Energy-Efficient Resource Allocation for Phantom Cellular Networks with Imperfect CSI

Amr M. Abdelhady, *Student Member, IEEE*, Osama Amin, *Senior Member, IEEE*, and Mohamed-Slim Alouini, *Fellow, IEEE*

Abstract—Multi-tier heterogeneous networks have become an essential constituent for next generation cellular networks. Meanwhile, energy efficiency (EE) has been considered a critical design criterion along with the traditional spectral efficiency (SE) metric. In this context, we study power and spectrum allocation for a two-tier phantom cellular network. The optimization framework includes both EE and SE. We consider densely deployed phantom cellular networks and model the EE optimization problem taking into consideration the inevitable interference in this setup and imperfect channel estimation impairments. To this end, we propose three resource allocation strategies aiming at optimizing this network EE performance metric. Furthermore, we investigate the effect of changing some system parameters on the performance of these strategies, such as phantom cells resource units share, number of deployed phantom cells within a macro cell, number of pilots, and the phantom cells transmission power budget. It is found that increasing the number of pilots will deteriorate the EE performance of the whole setup, while increasing phantom cells transmission power budget will not affect the EE of the whole setup significantly. In addition, we observed that it is always useful to allocate most of the network resource units to the phantom cells tier.

Index Terms—Energy efficiency, spectral efficiency, phantom cell, resource allocation, DC programming, imperfect CSI, pilots overhead, control overhead, EE metric.

I. INTRODUCTION

Since its inception, cellular communication networks have passed through numerous stages of evolution in terms of number of subscribers, services provided, mobile equipment technology, and network architecture. This fast-going evolution has led to unprecedented growth of data traffic requirements [1]. Consequently, researchers' efforts have been dedicated to maximize the overall network throughput. Recently, it has been reported that the massive data traffic flow increases the energy consumption tremendously which has the direct consequence of increasing the CO₂ footprint and limiting the lifetime of energy constrained devices [2], [3]. The previously mentioned consequences have shed the light on the importance of introducing the energy efficiency (EE) performance metric in cellular networks. Consequently, research interests are steered to adopt energy-aware network architectures and tune their operational parameters to optimize the EE performance.

Diverse energy-aware mechanisms have been explored in both academia and industry to achieve energy-efficient communications while meeting the ubiquitous data rate require-

ments. Among them, base-station cooperation that is implemented based on coordinated multi-point and relaying transmissions. In coordinated multi-point transmission, multiple base stations (BSs) coordinate their transmission to the same receiver using beamforming and joint processing to improve the transmission quality and reduce/cancel inter-cell interference [4]. Relaying transmission is used to extend the coverage through multi-hop transmission at reduced energy cost [3], [5], [6]. Heterogeneous cellular networks have been adopted recently not only to escalate the spectral efficiency, but also to reduce the energy consumption and deployment cost [7], [8]. Heterogeneous cellular networks are deployed using different types of BSs to improve the coverage/rate for the same or less energy cost.

Inspired by heterogeneous cellular networks architecture, phantom cellular networks were introduced by DOCOMO Inc. in [9] as a promising energy efficient heterogeneous network that improves the network sum-rate. The phantom cellular network consists of two tiers, where a macro cell (first tier) operates on a certain frequency band and has within its coverage a number of phantom cells (second tier) operating on a different frequency band. Phantom cells are responsible only for data transmission (D-Plane) while their control signaling (C-plane) is carried out by their supervising macro cell, which is known as (D-plane)-(C-plane) splitting. Recently, phantom cellular networks have been investigated in many research works [10]–[14]. The performance improvement limits of the phantom architecture are studied using stochastic geometry in terms of coverage probability, average throughput per-user and average rate in [12]. On the other hand, the phantom cells deployment optimization is studied in [13], where the BS density and number of antennas are optimized to minimize the average spatial energy consumption. As for resource allocation problem, we studied the resource allocation problem impact on the EE and spectral efficiency trade-off in [14]. Throughout our previous study, we assumed the case of sparse deployment where the interference was negligible and perfect channel state information (CSI) was available at the transmitter side.

In this paper, we consider the generalized problem of resource allocation of phantom cellular networks. Specifically, we consider the downlink transmission of a *densely* deployed phantom cellular network, where we take the effect of inter-cell interference between phantom cells in formulating the global EE metric. Furthermore, we assume the availability of *imperfect CSI* and model its impact on the signal-to-noise ratio, power consumption and overhead cost. To this end, we propose three resource allocation strategies that optimize

The authors are with Computer, Electrical, and Mathematical Sciences and Engineering (CEMSE) Division, King Abdullah University of Science and Technology (KAUST), Thuwal, Makkah Province, Saudi Arabia. Email: {amr.abdelhady, osama.amin, slim.alouini}@kaust.edu.sa.

the resource units (RUs) assignment and the corresponding power allocation to maximize the EE performance metric for the whole network. Throughout the proposed strategies, we convert the non-linear fractional objective function to a parametric non-fractional form. In the first strategy, we assign the RUs assuming average interference channel gains and equal power allocation at the interfering BSs. Then, we keep the RUs assignment fixed, and optimize the power allocation using difference of convex/concave (DC) functions programming. In the second strategy, we propose a tight lower bound on the spectral efficiency in order to reformulate the optimization problem to be in a form of DC programming in both RUs and the corresponding power allocation. In the third strategy, we assume fixed average interference channel gains and equal interferers power. Then, tune RUs assignment and power allocation variables jointly to maximize the simplified objective function. We provide for each strategy an iterative algorithm that computes the optimized spectral efficiency resources allocation in the first iteration, then proceed in the next iterations to find the energy-efficient resources allocation.

The rest of the paper is organized as follows. In Section II, we describe the phantom cellular network system model. Section III formulates the optimization problem. Section IV proposes the power allocation with fixed RUs assignment strategy. Section V proposes joint power and RUs allocation strategy. Section VI proposes joint power and RUs allocation assuming fixed average interference strategy. Then, we validate the performance improvement through simulation examples in Section VII. Finally, we conclude the paper in Section VIII.

Notations: $|\cdot|$ denotes the absolute value operator, $\mathbb{E}[\cdot]$ denotes the statistical expectation operator, and x^* denotes the optimized value of the variable x .

II. ENERGY EFFICIENCY OF PHANTOM CELLULAR NETWORKS

In this section, we describe the phantom cellular network system model and the downlink transmission scheme considering the pilots and control signaling overhead. Then, we formulate the global EE metric for the phantom cellular network.

A. Phantom Cellular Network System Model

Consider a system consisting of one macro cell that covers a certain region within which C_{Ph} phantom cells are deployed to serve a total of K users, as shown in Fig. 1. The users indexing is global and unique in the sense that each user has a unique index that is always referred to in any indexed variable. The total power available for transmission at the macro BS is P_M , with reciprocal power amplifier efficiency κ_M . On the other hand, the total power available for transmission at the phantom BS is P_{Ph} , with reciprocal power amplifier efficiency κ_{Ph} . We account for the macro BS and the phantom BS static circuits power consumption, backhaul links power losses, and signal processing by $P_{M,cr}$ and $P_{Ph,cr}$, respectively.

The RU is defined as the time concatenation of N_r Long Term Evolution (LTE)- defined resource blocks [15]. The RU spans $7 \times N_r$ Orthogonal Frequency Division Multiplexing (OFDM) symbol duration, and 12 subcarriers each of which

Fig. 1: System architecture and dense deployment layout

occupies 15 Khz of bandwidth, as shown in Fig 2. The N available RUs for the whole setup are divided into $N_M = N - N_{Ph}$ RUs for macro cell usage and N_{Ph} RUs for phantom cells usage, where each phantom cell uses the same set of RUs. In contrary to the user indexing convention, the indexing of RUs in this work is localized for each cell. Furthermore, the n -th RU in any phantom cell occupies exactly the same frequency band as its counterpart in other cells.

The proposed configuration has a total of $K(N_{Ph}C_{Ph} + N_M)$ possible logical data links between the users and BSs. On the other hand, the actual number of physical links is less due to the localization constraints of the phantom cells, where each phantom cell can only provide service to its local users. Furthermore, at each cell each RU can not be assigned to more than one user, i.e., time sharing is not allowed for RUs. We represent the spectrum allocation on a certain link by two sets of variables $s_{c,u}^n$ and $s_{0,u}^m$ where $c \in \{1, 2, \dots, C_{Ph}\}$, $u \in \{1, 2, \dots, K\}$, $n \in \{1, 2, \dots, N_{Ph}\}$, and $m \in \{1, 2, \dots, N_M\}$. We denote allocating the z -th RU of the x -th cell to the y -th user by setting $s_{x,y}^z = 1$. The macro cell is referred to by cell index 0, while phantom cells take cell indices varying from 1 to C_{Ph} , as shown in Fig. 1. We assume that any user in a given phantom cell can be served by this phantom cell and/or the macro cell. Similarly, we use the same indexing method for the power terms, where $p_{c,u}^n$ and $p_{0,u}^m$ denote to the power allocation variables for phantom cells and macro cell, respectively. The set of all power allocation variables is denoted by $\mathbf{P}$, while the set of all spectrum allocation variables is denoted by $\mathbf{S}$.

The proposed system setup captures different kinds of overheads, which can be observed from the structure of the macro cell RUs and their phantom cell counterparts illustrated in Fig. 2. We assume that both the phantom cells and the macro cell reserve N_p OFDM symbols for pilots transmission. However, in the macro cell case, we have additional $(C_{Ph} + 1) \times N_c$ more OFDM symbols reserved for control signaling where N_c represents the control signaling overhead per cell. We define α_p and α_c as the pilots and control overhead signaling portion in each RU, respectively. Consequently, the total overhead part

Fig. 2: Macro cell and phantom cell RU structure, respectively, from left to right.

of a phantom cell RU is α_p , whilst the total overhead part of the macro cell RU is $\alpha_{p,c} = \alpha_p + \alpha_c$, where $\alpha_p = \frac{N_p}{7N_t}$, and $\alpha_c = \frac{(C_{Ph}+1) \times N_c}{7N_t}$.

All channel impairments in terms of fading, shadowing, path loss affecting a link, along with the receiver thermal noise are integrated together into channel-to-noise terms: $\gamma_{c,u}^n$ for phantom cell links and $\gamma_{0,u}^m$ for macro cell links. The general channel-to-noise term $\gamma_{y,z}^x$ is expressed as $\gamma_{y,z}^x = |h_{y,z}^x|^2 G_{y,z} / \sigma_n^2$, where $h_{y,z}^x$ is the frequency domain channel of the link between the BS of the y -th cell and the z -th user using the x -th RU, $G_{y,z}$ encapsulates the path loss and shadowing effects between the BS of the y -th cell and z -th user using any RU and σ_n^2 is the additive white Gaussian noise (AWGN) variance. In the case of the imperfect CSI at the transmitters, $h_{y,z}^x$ becomes $\hat{h}_{y,z}^x$ and $\gamma_{y,z}^x$ becomes $\hat{\gamma}_{y,z}^x$, $\hat{\gamma}_{y,z}^x = |\hat{h}_{y,z}^x|^2 G_{y,z} / \sigma_n^2$ and $\hat{h}_{y,z}^x$ is the MMSE estimate of $h_{y,z}^x$. The impact of the non-perfect CSI estimation on achievable rate and EE performance loss will be shown in the next section.

We account for non-perfect CSI effects for a given link between the y -th BS and the z -th user, through the variance of the channel estimation error $\sigma_{\Delta h_{y,z}}^2$. Under the assumption of AWGN with zero mean and variance σ_n^2 and L -tap power delay channel model, $\sigma_{\Delta h_{y,z}}^2$ is expressed as [16],

$$\sigma_{\Delta h_{y,z}}^2 = \sum_{i=1}^L \frac{\sigma_{g_{y,z}}^2 \sigma_n^2}{\sigma_n^2 + \sigma_{g_{y,z}}^2 G_{y,z} p_y^{\text{pilot}}}, \quad (1)$$

where $p_0^{\text{pilot}} = N_p P_M$, $p_c^{\text{pilot}} = N_p P_{Ph}$ and $\sigma_{g_{y,z}}^2$ is the i -th tap channel variance of $h_{y,z}^x \forall x$.

The received signal by user z is given by

$$y_z = \sum_{n=1}^{N_{Ph}} \sum_{c=1}^{C_{Ph}} \sum_{u=1}^K h_{c,z}^n d_{c,u}^m + \sum_{m=1}^{N_M} \sum_{u=1}^K h_{0,z}^m d_{0,u}^m + n_z, \quad (2)$$

where $d_{y,z}^x$ is the transmitted symbol by the BS of the y -th cell to its z -th user using the x -th RU multiplied by $s_{y,z}^x$, and n_z is the AWGN with zero mean and variance σ_n^2 .

B. Global Energy Efficiency Metric

The global EE performance metric of the downlink transmission of phantom cellular network is defined as

$$\eta_{EE} = \frac{\eta_{SE}}{P_{\text{tot}}}, \quad (3)$$

where P_{tot} is the total power spent by the system and η_{SE} is the total spectral efficiency of the considered system expressed as

$$\eta_{SE} = \frac{1 - \alpha_{p,c}}{N \ln(2)} \sum_{m=1}^{N_M} \sum_{u=1}^K s_{0,u}^m \ln \left(1 + \frac{\hat{\gamma}_{0,u}^m p_{0,u}^m}{\beta_{0,u} G_{0,u} p_{0,u}^m + 1} \right) + \frac{1 - \alpha_p}{N \ln(2)} \sum_{n=1}^{N_{Ph}} \sum_{c=1}^{C_{Ph}} \sum_{u=1}^K s_{c,u}^n \times \ln \left(1 + \frac{\hat{\gamma}_{c,u}^n p_{c,u}^n}{\sum_{c' \neq c, u'} s_{c',u'}^n \hat{\gamma}_{c',u'}^n p_{c',u'}^n + \beta_{c,u} G_{c,u} p_{c,u}^n + 1} \right), \quad (4)$$

with $\beta_{y,z} = \sigma_{\Delta h_{y,z}}^2 / \sigma_n^2$. The first term in (4) represents the spectral efficiency contribution of the phantom cells and the second term represents the spectral efficiency contribution of the macro cell. As for P_{tot} , it is expressed as

$$P_{\text{tot}} = (1 - \alpha_{p,c}) \kappa_M \sum_{m=1}^{N_M} \sum_{u=1}^K s_{0,u}^m p_{0,u}^m + \alpha_{p,c} \kappa_M P_M + P_{M,cr} + (1 - \alpha_p) \kappa_{Ph} \sum_{n=1}^{N_{Ph}} \sum_{c=1}^{C_{Ph}} \sum_{u=1}^K s_{c,u}^n p_{c,u}^n + \alpha_p \kappa_{Ph} P_{Ph} + C_{Ph} P_{Ph,cr}. \quad (5)$$

The effect of overhead cost is captured in η_{SE} by scaling the spectral efficiency contribution of the phantom cells by $(1 - \alpha_p)$ and the spectral efficiency contribution of the macro cell by $(1 - \alpha_{p,c})$ to exclude the portion of signaling rate used for pilots and control transmission. In a similar fashion, we scale power terms of the phantom cells data transmission in P_{tot} by $(1 - \alpha_p)$, and the power terms of the macro cell data transmission by $(1 - \alpha_{p,c})$. However, we add other terms that represent the power of phantom cells' pilots transmission, which are scaled by α_p . As for the macro cell, we add other terms that represent the power used for both pilots and control signaling, which are scaled by $\alpha_{p,c}$.

The effect of non-perfect CSI on the achievable rate and consequently on the EE metric appears in two terms $\hat{\gamma}_{y,z}^x$ and $\beta_{y,z}$. We would like to emphasize that as $\beta_{y,z}$ increases, the achievable rate decreases, which is expected since $\beta_{y,z}$ increases with the increase of estimation error variance. This observation occurs when the power assigned for pilots transmission decreases and/or their number decreases. Moreover, the effect of imperfect channel estimation becomes more severe when the transmission on this ill-estimated channel is done at high power levels. Because as the variance of the channel estimation error becomes large, transmitting using large power will cause significant degradation for the SINR as the term $\beta_{y,z} G_{y,z} p_{y,z}^x$ in the denominator of SINR will be large.

III. ENERGY-EFFICIENT RESOURCE ALLOCATION STRATEGIES

In this section, we optimize $\mathbf{P}, \mathbf{S}$ to maximize the EE according to the following formulated optimization problem:

$$\begin{aligned}
 (\mathbf{P1}) \quad & \max_{\mathbf{P}, \mathbf{S}} \quad \eta_{EE} \\
 \text{subject to} \quad & C1: \sum_{u=1}^K \sum_{m=1}^{N_M} s_{0,u}^m p_{0,u}^m \leq P_M \\
 & C2: \sum_{u=1}^K \sum_{n=1}^{N_{Ph}} s_{c,u}^n p_{c,u}^n \leq P_{Ph}, \forall c|u \in U_c \\
 & C3: \sum_{u=1}^K s_{c,u}^n \leq 1, \forall c, n \\
 & C4: \sum_{u=1}^K s_{0,u}^m \leq 1, \forall m \\
 & C5: s_{c,u}^n \in \{0, 1\}, \forall c, u, n \\
 & C6: s_{0,u}^m \in \{0, 1\}, \forall u, m \\
 & C7: p_{c,u}^m \geq 0, \forall c, u, n \\
 & C8: p_{0,u}^m \geq 0, \forall u, m,
 \end{aligned}$$

where, we use C1 and C2 power constraints to guarantee that the total radiated power by any BS does not exceed its power budget. As for the RUs, we use C3 and C4 to indicate that the available RUs at every BS must be allocated to at most one user under the cell coverage. Moreover, both C5 and C6 constraints impose the fact that time sharing of RUs is prohibited in the proposed system. Finally, C7 and C8 are the constraints used to impose non-negative power constraints.

Solving the mixed integer nonlinear programming problem (P1) is computationally very expensive. A common approach is to relax integer variables and convert it to a continuous optimization problem, solve it, then discretize the solution. Such kind of approaches achieve a near-optimal performance in spectrum assignment problems where the discrete assignment parameters are relaxed in the interval $[0, 1]$ for large number of subcarriers [17]–[20]. In our optimization problem, relaxing $s_{y,z}^x$ converts (P1) into a nonlinear fractional problem, which is still a challenging problem even for power allocation only. In the following analysis, we adopt Dinkelbach method that converts the nonlinear fractional objective function in (P1) into the following parameterized nonlinear problem [21]

$$\begin{aligned}
 (\mathbf{P2}) \quad & \max_{\mathbf{P}, \mathbf{S}} \quad \mathcal{F}(q, \mathbf{P}, \mathbf{S}) = \eta_{SE} - qP_{tot} \\
 \text{subject to} \quad & C1 - C8,
 \end{aligned}$$

where q is the maximum EE, i.e., $q = \max \eta_{EE}$. Dinkelbach method states that if the EE optimal resource allocation is $(\mathbf{P}^*, \mathbf{S}^*)$, then $\mathcal{F}(q, \mathbf{P}^*, \mathbf{S}^*) = 0$ [21]. Although q is initially unknown, P2 is solved in terms of q , then it can be computed as shown in [21].

Throughout the following subsections, we develop three strategies with different degrees of performance improvement and complexity to optimize the resource allocation in order to maximize the EE considering the equivalent optimization

problem (P2). The first strategy optimizes the power allocation for a preassigned RUs to reduce the complexity. On the other hand, the second strategy introduces a joint optimization for both the power and the RUs to improve the EE performance over the PAFRUA strategy at a cost of increased complexity. On the other side of the performance-complexity trade-off, we propose the third strategy that jointly optimizes the power and RUs assignment to maximize the EE for a fixed average interference.

A. Power Allocation with Fixed Resource Units Assignment (PAFRUA) Strategy

The PAFRUA strategy aims at simplifying the optimization problem by solving only the power allocation for a preassigned RUs. The RUs within each phantom cell are assigned to the users experiencing the best SINR assuming equal power allocation. As a result, at the c -th phantom cell, the n -th RU is assigned to the u^* -th user such that $s_{c,u^*}^n = 1$ and $s_{c,u}^n = 0, u \neq u^*$, where u^* is found from

$$u^* = \arg \max_u \frac{\hat{\gamma}_{c,u}^n P_{Ph}/N_{Ph}}{\sum_{c' \neq c} \hat{\gamma}_{c',u}^n P_{Ph}/N_{Ph} + \beta_{c,u} G_{c,u} P_{Ph}/N_{Ph} + 1}. \quad (6)$$

As for the macro cell, the m -th RU is assigned to the u^* -th user such that $s_{0,u^*}^m = 1$ and $s_{0,u}^m = 0, u \neq u^*$, where u^* is found from

$$u^* = \arg \max_u \frac{\hat{\gamma}_{0,u}^m P_M/N_M}{\beta_{0,u} G_{0,u} P_M/N_M + 1}. \quad (7)$$

Then, the power is allocated by solving the following optimization problem

$$\begin{aligned}
 (\mathbf{P3}) \quad & \max_{\mathbf{P}} \quad \mathcal{F}(q, \mathbf{P}, \mathbf{S}^*) = \eta_{SE} - qP_{tot} \\
 \text{subject to} \quad & C1, C2, C7, C8.
 \end{aligned}$$

The objective function in (P3) is non-convex function in $\mathbf{P}$. One way to solve this problem is to convert it to a DC optimization problem by expressing η_{SE} as follows

$$\eta_{SE} = f(\mathbf{P}) - g(\mathbf{P}) \quad (8)$$

where $f(\mathbf{P})$ and $g(\mathbf{P})$ are defined, respectively, as

$$\begin{aligned}
 f(\mathbf{P}) = & \frac{1 - \alpha_{p,c}}{N \ln(2)} \sum_{m=1}^{N_M} \sum_{u=1}^K s_{0,u}^m \ln \left(1 + \frac{\hat{\gamma}_{0,u}^m p_{0,u}^m}{\beta_{0,u} G_{0,u} p_{0,u}^m + 1} \right) \\
 & + \frac{1 - \alpha_p}{N \ln(2)} \sum_{n=1}^{N_{Ph}} \sum_{c=1}^{C_{Ph}} \sum_{u=1}^K s_{c,u}^n \ln \left(1 + \hat{\gamma}_{c,u}^n p_{c,u}^n \right. \\
 & \left. + \beta_{c,u} G_{c,u} p_{c,u}^n + \sum_{c' \neq c, u'} s_{c',u'}^n \hat{\gamma}_{c',u'}^n p_{c',u'}^n \right) \quad (9)
 \end{aligned}$$

and

$$\begin{aligned}
 g(\mathbf{P}) = & \frac{1 - \alpha_p}{N \ln(2)} \sum_{n=1}^{N_{Ph}} \sum_{c=1}^{C_{Ph}} \sum_{u=1}^K s_{c,u}^n \ln \left(1 + \sum_{\substack{c' \neq c \\ u'}} s_{c',u'}^n \hat{\gamma}_{c',u'}^n p_{c',u'}^n \right. \\
 & \left. + \beta_{c,u} G_{c,u} p_{c,u}^n \right). \quad (10)
 \end{aligned}$$

To solve that DC program, we use the successive convex approximation (SCA). In this approach, we first approximate $g(\mathbf{P})$ using the first order representation of Taylor series at $\mathbf{P}'$. Then, we solve the reduced convex optimization problem recursively till convergence, i.e., $\mathbf{P} = \mathbf{P}'$. The SCA problem represents the inner layer of the problem solution, while the outer layer is performed by Dinkelbach method. The inner layer optimization is expressed as

$$(\mathbf{P4}) \quad \max_{\mathbf{P}} \quad \tilde{\mathcal{F}}(q, \tilde{\mathbf{P}}, \mathbf{S}^*) = f(\mathbf{P}) - \tilde{g}_{\mathbf{P}'}(\mathbf{P}) - qP_{\text{tot}}$$

subject to C1, C2, C7, C8,

where $\tilde{g}_{\mathbf{P}'}(\mathbf{P})$ is the approximated function of $g(\mathbf{P})$ after using Taylor series expansion around $\mathbf{P}'$, which is found to be

$$\tilde{g}_{\mathbf{P}'}(\mathbf{P}) = g(\mathbf{P}') + \sum_{n=1}^{N_{\text{Ph}}} \sum_{c=1}^{C_{\text{Ph}}} \sum_{u=1}^K (p_{c,u}^n - p_{c,u}'^n) \frac{\partial g}{\partial p_{c,u}^n} \Big|_{\mathbf{P}'}$$

with $\frac{\partial g}{\partial p_{y,z}^x}$ denoting the first derivative of $g(\mathbf{P})$ with respect to any general power term. One can derive the expression of $\frac{\partial g}{\partial p_{y,z}^x}$ to be

$$\frac{\partial g}{\partial p_{y,z}^x} = \left(\sum_{\substack{c=1, u=1 \\ c \neq y}}^{C_{\text{Ph}}, K} \frac{s_{c,u}^x s_{y,z}^x \hat{\gamma}_{y,u}^x}{\sum_{\substack{c'=1, u'=1 \\ c' \neq c, u' \neq u}} s_{c',u'}^x \hat{\gamma}_{c',u'}^x p_{c',u'}^x + \beta_{c,u} G_{c,u} p_{c,u}^x + 1} + \frac{s_{y,z}^x \beta_{y,z}}{\sum_{c' \neq y, u'} s_{c',u'}^x \hat{\gamma}_{c',u'}^x p_{c',u'}^x + \beta_{y,z} p_{y,z}^x + 1} \right) \frac{1 - \alpha_{\text{p}}}{N \ln(2)}. \quad (11)$$

As a result, to illustrate the first resource allocation strategy (PAFRUA), we propose algorithm I, which is summarized as follows:

- First we calculate the RUs allocation in step 3 and keep this allocation fixed throughout the algorithm.
- For initialization, the total available power of each BS is distributed uniformly in step 4 over all the activated communication links defined in step 3.
- The outer layer of Dinkelbach algorithm is implemented from step 5 to step 20, which aims to iteratively find the maximum EE, i.e. q . During the initialization phase, q is assigned zero value in step 2, to obtain the maximum spectral efficiency solution in step 13.
- The inner layer of Dinkelbach algorithm involves solving (P3) iteratively by SCA which is performed from step 6 to step 11. In each iteration of SCA, we solve the convex optimization problem (P4) formulated using the latest updated value of q and $\mathbf{P}'$ (which is the optimal solution of (P4) in the previous SCA iteration) using the interior point method to obtain $\mathbf{P}^*$. We then update the value of $\mathbf{P}'$ with $\mathbf{P}^*$ (the set of optimized power allocation variables) and solve again the new instance of (P4) where it is formulated using the updated value of $\mathbf{P}'$ and so on till a convergence is reached for the $\mathbf{P}'$ value as can be seen in step 9. In algorithm I, we used $\mathbf{P}_{\text{SE}}^*$ to denote the set of power allocation variables

maximizing SE, and $\mathbf{P}_{\text{EE}}^*$ to denote the set of power allocation variables maximizing EE.

Algorithm I

- 1: **Input** user assignments to phantom cells, ϵ_{outer} : maximum tolerable error for Dinkelbach outer loop, and ϵ_{inner} : maximum tolerable error for the SCA inner loop.
- 2: **Set** $q_{\text{error}} = \infty$, $i = 0$, $\mathbf{S} = \mathbf{0}$, $q^i = 0$ and $\mathbf{P}_{\text{inner loop error}} = \infty$
- 3: **Set** $s_{y,u^*(y,x)}^x = 1 \quad \forall (y, x)$, where $u^*(y, x)$ is computed from (6) and (7).
- 4: **Initialize** $p_{0,u}^m = s_{0,u}^m \frac{P_{\text{M}}}{N_{\text{M}}} \quad \forall u, m$, $p_{c,u}^n = s_{c,u}^n \frac{P_{\text{Ph}}}{N_{\text{Ph}}} \quad \forall u, n, c$
- 5: **while** $q_{\text{error}} > \epsilon_{\text{outer}}$ **do**
- 6: **while** $\mathbf{P}_{\text{inner loop error}} > \epsilon_{\text{inner}}$ **do**
- 7: **Solve** the convex optimization problem (P4) to get $\mathbf{P}^*$.
- 8: **Compute** $\mathbf{P}_{\text{inner loop error}}$ as follows
- 9:
$$\mathbf{P}_{\text{inner loop error}} = \sum_{n=1}^{N_{\text{Ph}}} \sum_{c=1}^{C_{\text{Ph}}} \sum_{u=1}^K (p_{c,u}^n - p_{c,u}'^n)^2 + \sum_{m=1}^{N_{\text{M}}} \sum_{u=1}^K (p_{0,u}^m - p_{0,u}'^m)^2$$
- 10: $\mathbf{P}' \leftarrow \mathbf{P}^*$
- 11: **end while**
- 12: **if** $i = 1$ **then**
- 13: **Set** $\mathbf{P}_{\text{SE}}^* = \mathbf{P}'$
- 14: **else**
- 15: **Set** $\mathbf{P}_{\text{EE}}^* = \mathbf{P}'$
- 16: **end if**
- 17: **Compute** $q_{\text{error}} = \tilde{\mathcal{F}}(q^i, \mathbf{P}', \mathbf{S}^*)$
- 18: $i \leftarrow i + 1$
- 19: **Compute** $q^i = \frac{\eta_{\text{SE}}}{P_{\text{tot}}}$, using (4) and (5).
- 20: **end while**
- 21: **Output** $\mathbf{P}_{\text{EE}}^*$, and $\mathbf{P}_{\text{SE}}^*$.

B. Joint Power and Resource Units Allocation (JPRUA) strategy

Throughout this subsection, we propose a joint power and RUs allocation strategy to maximize the EE of the phantom cellular networks. To this end, we relax the binary constraint on $\mathbf{S}$ variables by replacing it with the box constraints; C9 : $0 \leq s_{0,u}^m \leq 1 \quad \forall u, m$ and C10 : $0 \leq s_{c,u}^n \leq 1 \quad \forall c, u, n$. We assume that time sharing is implemented such that no intra-cell interference occurs. As a result (P2) is replaced with (P5)

$$(\mathbf{P5}) \quad \max_{\mathbf{P}, \mathbf{S}} \quad \mathcal{F}(q, \mathbf{P}, \mathbf{S}) = \eta_{\text{SE}} - qP_{\text{tot}}$$

subject to C1 – C4, C7 – C10.

One can derive that η_{SE} is not a concave function in both $\mathbf{S}$ and $\mathbf{P}$ variables. Moreover, it is not intuitive whether it can be expressed as a DC function of these variables. To solve such kind of problems in *interference-free* phantom cellular network, we used the following transformation in [14]

$$\tilde{p}_{y,z}^x = s_{y,z}^x p_{y,z}^x \quad \forall x, y, z, \quad (12)$$

to transform η_{SE} into a concave function in both variables. In the underlying interference-limited phantom cellular network, adopting (12) enables us to express the spectral efficiency as:

$$\eta_{SE}(\tilde{\mathbf{P}}, \mathbf{S}) = \frac{1 - \alpha_{p,c}}{N \ln(2)} \sum_{m=1}^{N_M} \sum_{u=1}^K s_{0,u}^m \ln \left(1 + \frac{\hat{\gamma}_{0,u}^m \tilde{p}_{0,u}^m}{\beta_{0,u} G_{0,u} \tilde{p}_{0,u}^m + 1} \right) + \frac{1 - \alpha_p}{N \ln(2)} \sum_{n=1}^{N_{Ph}} \sum_{c=1}^{C_{Ph}} \sum_{u=1}^K s_{c,u}^n \ln \left(1 + \frac{\hat{\gamma}_{c,u}^n \tilde{p}_{c,u}^n}{1 + \sum_{c' \neq c, u'} \frac{\hat{\gamma}_{c',u'}^n \tilde{p}_{c',u'}^n}{s_{c',u'}^n} + \beta_{c,u} G_{c,u} \tilde{p}_{c,u}^n / s_{c,u}^n} \right). \quad (13)$$

As a result, the spectral efficiency terms of the macro cell are concave in both $\mathbf{S}$ and $\mathbf{P}$ since it is interference-free system. On the other hand, the spectral efficiency terms of the phantom cellular systems are not concave (see Appendix A for the proof). One way to deal with this problem is to divide the interference terms in the phantom cells spectral efficiency expression by $s_{c,u}^n$, obtaining the following spectral efficiency lower bound

$$\eta_{SE}^{lb}(\tilde{\mathbf{P}}, \mathbf{S}) = \frac{1 - \alpha_{p,c}}{N \ln(2)} \sum_{m=1}^{N_M} \sum_{u=1}^K s_{0,u}^m \ln \left(1 + \frac{\hat{\gamma}_{0,u}^m \tilde{p}_{0,u}^m}{\beta_{0,u} G_{0,u} \tilde{p}_{0,u}^m + 1} \right) + \frac{1 - \alpha_p}{N \ln(2)} \sum_{n=1}^{N_{Ph}} \sum_{c=1}^{C_{Ph}} \sum_{u=1}^K s_{c,u}^n \ln \left(1 + \frac{\hat{\gamma}_{c,u}^n \tilde{p}_{c,u}^n}{1 + \sum_{c' \neq c, u'} \frac{\hat{\gamma}_{c',u'}^n \tilde{p}_{c',u'}^n}{s_{c',u'}^n} + \beta_{c,u} G_{c,u} \tilde{p}_{c,u}^n / s_{c,u}^n} \right). \quad (14)$$

Interestingly, the lower bound $\eta_{SE}^{lb}(\tilde{\mathbf{P}}, \mathbf{S})$ matches exactly the spectral efficiency expression using the transformation in (12) $\eta_{SE}(\tilde{\mathbf{P}}, \mathbf{S})$ at $s_{c,u}^n = 0, 1$. Moreover, η_{SE}^{lb} is proven to be a DC function in both $\tilde{\mathbf{P}}$ and $\mathbf{S}$ as shown in Appendix B.

Therefore, we adopt the approximation η_{SE}^{lb} in formulating the optimization problem (P5). Then, by using the DC function characteristics of η_{SE}^{lb} , we express it as $\eta_{SE}^{lb}(\tilde{\mathbf{P}}, \mathbf{S}) = f(\tilde{\mathbf{P}}, \mathbf{S}) - g(\tilde{\mathbf{P}}, \mathbf{S})$, where $f(\tilde{\mathbf{P}}, \mathbf{S})$ is defined as

$$f(\tilde{\mathbf{P}}, \mathbf{S}) = \frac{1 - \alpha_{p,c}}{N \ln(2)} \sum_{m=1}^{N_M} \sum_{u=1}^K s_{0,u}^m \ln \left(1 + \frac{\hat{\gamma}_{0,u}^m \tilde{p}_{0,u}^m}{\beta_{0,u} G_{0,u} \tilde{p}_{0,u}^m + 1} \right) + \frac{1 - \alpha_p}{N \ln(2)} \sum_{n=1}^{N_{Ph}} \sum_{c=1}^{C_{Ph}} \sum_{u=1}^K s_{c,u}^n \ln \left(1 + \frac{\hat{\gamma}_{c,u}^n \tilde{p}_{c,u}^n}{s_{c,u}^n} \right) + \sum_{c' \neq c, u'} \frac{\hat{\gamma}_{c',u'}^n \tilde{p}_{c',u'}^n}{s_{c',u'}^n} + \frac{\beta_{c,u} G_{c,u} \tilde{p}_{c,u}^n}{s_{c,u}^n}, \quad (15)$$

while $g(\tilde{\mathbf{P}}, \mathbf{S})$ is defined as

$$g(\tilde{\mathbf{P}}, \mathbf{S}) = \frac{1 - \alpha_p}{N \ln(2)} \left(\sum_{n=1}^{N_{Ph}} \sum_{c=1}^{C_{Ph}} \sum_{u=1}^K s_{c,u}^n \ln \left(1 + \sum_{c' \neq c, u'} \frac{\hat{\gamma}_{c',u'}^n \tilde{p}_{c',u'}^n}{s_{c,u}^n} + \frac{\beta_{c,u} G_{c,u} \tilde{p}_{c,u}^n}{s_{c,u}^n} \right) \right) \quad (16)$$

$$(P6) \quad \max_{\tilde{\mathbf{P}}, \mathbf{S}} \quad \mathcal{F}_2(q, \tilde{\mathbf{P}}, \mathbf{S}) = \tilde{\eta}_{SE}^{lb}(\tilde{\mathbf{P}}, \mathbf{S}) - q P_{tot}$$

subject to C3, C4, C9, C10,

$$C11 : \sum_{u=1}^K \sum_{m=1}^{N_M} \tilde{p}_{0,u}^m \leq P_M$$

$$C12 : \sum_{u=1}^K \sum_{n=1}^{N_{Ph}} \tilde{p}_{c,u}^n \leq P_{Ph}, \forall c$$

$$C13 : \tilde{p}_{c,u}^n \geq 0, \forall c, u, n$$

$$C14 : \tilde{p}_{0,u}^m \geq 0, \forall u, m.$$

where $\tilde{\eta}_{SE}^{lb}$ is the approximated spectral efficiency using the first order Taylor approximation of $g(\tilde{\mathbf{P}}, \mathbf{S})$ around $\mathbf{S}', \tilde{\mathbf{P}}'$ expressed as

$$\tilde{\eta}_{SE}^{lb}(\tilde{\mathbf{P}}, \mathbf{S}) = f(\tilde{\mathbf{P}}, \mathbf{S}) - g(\tilde{\mathbf{P}}, \mathbf{S}') - \sum_{n=1}^{N_{Ph}} \sum_{c=1}^{C_{Ph}} \sum_{u=1}^K \frac{\partial g(\tilde{\mathbf{P}}, \mathbf{S})}{\partial \tilde{p}_{c,u}^n} \bigg|_{\mathbf{S}', \tilde{\mathbf{P}}'} (\tilde{p}_{c,u}^n - \tilde{p}_{c,u}'^n) - \sum_{n=1}^{N_{Ph}} \sum_{c=1}^{C_{Ph}} \sum_{u=1}^K \frac{\partial g(\tilde{\mathbf{P}}, \mathbf{S})}{\partial s_{c,u}^n} \bigg|_{\mathbf{S}', \tilde{\mathbf{P}}'} (s_{c,u}^n - s_{c,u}'^n) \quad (17)$$

where $\frac{\partial g(\tilde{\mathbf{P}}, \mathbf{S})}{\partial \tilde{p}_{c,u}^n}$ and $\frac{\partial g(\tilde{\mathbf{P}}, \mathbf{S})}{\partial s_{c,u}^n}$ are expressed, respectively, as

$$\frac{\partial g(\tilde{\mathbf{P}}, \mathbf{S})}{\partial \tilde{p}_{y,z}^x} = \frac{\beta_{y,z} G_{y,z}}{\sum_{c' \neq y, u'} \hat{\gamma}_{c',z}^x \tilde{p}_{c',u'}^x / s_{y,z}^x + \beta_{y,z} G_{y,z} \tilde{p}_{y,z}^x / s_{y,z}^x + 1} + \sum_{c=1}^{C_{Ph}} \sum_{u=1}^K \frac{\hat{\gamma}_{y,u}^x}{\sum_{c' \neq c, u'} \hat{\gamma}_{c',u}^x \tilde{p}_{c',u'}^x / s_{c,u}^x + \beta_{c,u} G_{c,u} \tilde{p}_{c,u}^x / s_{c,u}^x + 1} \quad (18)$$

$$\frac{\partial g(\tilde{\mathbf{P}}, \mathbf{S})}{\partial s_{y,z}^x} = - \frac{\sum_{c' \neq y, u'} \hat{\gamma}_{c',z}^x \tilde{p}_{c',u'}^x + \beta_{y,z} G_{y,z} \tilde{p}_{y,z}^x}{\sum_{c' \neq y, u'} \hat{\gamma}_{c',z}^x \tilde{p}_{c',u'}^x + \beta_{y,z} G_{y,z} \tilde{p}_{y,z}^x + s_{y,z}^x} + \ln \left(\sum_{c' \neq y, u'} \hat{\gamma}_{c',z}^x \tilde{p}_{c',u'}^x / s_{y,z}^x + \beta_{y,z} G_{y,z} \tilde{p}_{y,z}^x / s_{y,z}^x + 1 \right) \quad (19)$$

To avoid the non-differentiability characteristics of $g(\tilde{\mathbf{P}}, \mathbf{S})$ with respect to $\mathbf{S}$ variables at zero values, we replace the two constraints C9, C10 with C15 : $\epsilon \leq s_{0,u}^m \leq 1 \quad \forall u, m$ and C16 : $\epsilon \leq s_{c,u}^n \leq 1 \quad \forall c, u, n$, where ϵ is a positive

very small number. Therefore, we consider the following optimization problem instead

$$(\text{P7}) \quad \max_{\tilde{\mathbf{P}}, \mathbf{S}} \quad \mathcal{F}_2(q, \tilde{\mathbf{P}}, \mathbf{S})$$

subject to C3, C4, C11, C12, C13, C14, C15, C16.

Similar to the PAFRUA strategy, the JPRUA strategy is performed through employing Dinkelbach's algorithm as the outer layer, and SCA as an inner layer. We propose Algorithm II to find the JPRUA solution, which is summarized as follows:

Algorithm II

```

1: Input user assignments to phantom cells,  $\epsilon_{\text{outer}}$ : maximum
   tolerable error for Dinkelbach's algorithm.
2: Set  $q_{\text{error}} = \infty$ ,  $i = 0$ ,  $\mathbf{S} = \mathbf{0}$  and  $q^i = 0$ 
3: Set  $s_{x,u}^{l*}(x, l) = \frac{1}{K} \quad \forall(x, l)$ 
4: Initialize  $\tilde{p}_{0,u}^m = s_{0,u}^m \frac{P_M}{N_M} \quad \forall u, m$ ,  $\tilde{p}_{c,u}^n = s_{c,u}^n \frac{P_{Ph}}{N_{Ph}} \quad \forall u, n, c$ 
5: while  $q_{\text{error}} > \epsilon_{\text{outer}}$  do
6:   while inner loop error  $> \epsilon_{\text{inner}}$  do
7:     Solve the convex optimization problem (P7) to
       get  $\tilde{\mathbf{P}}^*, \mathbf{S}^*$ .
8:     Compute inner loop error as:
       
$$\sum_{n=1}^{N_{Ph}} \sum_{c=1}^{C_{Ph}} \sum_{u=1}^K (p_{c,u}^{n*} - \tilde{p}_{c,u}^n)^2 + (s_{c,u}^{n*} - s_{c,u}^n)^2$$

       
$$+ \sum_{m=1}^{N_M} \sum_{u=1}^K (p_{0,u}^{m*} - \tilde{p}_{0,u}^m)^2 + (s_{0,u}^{m*} - s_{0,u}^m)^2$$

9:      $\tilde{\mathbf{P}}' \leftarrow \tilde{\mathbf{P}}^*, \mathbf{S}' \leftarrow \mathbf{S}^*$ 
10:   end while
11:   if  $i = 1$  then
12:     Set  $\tilde{\mathbf{P}}_{SE}^* = \tilde{\mathbf{P}}'$ ,  $\mathbf{S}_{SE}^* = \mathbf{S}'$ 
13:   else
14:     Set  $\tilde{\mathbf{P}}_{EE}^* = \tilde{\mathbf{P}}'$ ,  $\mathbf{S}_{EE}^* = \mathbf{S}'$ 
15:   end if
16:   Compute  $q_{\text{error}} = \mathcal{F}(q^i, \tilde{\mathbf{P}}', \mathbf{S})$ 
17:    $i \leftarrow i + 1$ 
18:   Compute  $q^i = \frac{\eta_{SE}^{\text{lb}}}{P_{\text{tot}}}$ , using (14) and (5).
19: end while
20: Output  $\tilde{\mathbf{P}}_{EE}^*, \tilde{\mathbf{P}}_{SE}^*, \mathbf{S}_{SE}^*$ , and  $\mathbf{S}_{EE}^*$ .

```

- Initialize the RUs and power allocation by doing steps 2, 3, and 4 of Algorithm II which distributes the total available power of each BS uniformly over RUs.
- Outer loop of Dinkelbach method from step 5 to step 19 that updates q iteratively till it reaches the maximum EE value upon convergence.
- Inner loop to find the solution of (P5) for a fixed value of q iteratively using SCA, where in each iteration, (P6) is solved using interior point method.

After finding $\tilde{\mathbf{P}}^*, \mathbf{P}^*$ is calculated by dividing $\tilde{\mathbf{P}}^*$ values by their corresponding continuous $\mathbf{S}$ values, finally the continuous $\mathbf{S}$ values are discretized to obtain a feasible solution for the original optimization problem according to the following rule: set $s_{y,z}^x = 1$ if $s_{y,z}^x > s_{y,z'}^x \quad \forall x, y, z$ where $z \neq z'$, $z, z' \in U_y$ otherwise set $s_{y,z}^x = 0$.

It is clear that the complexity of this resource allocation approach is higher than the previously mentioned approach

because here we have doubled the number of variables being optimized, number of constraints, and complicated the objective function.

C. Joint Power and Resource Units Allocation assuming Fixed Average Interference (JPRUAFAI) strategy

In the third strategy, we target a distributed algorithm where each cell will assume fixed interference that can be calculated for each cell independently by using the expected values of the interference channels and assuming that the other BSs will transmit with equal power distribution over all their RUs. Therefore, the spectral efficiency of the adopted system is expressed as

$$\eta_{SE,3} = \frac{1 - \alpha_{p,c}}{N \ln(2)} \sum_{m=1}^{N_M} \sum_{u=1}^K s_{0,u}^m \ln \left(1 + \frac{\hat{\gamma}_{0,u}^m p_{0,u}^m}{\beta_{0,u} G_{0,u} p_{0,u}^m + 1} \right) + \frac{1 - \alpha_p}{N \ln(2)} \sum_{n=1}^{N_{Ph}} \sum_{c=1}^{C_{Ph}} \sum_{u=1}^K s_{c,u}^n \ln \left(1 + \frac{\hat{\gamma}_{c,u}^n p_{c,u}^n}{\sum_{c' \neq c} \mathbb{E}\{\hat{\gamma}_{c',u}^n\} \frac{P_{Ph}}{N_{Ph}} + \beta_{c,u} G_{c,u} p_{c,u}^n + 1} \right) \quad (20)$$

Similar to the JPRUA strategy, we relax the binary constraints for $\mathbf{S}$ variables, and use the transformation of power variables $\tilde{p}_{y,z}^x = s_{y,z}^x p_{y,z}^x \quad \forall x, y, z$. As a result, the equivalent parameterized nonlinear non-fractional problem is expressed as

$$(\text{P8}) \quad \max_{\tilde{\mathbf{P}}, \mathbf{S}} \quad \mathcal{F}_3(q, \tilde{\mathbf{P}}, \mathbf{S}) = \eta_{SE,3} - q P_{\text{tot}}$$

subject to C3, C4, C9, C10, C11, C12, C13, C14

The Lagrangian formulation of (P8) is expressed as:

$$\mathcal{L} = \mathcal{F}_3(q, \tilde{\mathbf{P}}, \mathbf{S}) + \sum_{i=1}^{C_{Ph}} \lambda_i \left(P_{Ph} - (1 - \alpha_p) \kappa_{Ph} \sum_{n=1}^{N_{Ph}} \sum_{u=1}^K \tilde{p}_{i,u}^n - \alpha_p P_{Ph} \right) + \lambda_0 \left(P_M - (1 - \alpha_{p,c} \kappa_M) \sum_{m=1}^{N_M} \sum_{u=1}^K \tilde{p}_{0,u}^m - \alpha_{p,c} P_M \right) \quad (21)$$

where λ_0 represents the Lagrange multiplier associated with the power budget constraint of the macro cell, and λ_c represents the Lagrange multiplier associated with the power budget constraint of the phantom cell c .

Then, we express the dual problem of (P8) as

$$(\text{P9}) \quad \min_{\lambda \geq 0} \max_{\tilde{\mathbf{P}}, \mathbf{S}} \mathcal{L}(\lambda, \tilde{\mathbf{P}}, \mathbf{S}, q).$$

By applying the Karush-Kuhn-Tucker (KKT) conditions, the optimal power allocation can be found by

$$\frac{\partial \mathcal{L}}{\partial \tilde{p}_{c,u}^n} = 0, \quad \forall u, n, \quad \frac{\partial \mathcal{L}}{\partial \tilde{p}_{0,u}^m} = 0 \quad \forall u, m, \quad (22)$$

let $a = \sum_{c'=1, c' \neq c}^{P_h} \mathbb{E}\{\hat{\gamma}_{c',u}^n\} \frac{P_{\max}^{Ph}}{N_{Ph}} + 1$, $b = 2\beta_{c,u}G_{c,u} + \hat{\gamma}_{c,u}^n$, and $c = (\beta_{c,u}G_{c,u})^2 + \beta_{c,u}G_{c,u}\hat{\gamma}_{c,u}^n$. Thus, the optimal power allocation can be expressed as

$$p_{c,u}^{n*} = \left[\frac{-ab + \sqrt{a^2b^2 - 4c \left(a^2 - \frac{\hat{\gamma}_{c,u}^n a}{N \ln(2)(q\kappa_{Ph} + \lambda_c)} \right)}}{2c} \right]^+ \quad \forall c, u, n \quad (23)$$

$$p_{0,u}^{m*} = \left[\frac{-b + \sqrt{b^2 - 4c \left(1 - \frac{\hat{\gamma}_{0,u}^m}{N \ln(2)(q\kappa_M + \lambda_0)} \right)}}{2c} \right]^+ \quad \forall u, m \quad (24)$$

where $[x]^+ = \max(0, x)$, and x^* is the optimized value. Using the same argument in [14], [17], [20], [22], we allocate RUs according to the following rule

$$s_{0,u}^{m*} = \begin{cases} 1 & \text{if } \frac{\partial \mathcal{L}}{\partial s_{0,u}^m} = \max_j \frac{\partial \mathcal{L}}{\partial s_{0,j}^m}, j \in \{1 : K\} \text{ and } \frac{\partial \mathcal{L}}{\partial s_{0,u}^m} \geq 0 \\ 0 & \text{otherwise.} \end{cases} \quad (25)$$

$$s_{c,u}^{n*} = \begin{cases} 1 & \text{if } \frac{\partial \mathcal{L}}{\partial s_{c,u}^n} = \max_j \frac{\partial \mathcal{L}}{\partial s_{c,j}^n}, j \in U_c \text{ and } \frac{\partial \mathcal{L}}{\partial s_{c,u}^n} \geq 0 \\ 0 & \text{otherwise.} \end{cases} \quad (26)$$

where $\frac{\partial \mathcal{L}_{SE}}{\partial s_{0,u}^m}$ and $\frac{\partial \mathcal{L}_{SE}}{\partial s_{c,u}^n}$ are the first order derivatives with respect to $s_{0,u}^m$ and $s_{c,u}^n$ which are computed, respectively, from

$$\frac{\partial \mathcal{L}}{\partial s_{c,u}^n} = \frac{1 - \alpha_p}{N \ln(2)} \left(\ln \left(1 + \frac{\hat{\gamma}_{c,u}^n p_{c,u}^n}{a + \beta_{c,u}G_{c,u}p_{c,u}^n} \right) - \frac{\hat{\gamma}_{c,u}^n p_{c,u}^n a}{(a + \beta_{c,u}G_{c,u}p_{c,u}^n)(a + \beta_{c,u}G_{c,u}p_{c,u}^n + \hat{\gamma}_{c,u}^n p_{c,u}^n)} \right) \quad (27)$$

$$\frac{\partial \mathcal{L}}{\partial s_{0,u}^m} = \frac{1 - \alpha_{p,c}}{N \ln(2)} \left(\ln \left(1 + \frac{\hat{\gamma}_{0,u}^m p_{0,u}^m}{\beta_{0,u}G_{0,u}p_{0,u}^m + 1} \right) - \frac{\hat{\gamma}_{0,u}^m p_{0,u}^m}{(\beta_{0,u}G_{0,u}p_{0,u}^m + 1)(\beta_{0,u}G_{0,u}p_{0,u}^m + 1 + \hat{\gamma}_{0,u}^m p_{0,u}^m)} \right) \quad (28)$$

To find the power allocation solution from (24), (23), we need to know λ and q , which can be found iteratively using Algorithm III.

Throughout Algorithm III, we use the following notations to deal easily with both types of cells: the macro cell and the phantom cells. The generalized number of users per cell is defined as

$$N_l = \begin{cases} N_M & \text{if } l = 0 \\ N_{Ph} & \text{otherwise} \end{cases}, \quad (29)$$

and the power budget per cell is defined as

$$p_{\max}^l = \begin{cases} P_M & \text{if } l = 0 \\ P_{Ph} & \text{otherwise} \end{cases}. \quad (30)$$

It is worth noting that, the aforementioned solution of $\mathbf{S}$ satisfies the constraints C3 and C4. Now, we have the solution of $\max_{\mathbf{P}, \mathbf{S}} \mathcal{L}(\lambda, \mathbf{P}, \mathbf{S})$ for a certain value of λ , where λ is the vector consisting of $\lambda_0, \lambda_1, \dots, \lambda_{C_{Ph}}$. Hence, we solve the dual optimization problem by initializing λ , and computing $\mathbf{P}, \mathbf{S}$ accordingly. After that, we update the values of λ using the gradient descent method. The two previously mentioned computations are done repetitively till convergence, as can be found in Algorithm III.

Algorithm III

- 1: **Input** user assignments to phantom cells, ϵ_{outer} : maximum tolerable error for Dinkelbach's algorithm, ϵ_{inner} : maximum error tolerated in the complementary slackness of KKT condition.
- 2: **Set** $q_{\text{error}} = \infty$, $i = 0$ and $q^i = 0$
- 3: **while** $q_{\text{error}} > \epsilon_{\text{outer}}$ **do**
- 4: **Choose** λ such that we get a feasible power, and RUs allocation using (24), (23), (25) and (26).
- 5: **Set** $l = 0$
- 6: **while** $l \leq C_{Ph}$ **do**
- 7: **Set** $\text{CompSlack}_{\text{error}} = \infty$.
- 8: **while** $\text{CompSlack}_{\text{error}} > \epsilon_{\text{inner}}$ **do**
- 9: **Compute** $(\mathbf{P}, \mathbf{S})$ from (24), (23), (25) and (26)
- 10: **Compute** descent direction
- 11:
$$\vec{\lambda}_l = - \frac{p_{\max}^l - \sum_{u=1}^K \sum_{r=1}^{N_l} s_{l,u}^r \log_2(1 + \hat{\gamma}_{l,u}^r p_{l,u}^r)}{|p_{\max}^l - \sum_{u=1}^K \sum_{r=1}^{N_l} s_{l,u}^r \log_2(1 + \hat{\gamma}_{l,u}^r p_{l,u}^r)|}$$
- 12: **Compute** the descent step size λ_{step}
- 13: **Update** $\lambda_l \leftarrow \lambda_l + \lambda_{\text{step}} \vec{\lambda}_l$
- 14: **Compute** $\text{CompSlack}_{\text{error}}$ as:
- 15:
$$\left| \lambda_l \left(p_{\max}^l - \sum_{u=1}^K \sum_{r=1}^{N_l} s_{l,u}^r \log_2(1 + \hat{\gamma}_{l,u}^r p_{l,u}^r) \right) \right|$$
- 16: **end while**
- 17: $l \leftarrow l + 1$
- 18: **end while**
- 19: **if** $i = 1$ **then**
- 20: **Set** $\mathbf{P}_{SE}^* = \mathbf{P}, \mathbf{S}_{SE}^* = \mathbf{S}$
- 21: **else**
- 22: **Set** $\mathbf{P}_{EE}^* = \mathbf{P}, \mathbf{S}_{EE}^* = \mathbf{S}$
- 23: **end if**
- 24: **Compute** $q_{\text{error}} = \mathcal{F}(\mathbf{P}, \mathbf{S}, q^i)$
- 25: $i \leftarrow i + 1$
- 26: **Compute** $q^i = \frac{\eta_{SE}}{P_{\text{tot}}}$, using (20) and (5).
- 27: **end while**
- 28: **Output** $\mathbf{P}_{EE}^*, \mathbf{S}_{EE}^*, \mathbf{P}_{SE}^*$ and $\mathbf{S}_{SE}^*$.

The proposed algorithm consists of three nested loops to iteratively allocate the resources, which is summarized from inside to outside as follows,

- The first inner loop from step 8 to step 16 is responsible for solving the dual optimization problem (**P9**) using the gradient descent method to find λ_l for a given q and a cell l . The central part computes $(\mathbf{P}, \mathbf{S})$ for a given value of q and λ_l within a cell l as seen in step 9.
- The second inner loop from step 6 to step 18 is to switch between different cells.

- Finally, the outer loop from steps 3 to 27 is responsible for updating q based on Dinkelbach's method.

It can be deduced from the previous discussion that Algorithm III is a distributed algorithm to a great extent, as all calculations are cell-specific except for the update of q where there must be a central node to calculate the next update of q . All the provided algorithms in this section start with a zero initial value of q , which yields the optimized spectral efficiency solution. Thus, all the introduced algorithms can provide the optimized resource allocation strategies for both EE and SE optimization problems.

The complexity of this approach is the least of all the previously mentioned approaches because of the simplified version of the objective function and more importantly the existence of closed-form expressions for $\mathbf{P}^*$ in addition to the small number of dual variables that we are searching over numerically using gradient descent.

D. Computational complexity

The resource allocation strategies we provided, show a clear example for performance-complexity trade-off where it could be expected that best performance is accompanied with highest complexity. To quantify the complexity of each of the strategies, we define the number of power allocation variables in this setup as $\mathcal{N}$, where $\mathcal{N} = N_M K + \sum_{c=1}^{C_{Ph}} N_{Ph} |U_c| = (N_M + N_{Ph}) K$, where $|U_c|$ is the cardinality of the set U_c which is the set of users covered by phantom cell c , $\mathcal{M}_1, \mathcal{M}_2$ are the number of inequalities representing constraints of JPRUA and PAFRUA optimization problems respectively, and $\mathcal{F}$ is the cost of calculating the first and the second derivatives of the objective function and the functions used to express the constraints. Let I_x^D be the number of Dinkelbach algorithm iterations needed by strategy x to converge, I_x^{SCA} be the number of SCA iterations needed by strategy x to converge, and I^{grad} is number of iterations required to solve (P9). In most of the practical problems the number of iterations required for the interior point method algorithm to converge (α) is between 10 and 100 [23]. To simplify notation we will refer to JPRUA strategy by index 1, PAFRUA strategy by index 2, and JPRUAFAI strategy index 3 in the terms representing the number of iterations. Based on the previous definitions, the computational complexity of these strategies can be expressed as follows:

- JPRUA complexity:
 $O(I_1^D I_1^{SCA} \times \alpha \times \max\{(2\mathcal{N})^3, (2\mathcal{N})^2 \mathcal{M}_1, \mathcal{F}_1\})$
- PAFRUA complexity:
 $O(I_2^D I_2^{SCA} \times \alpha \times \max\{\mathcal{N}^3, \mathcal{N}^2 \mathcal{M}_2, \mathcal{F}_2\})$
- JPRUAFAI complexity:
 $O(I_3^D I^{\text{grad}} (C_{Ph} + 1) (3\mathcal{N}))$,

where $\mathcal{M}_1 = C_{Ph} N_{Ph} + N_m + \mathcal{N} + C_{Ph} + 1$, $\mathcal{M}_2 = \mathcal{N} + C_{Ph} + 1$, $\mathcal{F}_1 = 2(\mathcal{N} + \mathcal{M}_1) + 4(\mathcal{N} + \mathcal{M}_1)^2$, and $\mathcal{F}_2 = \mathcal{N} + \mathcal{M}_2 + (\mathcal{N} + \mathcal{M}_2)^2$.

TABLE I: Channel Model Parameters.

$h = h_{BS} = 20$ m	$f_c = 3500$ MHz	$\kappa_{Ph} = 5$	$W = 20$ m
$N_0 = -174$ dBm/Hz	$h_{UT} = 1.5$ m	$\kappa_M = 2.6$	

IV. SIMULATION RESULTS

In this section, we study the average global EE performance for the three resource allocation strategies developed in the aforementioned section. The benchmarks schemes are equal power allocation strategy and the JPRUA strategy with the relaxed continuous values of $\mathbf{S}$ (without discretization). The previously mentioned strategies are used to provide lower and upper bounds on the EE performance, respectively. Then, we can compare the proposed strategies relative performance with respect to them.

As for the simulation setup, we consider one macro cell having five phantom cells within its coverage. The macro cell is located at the origin with a coverage radius of 320m and the phantom cells locations are presented in Table A and their coverage radius is 77m. The radius is used to decide whether the user can be served by a given phantom cell or not. The channel estimation is performed using the same number of pilots at the macro and phantom cells. The maximum power is used for estimation and distributed equally among RUs. As for the users locations, we assume that users are random locations that follow a uniform distribution within each phantom cell coverage circle. We compute the average EE performance over at least 900 channel realizations. In all the simulations we assume that phantom cells are equally loaded with users (number of users in every phantom cell is the same). Unless otherwise stated, we use the simulation parameters values given in Table C. As for other simulation results, we use the parameters in Table C, unless otherwise specified. The adopted large scale channel model is similar to the one used in [9] where, the macro cell links experience zero-mean log-normal shadowing with 8 dB standard deviation and the Phantom cells links experience also a zero-mean log-normal shadowing but with 10 dB standard deviation [24]. The path loss for macro cell links is described by [9]

$$L_M = 35.3 + 37.6 \log_{10}(d_u) \text{ dB} \quad (31)$$

where d_u is the distance between the BS and user u in meters. On the other hand, the path loss for the links in the Phantom cells is described by [9], [25]

$$\begin{aligned} L_{Ph} = & 106.01 - 7.1 \log_{10}(W) + 7.5 \log_{10}(h) \\ & - (24.37 - 3.7(h/h_{BS})^2) \log_{10}(h_{BS}) - 3.2(\log_{10}(11.75h_{UT}))^2 \\ & + (43.42 - 3.1 \log_{10}(h_{BS}))(\log_{10}(d) - 3) + 20 \log_{10}(f_c) \end{aligned} \quad (32)$$

where W is the street width in meters, h_{BS} is the BS height in meters, h is average building height in meters, h_{UT} is the average user height in meters, f_c is the carrier frequency in MHz, and d is the distance between the BS and the user in meters. The small scale fading is modeled as multipath fading with a six-ray typical urban model as described in [24] with 3 Km/hr speed.

In the first simulation, we vary the number of deployed phantom cells from one cells to five cells and plot the EE of the proposed strategies along with the benchmarks, as depicted

Cell ID	Cell center x-coordinate	Cell center y-coordinate
1	0	0
2	-109.12	-109.12
3	-109.12	109.12
4	109.12	109.12
5	109.12	-109.12

Number of activated cells	Active Cells indices
1	1
2	1,2
3	1,2,3
4	1,2,3,4
5	1,2,3,4,5

P_M (W)	50	K	10	C_{Ph}	5	N	8
P_{Ph} (W)	1	ϵ	10^{-9}	N_{Ph}	4	N_M	4

Fig. 3: Energy efficiency vs number of active phantom cells

in Fig. 3. The indices of the activated cells for different values of number of phantom cells are indicated in Table B. From the simulation results, we observe first that the JPRUA strategy performance converges to its relaxed version benchmark as the number of deployed phantom cells increases. On the other hand, at $C_{Ph} = 2$, the discretization effect becomes critical and the EE performance of JPRUA strategy degrades with respect to the other strategies. The impact of increasing C_{Ph} on the relative performance of the JPRUA strategy and its relaxed version is similar to the increase of number of subcarriers in [17]–[20], i.e. relaxing S variables gives good results for large C_{Ph} values. As for the EE performance, increasing C_{Ph} has two opposite impacts on the EE performance. The increase of C_{Ph} improves the SE of the underlying system as a result of increasing the number of possible communication channels per user. On the other hand, the total fixed energy consumption increases as well as the total interference level of the network. The aforementioned contradicting effects balance each other and cause a saturation in the EE performance for the simulation scenario depicted in Fig. 7 and Fig. 3.

In the second simulation, we vary the maximum transmission power budget available from 1 Watt to 9 Watts for 4 phantom cells and monitor the EE performance of the whole setup as illustrated in Fig. 4. The simulation shows that as P_{Ph} increases, the general trend of the EE performance of the optimized strategies remains approximately the same. On

Fig. 4: Energy efficiency vs Phantom cells maximum power

the other hand, the equal power allocation strategy degrades the EE performance of the system because it forces using the whole budget even when it is not efficient to use more power. This means that, the optimal EE solution of the resource allocation problem is not making use of the whole available power. This simulation shows that regardless of the variation of P_{Ph} the optimality of each of the three proposed strategies does not change and the performance gaps is approximately constant between all the proposed strategies.

In the third simulation, we study the effect of increasing the number of pilots on the EE performance of the network for different strategies. To this end, we assume that the number of OFDM pilots changes from 1 to 5 OFDM symbols. We aim at observing the trade-off occurring due to non-perfect channel estimation impairment. More specifically, as the number of pilots increases, two opposite effects take place. The first one is reducing the channel estimation error, which improves the SINR and consequently the spectral efficiency. On the other hand, increasing number of pilots reduces the available time duration for data transmission, which reduces the spectral efficiency. Moreover, the pilots allocated power increases the BS energy consumption of the whole system. According to the maximum power budget parameters, the simulation results indicate that increasing number of pilots will be an energy inefficient choice. It is worth to mention that such trend depends mainly on the maximum power budget available for the pilot, which may result in a unimodal behavior for low pilot power budget as discussed in [26]. The simulation also indicates that the relative EE performance of the three strategies is independent of changing the number of pilots. Furthermore, the best EE performance can be achieved using JPRUA strategy with a relatively big gap from the second ranked strategy (PAFRUA), and JPRUAFAI that achieves the least EE performance. However, the performance gap between PAFRUA and JPRUAFAI is not large, which motivates employing either the JPRUAFAI strategy to avoid additional computational complexity, or the JPRUA strategy to achieve a significantly higher EE performance on the expense of a

Fig. 5: Energy efficiency vs Number of pilots used

Fig. 6: Energy efficiency vs Number of RUs assigned to Phantom cells tier

higher computational complexity.

In the fourth simulation, we investigate the relation between the RUs division between the macro cell tier and the phantom cells tier from an EE perspective as indicated in Fig. 6. We vary the number of RUs available at the Phantom BSs from 1 to 5 while the total number of RUs in the system is kept fixed. The simulation results show that it is always better to make the phantom cells share of RUs as large as possible to improve the EE of the system. Moreover, it indicates that for small number of RUs being reserved for phantom cells usage, the three proposed resource allocation strategies achieve very close EE performance which means there is no need for any added complexity and the network should use JPRUAFAI strategy. However, for large phantom RUs share using JPRUA strategy gives a significant added value in terms of EE performance. Interestingly, this simulation shows that the EE performance gap between JPRUA and its upper bound JPRUA diminishes as $\frac{N_{Ph}}{N}$ increases, which motivates using JPRUA strategy and increasing $\frac{N_{Ph}}{N}$ to get significant EE performance enhancement. The closeness in terms of performance of the JPRUA strategy with and without $\mathbf{S}$ variables relaxation matches the results in [17]–[20] when large number of subcarriers is used.

In the fifth simulation, we study how the number of users in the setup affects the performance of the three proposed strategies. The simulation results presented in Fig. 7 show that all the proposed strategies perform better as the number of users increases. Because this increases the probability of finding users having good channel conditions and consequently, increases the average energy efficiency of the network. It is expected that this curve saturates as number of users gets very large as the probability to find better channel conditions becomes very low. In the sixth simulation we use a different setup where only four phantom cells are placed symmetrically around the origin with tangential virtual coverage circles, and users are associated to the nearest phantom cell (coverage circles are not used in users-cells association anymore). We place two users in each of the four quadrants around the macro

Fig. 7: Energy efficiency vs Total number of users

cell where all the users are located at the same distance from the macro cell BS but the azimuth angles of the user locations are drawn randomly from a uniform distribution spanning the values from 0 to $\pi/2$. In that simulation we monitor two metrics; the average EE performance of the whole setup shown in Fig. 8 and the average channel estimation error variance of the link between any of the users and all the BSs presented in Fig. 9.

The sixth simulation results show that at first as users get further from the macro BS they get closer to their serving phantom cells so the average EE performance increases till it reaches a peak where users are on average located at their serving BS. Then as users move away more from the macro cell they move further from their serving phantom cell and consequently the average EE performance deteriorates. This proportionality between average EE and the distance between users and their serving phantom cells BSs is based on two facts; pathloss increases as that distance increases, moreover, as the distance increases between a user and a given BS the

Fig. 8: Energy efficiency vs Distance of users from macro cell

Fig. 9: Variance of channel estimation error vs Distance of users from macro cell

channel between that BS and the user will get worse. It is clear from Fig. 9 that as the user approaches phantom cell 1 (serving cell) channel estimation error reaches its minimum while the estimation error of the channels between that user and the interfering BS decreases as the user moves away from all these BSs.

V. CONCLUSION

In this paper, we formulated the EE resource allocation problem for the densely deployed phantom cellular network, taking into account non-perfect CSI at the transmitters. The problem is a mixed integer nonlinear fractional programming problem. To solve this problem, we proposed three resource allocation strategies to maximize the EE performance subject to the constraints imposed by the system architecture. In the proposed resource allocation strategies, we first transformed the optimization problem from its fractional programming form to its parametric form. Then, we proposed different

resource allocation strategies with different degrees of complexity and distributability. The PAFRUA strategy simplifies the optimization problem and optimizes the power allocation for given RUs allocation configuration that was previously calculated to maximize the average SINR. On the other hand, the JPRUA strategy optimizes both variables through relaxing the integer variables and proposing a lower bound that is expressed as a DC function of the design variables. Different from the aforementioned strategies that are implemented in a central fashion, the JPRUAFAI strategy is considered a quasi-distributed scheme. The latter strategy has the lowest computational complexity since it optimizes an interference free version of the problem and uses closed form expressions for the power allocation variables.

The simulation results have shown the superiority of the JPRUA strategy in terms of performance where the number of variables existing in the problem becomes large and the objective functions of the two other strategies deviate significantly from the original objective function. Furthermore, simulation results showed that it is enough to use 1 OFDM symbol for pilots transmission, because as their number increases, the EE of the system exhibits deterioration. In addition, raising the maximum power available for phantom cells transmission degrades the EE performance of the system. Finally, the last simulation indicated that the ratio used to divide RUs between the macro cell tier and the phantom cell tier BSs should be biased towards phantom cells as this will be very beneficial for the EE of the whole setup.

VI. ACKNOWLEDGMENT

This work was made possible by NPRP grant no 6-001-2-001 from the Qatar National Research Fund (A member of The Qatar Foundation). The statements made herein are solely the responsibility of the authors.

APPENDIX A

In this appendix we prove the non-concavity of η_{SE} expressed in eq.(13).

If we consider a scenario where we have two phantom cells each of which hosts only one user, and both users are using the same RU so they are interfering on each other. By neglecting the term representing the macro cell contribution in the η_{SE} of eq.(4)

We will have

$$\eta_{SE}^{Ph} = s_1 \ln \left(1 + \frac{\gamma_{1,1} \tilde{p}_1 / s_1}{1 + \gamma_{2,1} \tilde{p}_2 + \beta_{1,1} \tilde{p}_1 / s_1} \right) + s_2 \ln \left(1 + \frac{\gamma_{2,2} \tilde{p}_2 / s_2}{1 + \gamma_{1,2} \tilde{p}_1 + \beta_{2,2} \tilde{p}_2 / s_2} \right)$$

By computing the second derivative of η_{SE}^{Ph} with respect to $\tilde{p}_2$

$$\frac{\partial^2 \eta_{SE}^{Ph}}{\partial \tilde{p}_2^2} = \frac{s_1^3 \gamma_{1,1} \gamma_{2,1}^2 (2s_1 + \tilde{p}_1 (2\beta_{1,1} + \gamma_{1,1}) + 2s_1 \tilde{p}_2 \gamma_{2,1})}{(s_1 + \beta_{1,1} \tilde{p}_1 + s_1 \tilde{p}_2 \gamma_{2,1})^2 (s_1 + \tilde{p}_1 (\beta_{1,1} + \gamma_{1,1}) + s_1 \tilde{p}_2 \gamma_{2,1})^2} - \frac{s_2^3 \gamma_{2,2} (1 + \gamma_{1,2} \tilde{p}_1) (2\beta_{2,2} \tilde{p}_1 (\beta_{2,2} + \gamma_{2,2}))}{(s_2 + \beta_{2,2} \tilde{p}_2 + s_2 \tilde{p}_1 \gamma_{1,2})^2 (s_2 + \tilde{p}_2 (\beta_{2,2} + \gamma_{2,2}) + s_2 \tilde{p}_1 \gamma_{1,2})^2} - \frac{s_2 (2\beta_{2,2} + \gamma_{2,2}) + s_2 \tilde{p}_1 (2\beta_{2,2} + \gamma_{2,2}) \gamma_{1,2}}{(s_2 + \beta_{2,2} \tilde{p}_2 + s_2 \tilde{p}_1 \gamma_{1,2})^2 (s_2 + \tilde{p}_2 (\beta_{2,2} + \gamma_{2,2}) + s_2 \tilde{p}_1 \gamma_{1,2})^2}$$

It can be noticed that by substituting with $s_2 = 0$ and $s_1 > 0$ in the previous equation, $\frac{\partial^2 \eta_{SE}^{Ph}}{\partial p_2^2} \Big|_{s_2=0} > 0$ which means that η_{SE}^{Ph} is not concave in $\tilde{p}_2$ for a part of the feasibility region of our optimization problem and this proves its non-concavity.

APPENDIX B

In this appendix we prove that $\eta_{SE}^{lb}(\tilde{\mathbf{P}}, \mathbf{S})$ can be expressed as DC functions in both $\mathbf{S}, \mathbf{P}$.

$$\eta_{SE}^{lb} = \eta_{SEPh}^{lb} + \eta_{SEM}^{lb} \quad (33)$$

$$\eta_{SEPh}^{lb}(\tilde{\mathbf{P}}, \mathbf{S}) = h_1(\tilde{\mathbf{P}}, \mathbf{S}) - h_2(\tilde{\mathbf{P}}, \mathbf{S}) \quad (34)$$

It can be observed that η_{SEM}^{lb} is a positive linear combination of functions of the form $g(x, y) = y \log \left(1 + \frac{ax}{1 + \frac{bx}{y}} \right)$ which is the perspective transform of the function $g_2(x) = \log \left(1 + \frac{ax}{1 + bx} \right)$. It can be noticed that $g_2(x)$ is the composition of the concave function $\log(x)$ with the concave function $1 + \frac{ax}{1 + bx}$ which proves the concavity of $g_2(x)$, consequently the concavity of $g(x, y)$ and the concavity of η_{SEM}^{lb} .

$$h_1(\tilde{\mathbf{P}}, \mathbf{S}) = \frac{1 - \alpha_p}{N \ln(2)} \sum_{n=1}^{N_{Ph}} \sum_{c=1}^{C_{Ph}} \sum_{u=1}^K s_{c,u}^n \ln \left(1 + \frac{\hat{\gamma}_{c,u}^n \tilde{p}_{c,u}^n}{s_{c,u}^n} + \sum_{\substack{c' \neq c \\ u'}} \frac{\hat{\gamma}_{c',u'}^n \tilde{p}_{c',u'}^n}{s_{c,u}^n} + \frac{\beta_{c,u} \tilde{p}_{c,u}^n}{s_{c,u}^n} \right) \quad (35)$$

$$h_2(\tilde{\mathbf{P}}, \mathbf{S}) = \frac{1 - \alpha_p}{N \ln(2)} \left(\sum_{n=1}^{N_{Ph}} \sum_{c=1}^{C_{Ph}} \sum_{u=1}^K s_{c,u}^n \ln \left(1 + \frac{\beta_{c,u} \tilde{p}_{c,u}^n}{s_{c,u}^n} + \sum_{\substack{c' \neq c \\ u'}} \frac{\hat{\gamma}_{c',u'}^n \tilde{p}_{c',u'}^n}{s_{c,u}^n} \right) \right) \quad (36)$$

It can be noted that both $h_1(\tilde{\mathbf{P}}, \mathbf{S}), h_2(\tilde{\mathbf{P}}, \mathbf{S})$ are positive linear combinations of functions of the following form:

$$f(x, y, z) = x \ln \left(1 + a \frac{y}{x} + \frac{\mathbf{b}^T \mathbf{z}}{x} \right) \quad (37)$$

In fact, $f(x, y, z)$ is the perspective transform of $f_2(y, z)$, where the perspective transform of a function $U(\mathbf{k})$ is defined as $T(\mathbf{k}, x) = x f_2 \left(\frac{\mathbf{k}}{x} \right)$. It is proven in [23] that the perspective transform maintains convexity/concavity of the function being transformed. So the convexity of $f(x, y, z)$ follows the convexity of $f_2(y, z)$, where

$$f_2(y, z) = \ln \left(1 + ay + \mathbf{b}^T \mathbf{z} \right) \quad (38)$$

Now, $f_2(y, z)$ is concave in both y, z because it is the composition of a concave function $\ln(\cdot)$ with the positive linear combination $1 + ay + \mathbf{b}^T \mathbf{z}$.

VII. APPENDIX C

In this section, we provide the proof for eq. 12:

$$\begin{aligned} g(\mathbf{P}) &= \frac{1 - \alpha_p}{N \ln(2)} \sum_{n=1}^{N_{Ph}} \sum_{c=1}^{C_{Ph}} \sum_{u=1}^K s_{c,u}^n \ln \left(1 + \sum_{\substack{c' \neq c \\ u'}} s_{c',u'}^n \hat{\gamma}_{c',u'}^n p_{c',u'}^n + \beta_{c,u} G_{c,u} p_{c,u}^n \right) \\ g(\mathbf{P}) &= \frac{1 - \alpha_p}{N \ln(2)} \sum_{n=1}^{N_{Ph}} \sum_{\substack{c=1 \\ c \neq y}}^{C_{Ph}} \sum_{u=1}^K s_{c,u}^n \ln \left(1 + \sum_{\substack{c' \neq c \\ u'}} s_{c',u'}^n \hat{\gamma}_{c',u'}^n p_{c',u'}^n + \beta_{c,u} G_{c,u} p_{c,u}^n \right) \\ &+ \frac{1 - \alpha_p}{N \ln(2)} \sum_{n=1}^{N_{Ph}} \sum_{u=1}^K s_{y,u}^n \ln \left(1 + \sum_{c' \neq y, u'} s_{c',u'}^n \hat{\gamma}_{c',u'}^n p_{c',u'}^n + \beta_{y,u} G_{y,u} p_{y,u}^n \right) \\ \frac{\partial g}{\partial p_{y,z}^x} &= \frac{1 - \alpha_p}{N \ln(2)} \sum_{n=1}^{N_{Ph}} \sum_{\substack{c=1 \\ c \neq y}}^{C_{Ph}} \sum_{u=1}^K \frac{\partial}{\partial p_{y,z}^x} s_{c,u}^n \ln \left(1 + \sum_{\substack{c' \neq c \\ u'}} s_{c',u'}^n \hat{\gamma}_{c',u'}^n p_{c',u'}^n + \beta_{c,u} G_{c,u} p_{c,u}^n \right) \\ &+ \frac{1 - \alpha_p}{N \ln(2)} \sum_{n=1}^{N_{Ph}} \sum_{\substack{c=1 \\ c \neq y}}^{C_{Ph}} \sum_{u=1}^K \frac{\partial}{\partial p_{y,z}^x} s_{c,u}^n \ln \left(1 + \sum_{\substack{c' \neq c \\ u'}} s_{c',u'}^n \hat{\gamma}_{c',u'}^n p_{c',u'}^n + \beta_{c,u} G_{c,u} p_{c,u}^n \right) \\ \frac{\partial g}{\partial p_{y,z}^x} &= \frac{1 - \alpha_p}{N \ln(2)} \sum_{n=1}^{N_{Ph}} \sum_{\substack{c=1 \\ c \neq y}}^{C_{Ph}} \sum_{u=1}^K \frac{\partial}{\partial p_{y,z}^x} s_{c,u}^n \ln \left(1 + \sum_{\substack{c' \neq c \\ u'}} s_{c',u'}^x \hat{\gamma}_{c',u'}^x p_{c',u'}^x + \beta_{c,u} G_{c,u} p_{c,u}^x \right) \\ &+ \frac{1 - \alpha_p}{N \ln(2)} \sum_{u=1}^K \frac{\partial}{\partial p_{y,z}^x} s_{y,u}^x \ln \left(1 + \sum_{\substack{c' \neq y \\ u'}} s_{c',u'}^x \hat{\gamma}_{c',u'}^x p_{c',u'}^x + \beta_{y,u} G_{y,u} p_{y,u}^x \right) \\ \frac{\partial g}{\partial p_{y,z}^x} &= \frac{1 - \alpha_p}{N \ln(2)} \sum_{\substack{c=1 \\ c \neq y}}^{C_{Ph}} \sum_{u=1}^K \frac{\partial}{\partial p_{y,z}^x} s_{c,u}^x \ln \left(1 + \sum_{\substack{c' \neq c \\ u'}} s_{c',u'}^x \hat{\gamma}_{c',u'}^x p_{c',u'}^x + \beta_{c,u} G_{c,u} p_{c,u}^x \right) \\ &+ \frac{1 - \alpha_p}{N \ln(2)} \sum_{\substack{u=1 \\ u \neq z}}^K \frac{\partial}{\partial p_{y,z}^x} s_{y,u}^x \ln \left(1 + \sum_{\substack{c' \neq y \\ u'}} s_{c',u'}^x \hat{\gamma}_{c',u'}^x p_{c',u'}^x + \beta_{y,u} G_{y,u} p_{y,u}^x \right) \\ &+ \frac{1 - \alpha_p}{N \ln(2)} \frac{\partial}{\partial p_{y,z}^x} s_{y,z}^x \ln \left(1 + \sum_{\substack{c' \neq y \\ u'}} s_{c',u'}^x \hat{\gamma}_{c',u'}^x p_{c',u'}^x + \beta_{y,z} G_{y,z} p_{y,z}^x \right) \\ \frac{\partial g}{\partial p_{y,z}^x} &= \frac{1 - \alpha_p}{N \ln(2)} \left(\sum_{\substack{c=1 \\ c \neq y}}^{C_{Ph}} \sum_{u=1}^K \frac{s_{c,u}^x s_{y,z}^x \hat{\gamma}_{y,u}^x}{\sum_{\substack{c' \neq c \\ u'}} s_{c',u'}^x \hat{\gamma}_{c',u'}^x p_{c',u'}^x + \beta_{c,u} G_{c,u} p_{c,u}^x + 1} \right. \\ &\left. + \frac{s_{y,z}^x \beta_{y,z}}{\sum_{\substack{c' \neq y \\ u'}} s_{c',u'}^x \hat{\gamma}_{c',u'}^x p_{c',u'}^x + \beta_{y,z} G_{y,z} p_{y,z}^x + 1} \right) \end{aligned}$$

REFERENCES

- [1] S. Gringeri, E. B. Basch, and T. J. Xia, "Technical considerations for supporting data rates beyond 100 Gb/s," *IEEE Commun. Mag.*, vol. 50, no. 2, pp. S21–S30, Feb. 2012.
- [2] A. Fehske, G. Fettweis, J. Malmodin, and G. Biczók, "The global footprint of mobile communications: The ecological and economic perspective," *IEEE Commun. Mag.*, vol. 49, no. 8, pp. 55–62, Aug. 2011.
- [3] O. Amin, S. Bavarian, and L. Lampe, "Cooperative techniques for energy-efficient wireless communications," in *Green Radio Communication Networks*. Cambridge University Press, 2012, pp. 125–149.
- [4] B. Du, C. Pan, W. Zhang, and M. Chen, "Distributed energy-efficient power optimization for CoMP systems with max-min fairness," *IEEE Commun. Lett.*, vol. 18, no. 6, pp. 999–1002, Jun 2014.
- [5] Z. Chang, T. Ristaniemi, and Z. Niu, "Radio resource allocation for collaborative OFDMA relay networks with imperfect channel state information," *IEEE Trans. Wireless Commun.*, vol. 13, no. 5, pp. 2824–2835, May 2014.
- [6] Z. Chang, Q. Zhang, X. Guo, and T. Ristaniemi, "Energy-efficient resource allocation for OFDMA two-way relay networks with imperfect CSI," *EURASIP Journal on Wireless Communications and Networking*, vol. 2015, no. 1, p. 225, 2015.

- [7] R. Hu and Y. Qian, "An energy efficient and spectrum efficient wireless heterogeneous network framework for 5G systems," *IEEE Commun. Mag.*, vol. 52, no. 5, pp. 94–101, Nov. 2014.
- [8] Z. Chang, Z. Han, and T. Ristaniemi, "Energy efficient optimization for wireless virtualized small cell networks with large scale multiple antenna," *IEEE Trans. Commun.*, DOI:10.1109/TCOMM.2017.2666182, 2017.
- [9] H. Ishii, Y. Kishiyama, and H. Takahashi, "A novel architecture for LTE-B: C-plane/U-plane split and phantom cell concept," in *Proc. Global Communications Conference (GLOBECOM)*, California, 2012, pp. 624–630.
- [10] B. Yu, L. Yang, H. Ishii, and S. Mukherjee, "Dynamic TDD support in macrocell-assisted small cell architecture," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 6, pp. 1201–1213, 2015.
- [11] S. Mukherjee and H. Ishii, "Energy efficiency in the phantom cell enhanced local area architecture," in *Wireless Communications and Networking Conference (WCNC), 2013 IEEE*, Shanghai, 2013, pp. 1267–1272.
- [12] H. Ibrahim, H. Elsayy, U. T. Nguyen, and M.-S. Alouini, "Modeling virtualized downlink cellular networks with ultra-dense small cells," in *IEEE International Conference on Communications (ICC)*, London, 2015, pp. 5360–5366.
- [13] Z. Chen, L. Qiu *et al.*, "Energy-efficient combination of small cells and multi-antenna under separation architecture," *IEEE Commun. Lett.*, vol. 19, no. 9, pp. 1572–1575, Sep. 2015.
- [14] A. M. Abdelhady, O. Amin, and M.-S. Alouini, "Resource allocation for phantom cellular networks: Energy efficiency vs spectral efficiency," in *International Conference on Communications (ICC)*, Kuala Lumpur, 2016.
- [15] S. Sesia, I. Toufik, and M. Baker, *LTE: the UMTS long term evolution*. Wiley Online Library, 2009.
- [16] S. Ohno and G. B. Giannakis, "Capacity maximizing MMSE-optimal pilots for wireless OFDM over frequency-selective block Rayleigh-fading channels," *IEEE Trans. Inf. Theory*, vol. 50, no. 9, pp. 2138–2145, Sep. 2004.
- [17] W. Yu and R. Lui, "Dual methods for nonconvex spectrum optimization of multicarrier systems," *IEEE Trans. Commun.*, vol. 54, no. 7, pp. 1310–1322, Jul. 2006.
- [18] K. Seong, M. Mohseni, and J. M. Cioffi, "Optimal resource allocation for OFDMA downlink systems," in *IEEE International Symposium on Information Theory (ICIT)*, Washington, 2006, pp. 1394 – 1398.
- [19] K. T. K. Cheung, S. Yang, and L. Hanzo, "Achieving maximum energy-efficiency in multi-relay OFDMA cellular networks: A fractional programming approach," *IEEE Trans. Commun.*, vol. 61, no. 7, pp. 2746–2757, Jul. 2013.
- [20] E. Bedeer, A. Alorainy, M. Hossain, O. Amin, and M.-S. Alouini, "Fairness-aware energy-efficient resource allocation for AF cooperative OFDMA networks," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 12, pp. 2478–2493, Dec. 2015.
- [21] W. Dinkelbach, "On nonlinear fractional programming," *Management Science*, vol. 13, no. 7, pp. 492–498, Mar. 1967.
- [22] C. Y. Wong, R. S. Cheng, K. B. Lataief, and R. D. Murch, "Multiuser OFDM with adaptive subcarrier, bit, and power allocation," *IEEE J. Sel. Areas Commun.*, vol. 17, no. 10, pp. 1747–1758, Oct. 1999.
- [23] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge university press, 2004.
- [24] 3GPP, "Spatial channel model for Multiple Input Multiple Output (MIMO) simulations," 3rd Generation Partnership Project (3GPP), TR 25.996, 12 2009. [Online]. Available: <http://www.3gpp.org/ftp/Specs/html-info/25996.htm>
- [25] M. Series, "Guidelines for evaluation of radio interface technologies for IMT-advanced," *Report ITU*, pp. 2135–1, 2009.
- [26] O. Amin, E. Bedeer, M. H. Ahmed, O. Dobre, and M.-S. Alouini, "Opportunistic energy-aware amplify-and-forward cooperative systems with imperfect CSI," *IEEE Trans. Veh. Technol.*, vol. 65, no. 7, pp. 4875–4886, Jul. 2016.

Amr M. Abdelhady (S'16) received B. Sc. degree with honors in Communications and Computer Engineering from Cairo University, Giza, Egypt, in 2012, M. Sc. degree in Electrical Engineering from King Abdullah University of Science and Technology (KAUST), Thuwal, Makkah, Kingdom of Saudi Arabia in 2016. Currently, he is a PhD student with King Abdullah University of Science and Technology. His general research interests lie in communications theory, signal processing for communications with special emphasis on optical wireless communications systems. Specific research areas include visible light communications, green communications, energy harvesting, cognitive radio, and cooperative communications.

Osama Amin (S'07, M'11, SM'15) received B. Sc. degree in Electrical and Electronics Engineering from Aswan University, Aswan, Egypt, in 2000, M. Sc. degree in Electrical Engineering from Assiut University, Assiut, Egypt in 2004 and Ph.D. degree in Electrical and Computer Engineering, University of Waterloo, Canada in 2010. In June 2012, he joined Assiut University as an Assistant Professor in the Electrical and Electronics Engineering department. Currently, he is with King Abdullah University of Science and Technology (KAUST), Thuwal, Makkah, Kingdom of Saudi Arabia. His general research interests lie in communications systems and signal processing for communications with special emphasis on wireless applications. Specific research areas include green communications, cognitive radio, cooperative communications, and channel estimation.

Dr. Amin is an Editor for the *IEEE Communications Letters*. He has served as a technical program committee (TPC) member for *ICC*, *IEEE VTC*, *CROWNCOM*, *PIMRC*, and *ISSPIT* conferences. He has served also as co-organizer and a co-chair of the Next Generation Green ICT and 5G Networking (*GreeNets*) 2015 in the IEEE International Conference on Ubiquitous Wireless Broadband (*ICUWB*) in Montreal, Canada.

Mohamed-Slim Alouini was born in Tunis, Tunisia. He received the Ph.D. degree in Electrical Engineering from the California Institute of Technology (Caltech), Pasadena, CA, USA, in 1998. He served as a faculty member in the University of Minnesota, Minneapolis, MN, USA, then in the Texas A&M University at Qatar, Education City, Doha, Qatar before joining King Abdullah University of Science and Technology (KAUST), Thuwal, Makkah Province, Saudi Arabia as a Professor of Electrical Engineering in 2009. His current research interests

include the modeling, design, and performance analysis of wireless communication systems.