown in Figures 12 and 13, respectively. Surprisingly, the removal of sill A had little effect on the structure of the horizontal velocity field, as shown by comparing Figures 12a and 12c. However, Figures 12b and 12d show a clear difference in the shape of the isotherms. Although the first-mode group of solitary waves at 70 km is not affected, the second-mode wave experiences a lift from front to back of the isotherms at a 200 m depth near 32 km. This change has negligible consequences though, because it does not result in the formation of a new wave or the elimination of an existing wave. A similar conclusion is reached regarding the removal of sill B, as shown in Figure 13. The first- and second-mode waves remain intact, with the exception that the isotherms are lifted near 50 km in Figure 13b but not in Figure 13d.

Since the local generation mechanism hinges on tidal beams that reflect from the bottom topography, in experiment R7 we used the same configuration as R3, but extended the running time to 10 tidal periods in order to allow for such beams to fully develop. Figure 14 shows the change in vertical displacement ζ of the isotherm from its equilibrium depth of 80 m during the time span 113–124 h. The red and green arrows, respectively, indicate the first- and second-mode baroclinic tides generated at the last tidal period. The green dashed arrows indicate the second-mode baroclinic tides generated during the previous two tidal periods. The ISWs generation sites, propagation speed and vertical structure of the first baroclinic modes are in good agreement with the properties estimated from the SAR images in section 2.1. However, looking at the Hovmöller diagram, there is no evidence that a tidal beam hits the thermocline in the far field. Thus, the conjecture that the ISWs arise in the far field via a local generation mechanism is not supported by our simulations.

We also calculated the slope parameter $\alpha = \frac{dh/dx}{\sqrt{(\omega^2 - f^2)/(N^2 - \omega^2)}}$, the bottom steepness profile divided by the tidal beam characteristic slope. As before, ω is the frequency of the M_2 tidal component f is the Coriolis parameter and N is the buoyancy frequency. The results are shown in Figure 15 together with the bottom

Figure 13. Comparison between cases R4 and R6 for studying the effect of sills (a) horizontal velocity field in R4; (b) isotherm field in the upper layer (0–250 m) in R4; (c) horizontal velocity field in R6; (d) isotherm field in the upper layer (0–250 m) in R6. Removing sill B has little effect on the generation of ISWs.

Figure 14. Horizontal profiles of the isotherm 25°C taken at 1 h time intervals during the time span 113–124 h. Isotherm displacements are calculated using the formula depicted on the vertical axis. ζ refers to the profile of isotherm 25°C and the number n is varied from the top to the bottom $n = 1, 2, \dots, 12$. The red and green arrows indicate the first and second baroclinic modes of internal waves, respectively.

Figure 15. (top) Topography used in case R3 and (bottom) the corresponding slope parameter α . Red line indicates the critical value $\alpha = 1$.

topography. The red dashed line indicates the critical value $\alpha = 1$. Clearly, the topography is supercritical at the edge of the western shelf, so a tidal beam generated at the break is expected to propagate downward toward the bottom and reflect there. Dissipation of beam energy is a possible explanation for not seeing any such reflected beams reaching the thermocline in the far field.

In summary, the results of simulation R7 indicate that tidal beams do not contribute to the generation process of ISWs in the southern Red Sea. We recall that the conjecture of a local mechanism is mainly based on the observed ISWs being generated near the center of the domain, far from the shelf break. According to our simulation results, however, what actually happens is that the initial wave depression induced by the interaction of the barotropic tide with the shelf, travels away from the shelf break before steepening and breaking into a packet of ISWs near the center of the domain. It should be noted that the world's largest known oceanic internal waves in the South China Sea result from a similar generation process: instead of arising in the immediate vicinity of the Luzon Strait via a hydraulic lee wave mechanism, these waves firstly emerge to the west in the form of a broad, energetic, spatially coherent, nearly sinusoidal internal tide. Then, as nonlinear steepening overcomes long-wave dispersion, the internal tide starts to steepen and ultimately breaks down into ISWs [Li et al., 2011; Alford et al., 2015]. This generation process is also distinct from that described by Gerkema and Zimmerman [1995] in shallow water, where a disturbance forms over the underlying ridge and then quickly steepens and breaks into ISWs as it is carried away by an ebb tidal current.

4.4. Breaking and Reflection

As reported in previous studies, in most sites where large-amplitude ISWs are observed, these waves propagate to the shore and dissipate through smaller-scale processes. In the southern Red Sea, the fate of the ISWs remains unclear. According to Da Silva et al. [2012], no ISWs have been observed on shelf areas, so most likely they stay in the trench, either dissipating or being reflected when they meet the slope. In fact, on two occasions, Da Silva et al. [2012] reported satellite images showing ISWs reflecting off shelves.

In R0, we simulated the outcome of ISWs propagating toward the eastern slope, and analyzed their fate. Eight snapshots plotted in Figure 16 outline the process from when this group of ISWs meets the steep slope until it dissipates. In Figure 16a, the leading wave is at 70 km with an amplitude of more than 50 m,

Figure 16. Eight snapshots show the reflection process when a group of west-to-east propagating ISWs meet the slope.

followed by two waves with smaller amplitudes. As it propagated to the east, the leading wave first steepened and its wavelength halved, as is shown in Figure 16b. Figures 16c and 16d illustrate three solitary waves that all run into the slope, causing strong mixing and the generation of a large depression along the slope. Note that the tidal current is eastward and increasing during the shoaling process shown in the eight snapshots. Figures 16e and 16f show that the mixing starts to settle and the depression is released as the wave propagates westward, while in Figures 16g and 16h, the depression continues to propagate and steepens into a solitary wave. Because of the limitations of our 2-D modeling setup and the complexity of the breaking process, it is difficult to simulate the whole dissipation process precisely. However, our approach still provides a rough overview of the process: relatively low-energy solitary waves may dissipate by a mixing process upon encountering a slope; moreover, if the amplitude of the waves is large enough, a relatively small depression can be generated and then released. Meanwhile, the stratification on the shelf in the winter time is quite weak, which results in most wave energy being dissipated or reflected.

4.5. Sensitivity Experiment Under Summer Stratification

To explore the sensitivity of the generation of ISWs to stratification, experiment R8 was conducted under summer stratification conditions. Figure 17 shows five profiles of temperature obtained from different CTD observations and one buoyancy frequency profile. The location of the blue profile is about 80 km northwest to the locations of the other profiles. According to Yao *et al.* [2014], the Gulf of Aden intermediate water enters the Red Sea through the strait of Bab el Mandeb and flows northward up to latitude of 24°N. This summer intrusion results in a stable three-layer structure, as depicted in the buoyancy frequency profile in Figure 17, which is consistent with the structure reported by Da Silva *et al.* [2012]. At the location where ISWs are generated, the black profile mostly represents the stratification of seawater environment in summer and was therefore configured in R8 to investigate the generation process of ISWs. The green, red and yellow profiles represent the winter stratification, which shows little difference from the climatology mean of WOA data plotted in Figure 4.

Figure 17. Profiles of temperature from CTD observations and calculated buoyancy frequency N at profile “19821003.” Locations of different profiles are indicated with stars in different colors.

A comparison of the ISWs generated under different stratifications is shown in Figure 18. The left plot shows results from R3 with the winter stratification, while the right plot shows results from R8 with the summer stratification. Even with the three-layer stratification, a group of ISWs is generated. The maximum amplitude of the ISWs is about 35 meters. *Da Silva et al.* [2012] reported that all the ISWs seen in satellite images were during the boreal winter months (November to May), but no apparent ISW signatures were observed during summer months (June to October). This was explained by the poor stratification due to the intrusion of Gulf of Aden intermediate water. However, the results of our experiment suggest that ISWs could still be generated under summer stratification, but the amplitudes are reduced by about 30%.

5. Summary

Satellite observations recently revealed trains of ISWs in the off-shelf region between 16.0°N and 16.5°N in the southern Red Sea. The estimated peak amplitude is more than 70 meters. The generation sites of these waves are beyond the middle of the Red Sea, 45 km away from the shelf. This raises the possibility of a local generation mechanism, by a remotely generated tidal beam that reflects from the bottom and then impinges on the thermocline from below. In addition, the tide in the Red Sea is considered weak, which casts doubt on whether tide-topography interaction can explain the generation of ISWs in the southern Red Sea.

However, our analysis of sea level elevation from five tide stations and the spatial distribution of the elevation and phase of the M_2 component in the Red Sea, indicates that the southern Red Sea exhibits both steep bathymetry and a relatively strong tidal current; these are favorable conditions for the generation of large-amplitude ISWs via tide-topography interaction in the southern Red Sea.

2-D numerical simulations with a nonlinear, nonhydrostatic MITgcm were conducted, in a first attempt to numerically explore this phenomenon. The results of our simulations are consistent with the properties analyzed from satellite observations, which identify the ISWs as being generated beyond the center of the trough, with the half wavelength of the head wave about 2 km. The computed maximum amplitude is around 70 meters, close to 72.5 m that is estimated from SAR image, and the predicted first-mode wave

Figure 18. Comparison of ISWs generated under different stratifications: at (left) winter time and (right) summer time. Both snapshots are taken at time $t = 1.91 T$.

speed is 1.50 m s^{-1} , slightly slower than the 1.53 m s^{-1} estimated from the solution of BVP of internal waves.

The simulation results moreover suggest that tide-topography interaction is the dominant generation mechanism of ISWs in the Red Sea. Local generation due to a remotely generated tidal beam, on the other hand, turns out not to be a relevant factor. Instead of evolving from the disturbed thermocline due to a locally impinging tidal beam, ISWs are generated from the depression formed near the shelf. As this depression propagates out of the shelf, it gradually steepens and ultimately breaks into a group of ISWs. It should be noted that even though the process of generation and evolution of the ISWs has been clearly identified, the precise roles of different factors, such as the strength of the initial internal tide and the effect of rotation, during the interaction of the internal tide with the barotropic tide, in this particular situation are still not entirely understood. These important questions will be pursued in our future work.

Our simulations also depict the reflection of ISWs at the eastern slope, which is reported to be observed in satellite images [Da Silva et al., 2012]. Finally, the sensitivity experiment under summer stratification conditions suggests that ISWs could be also generated even with the three-layer structure of stratification that characterizes the summer water exchange with the Gulf of Aden.

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