Downlink Error Rates of Half-duplex Users in Full-duplex Networks over a Laplacian Inter-User Interference Limited and EGK fading

Hamza Soury, *Student Member, IEEE*, Hesham ElSawy, *Member, IEEE*,
and Mohamed-Slim Alouini, *Fellow, IEEE*

Abstract—This paper develops a mathematical framework to study downlink error rates and throughput for half-duplex (HD) terminals served by a full-duplex (FD) base station (BS). The developed model is used to motivate long term pairing for users that have non-line of sight (NLOS) interfering link. Consequently, we study the interferer limited problem that appears between NLOS HD users-pair that are scheduled on the same FD-channel. The distribution of the interference is first characterized via its distribution function, which is derived in closed form. Then, a comprehensive performance assessment for the proposed pairing scheme is provided by assuming Extended Generalized- $\mathcal{K}$ (EGK) fading for the downlink and studying different modulation schemes. To this end, a unified closed form expression for the average symbol error rate is derived. Furthermore, we show the effective downlink throughput gain harvested by the pairing NLOS users as a function of the average signal-to-interference-ratio when compared to an idealized HD scenario with neither interference nor noise. Finally, we show the minimum required channel gain pairing threshold to harvest downlink throughput via the FD operation when compared to the HD case for each modulation scheme.

Index Terms—Interference limited, complex Gaussian, characteristic function, Laplace distribution, symbol error rate, EGK fading

I. INTRODUCTION

In-band full-duplex (FD) communication is introduced as a promising technology that would provide several benefits to cellular networks. Compared to its half-duplex (HD) counterpart, FD communication has the potential to improve spectrum utilization, increase link capacity, enhance physical layer security, and reduce relaying latency [1]. The aforementioned benefits brought by FD communication are all derived from its ability to simultaneously transmit and receive within the same frequency band. The ability of simultaneous transmission and reception for FD transceivers emerges from recent advances in radio frequency circuit design that enables sufficient self-interference (SI) cancellation [2], [3], and hence, eliminates the necessity for transmission/reception orthogonalization employed by HD transceivers [1], [4]–[6]. To reap the aforementioned benefits of FD communication, FD enabled transceivers are required at both sides of the

The authors are with King Abdullah University of Science and Technology (KAUST), Thuwal, Makkah Province, Kingdom of Saudi Arabia. The authors are members of the KAUST Strategic Research Initiative (SRI) in Uncertainty Quantification. E-mail: {soury.hamza, hesham.elsawy, slim.alouini}@kaust.edu.sa.

Fig. 1: Channel assignment in the 3NT FD and HD schemes.

communication link, which is hard to realize in the context of cellular networks.

Cellular networks operators can only enforce FD upgrade in their networks from the base stations' (BS) side and do not have direct access to upgrade the users equipment (UE) side. It has been shown in [7] that the residual self interference amplitude can be degraded significantly from -20dB to -60dB for frequencies between 2.35GHz and 2.45GHz. However, implementing SI cancellation at the UE can be expensive in terms of complexity, power consumption, and/or terminal price. Hence, backward compatibility between FD BSs and HD UEs is required especially at the early FD rollout phase. In this context, the 3-node topology (3NT) is proposed to serve HD users via FD BSs and yet benefit from FD communication [8], [9]. In the 3NT, the BS groups the UEs into pairs and simultaneously reuses the uplink channel of one UE in the downlink direction of the other UE in each pair, as shown in Fig. 1. Compared to the HD case, the 3NT requires half the number of channels to serve the same number of users, and hence, the bandwidth (BW) occupied by each channel can be doubled [10].

Doubling the channel BW directly implies doubling the transmission rate, but not necessarily doubling the throughput. This is because the BW improvement, offered by the 3NT, comes at the expense of creating SI at the BS side and inter-user interference at the UE side, which impose decoding errors. The BS exactly knows the interfering codeword and can accurately estimate the SI channel, and hence, SI can be sufficiently suppressed using SI cancellation techniques [1],

[4]–[6]. However, the UE does not have information about the signal being transmitted by the interfering UE, and hence, interference cancellation is not always viable. The authors in [8] proposed an interference alignment technique to eliminate the inter-user interference between the UEs pair coupled on the same channel. The authors in [11] mitigate the inter-user interference via an optimal channel allocation and power control that maximizes the system performance. In addition, in [12], [13] the authors investigate the user scheduling and power allocation by maximizing the system sum rate. However, the algorithms in [8], [11]–[13] require the instantaneous mutual channel state information between UEs, which is hard to estimate and/or communicate. A more practical way to mitigate the inter-user interference is to pair interfering users in the 3NT based on their statistical mutual channel conditions, which requires statistical studies for the inter-user interference problem.

A. Motivation and objectives

This paper proposes pairing users according to their long term statistical channel conditions.¹ Such user pairing criterion only requires the statistical information about the mutual channel gains between the users rather than the instantaneous channel gains, which will relief the computational complexity of the scheduling procedure and reduce signaling overhead. Particularly, we propose pairing users that have non-line of sight (NLOS) mutual interfering links. To support such pairing methodology, a comprehensive performance assessment is provided. First, assuming Rayleigh fading on the interference link², the inter-user interference is characterized by deriving closed form expressions for its probability density function (PDF), cumulative density function (CDF), and characteristic function (CHF). Then, the error rate performance is assessed for several downlink fading channels and modulation schemes. Specifically, the interference distribution is exploited to characterize the error rates for binary phase shift keying (BPSK), pulse amplitude modulation (PAM), quadrature amplitude modulation (QAM), and M phase shift keying (MPSK) modulations over Extended Generalized- $\mathcal{K}$ (EGK) fading channel [15]. A unified closed-form expression for the average symbol error rate (SER) is derived. To this end, the throughput of the proposed pairing scheme is characterized and compared to its HD counterpart. Finally, we provide insights for the modulation scheme selection based on the average SIR. To the best of our knowledge, our paper is the first to advocate long term pairing schemes for HD users served by FD BS and provide comprehensive performance assessment.

B. Summary of Contributions

It ought to be mentioned that outage probability and achievable rate analysis for the 3NT cellular networks are conducted in [16]–[19]. However, different from [16]–[19], this paper is the first to obtain a closed form expression for the inter-user

interference distribution, derive exact error rates for different modulation schemes, present a unified expression for error rates for different modulation schemes, and show the explicit effect of inter-user interference on the throughput of each modulation scheme. From the mathematical perspective, this paper offers an explicit form of the average probability of error of the above system under an EGK fading channel [15]. The EGK is selected on the useful link for its flexibility to capture different fading environments as special cases [20], including Rayleigh, Nakagami- m , Generalized Nakagami- m (GNM), Gamma, Weibull, and others as can be seen in Table II.

The remaining of the paper is organized as follows. Section II presents the system model and assumptions. The distribution of the inter-user interference is derived in Section III. The conditional error rates are analyzed in Section IV, i.e., conditioning over the useful link fading. Then, the average error rates are studied in Section V. Section VI focuses on the throughput performance and Section VII presents and discusses selected numerical results. Finally, Section VIII concludes the paper.

II. SYSTEM MODEL

This paper explicitly focuses on the inter-user interference problem for NLOS HD users paired on the same channel imposed in the 3NT shown in Fig. 1. Therefore, we consider a single cell scenario with an FD BS and two NLOS HD UEs. Without loss in generality, we assume that spectrum can be either divided into four non-overlapping channels that are assigned in HD mode or two non-overlapping channel that are assigned using the 3NT FD mode, as shown in Fig. 1. In the HD case, each user is assigned an interference free bandwidth (BW) B Hz in each of the uplink and downlink directions. On the other hand, the 3NT FD mode doubles the BW to $2B$ in each of the uplink and downlink directions at the expense of SI in the uplink and inter-user interference in the downlink. Without loss of generality, we focus on the downlink performance of one of the UEs. The complex baseband received signal at the test UE can be expressed as

$$r = \sqrt{P_d \mathcal{A}_d} h s + \sqrt{P_u \mathcal{A}_u} g x + n, \quad (1)$$

where P_d is the downlink transmit power, $\mathcal{A}_d$ is a constant that captures downlink large-scale power attenuation, s is the intended symbol drawn from a unit power constellation, h is the intended channel with unit-mean EGK distributed power gain, P_u is the power transmitted by the interfering UE, $\mathcal{A}_u$ is a constant that captures large-scale interference power attenuation, x is the inter-user interfering Gaussian symbol, g is the unit variance circularly symmetric complex Gaussian mutual channel gain between the UEs and n is the additive white noise. The random variables h , s , g , x , and n are assumed to be independent.

Remark 1 The circularly symmetric Gaussian distribution of g models a severe Rayleigh fading scenario, which occurs in NLOS scattering rich environments. The Rayleigh fading assumption on the interference channel is a direct consequence of the proposed NLOS pairing criterion, which can be realized

¹ This work is presented in part in [14].

² Rayleigh fading environment reflects NLOS links in rich scattering environment.

by multi-user diversity in which the BS would pair only NLOS UEs on the same uplink/downlink channels. It is worth noting that the analysis in [16]–[19] is based on the Rayleigh fading assumption on both the interfering and the useful links for analytical tractability.

Remark 2 Unlike $|g|$, which is assumed Rayleigh by virtue of the NLOS user pairing and multi-user diversity, the proposed pairing scheme do not put any constraint on the downlink link channel fading. Hence, $|h|$ is modeled using the general EGK fading distribution to provide comprehensive performance assessment for the NLOS pairing criterion under diverse types of fading channels that can appear between the user and the serving BS.

Remark 3 The Gaussian signaling abstraction for the interfering symbols is used in the literature and is shown to have negligible effect on the error rate performance [21], [22]. Furthermore, gaussian signals provide accurate approximation for the interference generated from a single node transmitting on a faded channel [21]. This assumption is also validated in Section VI of this paper.

Remark 4 The intended signal power is P_d and the interfering signal power is P_u because we study the downlink transmission in which the intra-mode interference comes from an uplink UE, as shown in Fig. 1.

For analytical tractability, we assume that the inter-user interference dominates the noise, and hence, the noise term is ignored in the analysis. The accuracy of this approximation is validated in the results section with realistic noise power. Ignoring the white noise, the received signal r can be rewritten as

$$r \approx \sqrt{P_d A_d} h s + \sqrt{P_u A_u} g x. \quad (2)$$

The received signal r is a complex valued random variable with real and imaginary components, which are represented as $r = \Re(r) + j\Im(r)$, where $\Re(\cdot)$ and $\Im(\cdot)$ denote the real and imaginary parts of a complex variable, respectively, and $j = \sqrt{-1}$ is the imaginary unit. To study (2), we first focus on the interference term, which can be considered as an additive complex perturbation for the useful symbol s . In the next section, we study the normalized inter-user interference, denoted as $Z = gx$. The normalized inter-user interference Z is characterized by closed form expressions for its PDF and CDF.

III. DISTRIBUTION OF THE INTER-USER INTERFERENCE

In what follows subscripts r and i denote the real and imaginary parts, respectively.

Since g and x are both zero-mean unit-variance complex Gaussian random variables, their real part and imaginary part are independent Gaussian random variables with zero-mean and variance $\frac{1}{2}$. The inter-user Interference Z is represented as

$$\begin{aligned} Z &= gx \\ &= (\Re(g)\Re(x) - \Im(g)\Im(x)) + j(\Re(g)\Im(x) + \Im(g)\Re(x)). \end{aligned} \quad (3)$$

The distribution of Z can be obtained using the relations between the CHF, the PDF, and the CDF. To benefit from these relations, the CHF, denoted by $\varphi_Z(t_1, t_2)$, should be derived first. Thus, Lemma 1 gives the CHF of the inter-user interference.

Lemma 1 The inter-user interference from a UE transmitting Gaussian signals over a Rayleigh faded channel has the following characteristic function

$$\varphi_Z(t_1, t_2) = \frac{4}{4 + t_1^2 + t_2^2} = \frac{1}{1 + \frac{|\mathbf{t}|^2}{4}}, \quad (4)$$

where the vector $\mathbf{t} = (t_1, t_2)$.

Proof: See Appendix A

Remark 5 The expression in (4) is a special case of the bivariate Laplace distribution [23]. Furthermore, the CHF of Z is independent of the angle between the real and imaginary components. Consequently, (4) shows that Z follows a circularly symmetric complex Laplace distribution.

From the CHF of Z given in Lemma 1, the CDF of Z is given in the following theorem.

Theorem 1. *The inter-user interference from a UE transmitting Gaussian signals over a Rayleigh faded channel has the following CDF*

$$\begin{aligned} F_Z(z_r, z_i) &= 1 - \frac{1}{2} (e^{-2z_r} + e^{-2z_i}) \\ &+ \frac{1}{4\pi} G_{1,0,2,1,2,1}^{0,1,0,2,0,2} \left[\frac{1}{z_r^2}, \frac{1}{z_i^2} \middle| \begin{matrix} 0 \\ 0 \end{matrix} \middle| \begin{matrix} \frac{1}{2}, 1 \\ \frac{1}{2}, 1 \end{matrix} \right], \end{aligned} \quad (5)$$

where $G_{1,0,2,1,2,1}^{0,1,0,2,0,2}[\cdot, \cdot]$ is the bivariate Meijer's G function (BMGF), an extension of the Meijer's G function, that is defined in [24], where an optimal code in Mathematica was provided too.

Proof: See Appendix B.

It is important to note that the PDF of the product of two independent complex Gaussian random variables has been studied before in [25], [26]. However, the expression in Theorem 1 presents a novelty, since this is the first time the CDF of the product of two independent complex Gaussian random variables is described in closed form. Consequently, the PDF of the normalized inter-user interference (i.e., Z) is given in the following corollary.

Corollary 1.1 The inter-user interference from a UE transmitting Gaussian signals over a Rayleigh faded channel has the following PDF

$$f_Z(z_r, z_i) = \frac{2}{\pi} K_0 \left(2\sqrt{z_r^2 + z_i^2} \right), \quad (6)$$

where $K_0(\cdot)$ is the 0-th order modified Bessel function of the second kind [27, Eq. (9.6.21)].

Proof: See Appendix C

Remark 6 The PDF expression in (6) clearly shows that Z does not have independent real and imaginary components because the joint PDF in (6) cannot be expressed as the product of two marginal Laplace PDFs.

Remark 7 The problem of decoding in the presence of additive Laplacian perturbation (i.e., noise or interference) has been widely addressed in the literature [28]–[32]. However, a common assumption in this literature is that the Laplacian perturbation has independent real and imaginary components. Hence, the work presented in this paper contributes to the literature by modeling the case with dependent real and imaginary components. Furthermore, this paper rigorously derive the Laplacian distribution of the interference from the system model rather than assuming a Laplacian perturbation.

Exploiting the interference characterization in this section, the error rate performance is analyzed in the next sections.

IV. CONDITIONAL ERROR RATES

This section characterizes the downlink decoding errors that may occur due to the Laplacian inter-user interference with the PDF shown in (6). In particular, we evaluate the probability of BPSK, PAM, QAM, and MPSK. At first, the conditional SER (i.e., conditioning on the intended channel gain) as a function of the signal-to-interference ratio (SIR), denoted as $\gamma = \frac{P_d A_d |h|^2}{P_u A_u}$, is investigated. The conditional error rate analysis can also be considered as the error rate over unfaded downlink channel³. By the end of this section, a unified expression of the conditional SER is presented.

For the sake of organized presentation, we classify the studied modulation schemes into two sets, namely, (i) one dimensional constellations (1D) (i.e. BPSK and M-PAM) and (ii) two dimensional constellations (2D) (i.e. M-QAM and MPSK). Then, the analysis of each class is presented in a separate section.

A. Error Rate for 1D Constellations

The conditional bit error rate (BER) for BPSK is giving is the following Theorem.

Theorem 2. *The conditional downlink bit error rate in 3NT with Laplacian inter-user interference and maximum likelihood detector for the BPSK modulation is given by*

$$P_{BPSK}(e|\gamma) = \frac{1}{2} e^{-2\sqrt{\gamma}}. \quad (7)$$

Proof: Since the BPSK transmits real symbols $s = \pm E_s$, the conditional PDF of the received signal r is obtained from (6) as $f_r(r) = e^{-2|r \mp E_s|}$. Moreover, the optimal decision is obtained by minimizing the distance $d(r, \pm E_s)$. Given that the BPSK symbols are equiprobable, the probability of error can be computed as $Pr(e|\gamma) = \int_0^\infty f_r(r|S = -E_s) dr$. Then (7) is obtained by setting $E_s = 1$ due to the assumed unit energy constellation. ■

Based on Theorem 2, the probability of error of an MPAM modulation is characterized in the following corollary

Corollary 2.1 The conditional downlink SER in 3NT with Laplacian inter-user interference and maximum likelihood detector for the MPAM modulation is given by

$$P_{PAM}(e|\gamma) = \left(1 - \frac{1}{M}\right) e^{-2\sqrt{\frac{3\gamma}{M^2-1}}}, \quad (8)$$

³An important application for unfaded channels appears in the context of massive MIMO systems due to the channel hardening effect [33].

where the exponent argument $\sqrt{\frac{3\gamma}{M^2-1}}$ is obtained by calculating the average energy per symbol for MPAM.

Proof: The corollary directly follows from the BER of a BPSK modulation as $P_{PAM}(e) = 2\left(1 - \frac{1}{M}\right) P_{BPSK}(e)$. Hence, the conditional SER of MPAM modulation can be deduced from (7). ■

B. Error Rate for 2D Constellations

In this section we consider 2D constellations formed by in-phase and quadrature phase components. Particularly, we focus on the M-QAM and the MPSK constellations. While the exact closed-form expressions of the SERs for the M-QAM and the MPSK schemes are derived, we also present simplified approximations in order to present a unified error rate expression for all of the considered modulation schemes.

1) *SER for Rectangular QAM:* A rectangular M -QAM is formed by one in-phase M_I -PAM and orthogonal quadrature phase M_Q -PAM, which means that $M = M_I M_Q$. The rectangular QAM modulation is a generalization of the square QAM that can cover more signal constellations. Therefore, the SER of rectangular QAM modulation is given in the following theorem.

Theorem 3. *The conditional downlink SER in 3NT with Laplacian inter-user interference and maximum likelihood detector for the rectangular QAM modulation is given by*

$$P_{QAM}(e|\gamma) = \rho(M_I) e^{-2\sqrt{\frac{3\gamma}{d}}} + \rho(M_Q) e^{-2\tau\sqrt{\frac{3\gamma}{d}}} - \frac{1}{\pi} \rho(M_I) \rho(M_Q) G_{1,0,2,1,2,1}^{0,1,0,2,0,2} \left[\frac{d^2}{3\gamma}, \frac{d^2}{3\tau^2\gamma} \middle| 0 \right. \left. \begin{matrix} \frac{1}{2}, 1 \\ 0 \end{matrix} \middle| \frac{1}{2}, 1 \right], \quad (9)$$

where $d = \sqrt{(M_I^2 - 1) + \tau^2 (M_Q^2 - 1)}$, τ is the in-phase-to-quadrature decision distance ratio defined as $\tau = \frac{d_Q}{d_I}$, and $\rho(x) = 1 - \frac{1}{x}$. The terms d_I and d_Q represent the in-phase and quadrature phase decision distance, respectively.

Proof: See Appendix D. ■

A special case of Theorem 3 is the square QAM, where $M_I = M_Q = \sqrt{M}$ and $\tau = 1$. Using the double Mellin-Barnes contour integral of the BMGF [34] and the expression of the SER in Theorem 3, the SER of square QAM can be reduced to

$$P_{SQAM}(e|\gamma) = 2\rho(\sqrt{M}) e^{-\sqrt{\frac{6\gamma}{M-1}}} - \frac{1}{\pi} \rho(\sqrt{M})^2 G_{2,1,1,2,1,0}^{0,2,2,0,0,1} \left[\frac{2(M-1)}{3\gamma}, 1 \middle| \frac{1}{2}, 1 \right. \left. \begin{matrix} 1 \\ \frac{1}{2}, 0 \end{matrix} \middle| 0 \right]. \quad (10)$$

The error rate for the rectangular M-QAM modulation given in Theorem 3 is expressed in terms of the BMGF, which imposes high computational complexity. Therefore, we propose the following simplifying approximation.

Proposition 1 The conditional downlink SER in 3NT with Laplacian inter-user interference and maximum likelihood

detector for the rectangular QAM modulation can be approximated by

$$P_{QAM}^A(e|\gamma) = \rho(M_I)e^{-2\frac{\sqrt{3\gamma}}{d}} + \rho(M_Q)e^{-2\tau\frac{\sqrt{3\gamma}}{d}} - \rho(M_I)\rho(M_Q)e^{-2(1+\tau)\frac{\sqrt{3\gamma}}{d}}. \quad (11)$$

In the case of square QAM modulation, the SER approximation reduces to

$$P_{SQAM}^A(e|\gamma) = 2\rho(\sqrt{M})e^{-\sqrt{\frac{6\gamma}{M-1}}} - \rho(\sqrt{M})^2e^{-2\sqrt{\frac{6\gamma}{M-1}}}. \quad (12)$$

Proof: The approximation is obtained by considering a QAM modulation formed by two orthogonal (approximated as independent in presence of Laplace interference) PAMs. Thereby, a correct detection of the M -QAM appears only for a correct detection in the M_I -PAM and M_Q -PAM. Consequently, the probability of error of M -QAM may be approximated as

$$P_{QAM}^A(e) = 1 - (1 - P_{PAM_I}(e))(1 - P_{PAM_Q}(e)) = P_{PAM_I}(e) + P_{PAM_Q}(e) - P_{PAM_I}(e)P_{PAM_Q}(e). \quad (13)$$

Then (11) is obtained by substituting (8) in (13). ■

2) *SER of MPSK:* This section considers the case where the transmitted signal s is modulated via an MPSK scheme with M being a power of 2 and $M \geq 4$, the BER of the BPSK case, i.e. $M = 2$, was derived in Theorem 7. The MPSK symbols are distributed uniformly over the unit circle and all symbols are equiprobable. Unlike the previous modulation schemes, the decision regions for the MPSK symbols are not known a priori and have to be characterized for each complex perturbation. Let I and Q be the normalized in-phase and quadrature phase components of the received signal, then the likelihood function and decision criterion under the complex Laplacian perturbation given in (6) is characterized in the following lemma.

Lemma 2 The likelihood function for the k -th symbol s_k in an MPSK constellation under the complex Laplacian perturbation given in (6) can be expressed as

$$\ell_k = R \cos(\psi - \varphi_k), \quad (14)$$

where $R = \sqrt{I^2 + Q^2}$ and $\psi = \arctan\left(\frac{Q}{I}\right)$. Consequently, the decision criterion for the MPSK received symbols can be defined by selecting the symbol s_k which minimizes $|\psi - \varphi_k|$, where $\varphi_k = \frac{2k\pi}{M}$.

Proof: According to the PDF of Z in (6), the likelihood function can be written as

$$\ell_k = \frac{2\gamma}{\pi} K_0\left(2\sqrt{\gamma}\sqrt{(I - \cos \varphi_k)^2 + (Q - \sin \varphi_k)^2}\right), \quad (15)$$

where

$$\begin{cases} I &= s_r^{(k)} + \frac{z_r}{\sqrt{\gamma}} &= \cos \varphi_k + \frac{z_r}{\sqrt{\gamma}} \\ Q &= s_i^{(k)} + \frac{z_i}{\sqrt{\gamma}} &= \sin \varphi_k + \frac{z_i}{\sqrt{\gamma}}. \end{cases} \quad (16)$$

From [27, Eqs. (9.6.27) & (9.6.1)], we know that $\frac{dK_0(x)}{dx} = -K_1(x)$ and that $K_1(x)$ is positive for $x > 0$, $K_1(\cdot)$ is the 1-st order modified Bessel function of the second kind. Hence,

$K_0(\cdot)$ is a monotonically decreasing function. Consequently, the likelihood function ℓ_k is maximized by minimizing the Bessel function argument, i.e., minimizing the distance $(I - \cos \varphi_k)^2 + (Q - \sin \varphi_k)^2$. Using some algebraic simplification, ℓ_k can be expressed as follows

$$\begin{aligned} \ell_k &= I \cos \varphi_k + Q \sin \varphi_k \\ &= R (\cos \psi \cos \varphi_k + \sin \psi \sin \varphi_k), \end{aligned}$$

which gives (14) and completes the proof. ■

An illustrative example for the MPSK decision regions, with $M = 8$, that maximizes the likelihood function ℓ_k is shown in Fig.2. Following similar methodology to [35], the decision regions are obtained by defining the two coordinates rotations (X_1, Y_1) , with rotation angle $\phi - \theta$, and (X_2, Y_2) , with rotation angle $\phi + \theta$, where ϕ is the symbol phase and $\theta = \frac{\pi}{M}$. The circular symmetry of the interference PDF in (6) implies that all symbols have similar error probability. Hence, the total error probability can be deduced by considering a generic single symbol, with angle φ lies between 0 and $\frac{\pi}{2}$, and the error probability can be explicitly expressed in terms of Y_1 and Y_2 as

$$Pe(\gamma, \phi) = \Pr[Y_1 \geq 0] + \Pr[Y_2 \leq 0] - \Pr[Y_1 \geq 0, Y_2 \leq 0]. \quad (17)$$

where Y_1 and Y_2 are obtained from the aforementioned rotation as

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} -\sin(\phi - \theta) & \cos(\phi - \theta) \\ -\sin(\phi + \theta) & \cos(\phi + \theta) \end{bmatrix} \begin{bmatrix} I \\ Q \end{bmatrix}. \quad (18)$$

Based on (17) and (18), the SER for the MPSK constellation is characterized in the following theorem

Theorem 4. The probability of error detection of one symbol with phase φ using maximum likelihood detector is given as follows

$$P_e(\gamma, \varphi) = e^{-2\delta\sqrt{\gamma}} - \frac{\delta}{2\sqrt{\pi}} G_{1,1,1,1,2,0}^{0,1,1,0,0,2} \left[\delta^2, \frac{4\delta^2(1 - \delta^2)}{\gamma(\cos^2 \varphi - \delta^2)} \left| \begin{matrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & 0 \end{matrix} \right| \begin{matrix} 1, 0 \\ - \end{matrix} \right], \quad (19)$$

where $\delta = \sin\left(\frac{\pi}{M}\right)$ and $\varphi = \frac{2k\pi}{M}$.

Proof: See Appendix E. ■

Once the probability of error of a single symbol is obtained, the total probability of error for MPSK can be deduced directly in Corollary 4.1.

Corollary 4.1 The conditional downlink SER in 3NT with Laplacian inter-user interference and maximum likelihood detector for the MPSK modulation is given by

$$P_{MPSK}(e|\gamma) = e^{-2\delta\sqrt{\gamma}} - \frac{2\delta}{M\sqrt{\pi}} \times \sum_{k=0}^{\frac{M}{4}-1} G_{1,1,1,1,2,0}^{0,1,1,0,0,2} \left[\delta^2, \frac{4\delta^2(1 - \delta^2)}{\gamma\left(\cos\left(\frac{2k\pi}{M}\right)^2 - \delta^2\right)} \left| \begin{matrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & 0 \end{matrix} \right| \begin{matrix} 1, 0 \\ - \end{matrix} \right]. \quad (20)$$

Proof: The total probability of error of the MPSK modulation can be computed as the average of the probability of

Fig. 2: Decision regions using maximum likelihood detector for 8-PSK.

error over all the symbols. Since the perturbation PDF in (6) is circular symmetric, we can focus our study to the region between 0 and $\frac{\pi}{2}$ and the total SER of the MPSK can be written as

$$P_{MPSK}(e|\gamma) = \frac{4}{M} \sum_{k=0}^{\frac{M}{4}-1} P_e(\gamma, \varphi_k) = \frac{4}{M} \sum_{k=0}^{\frac{M}{4}-1} P_e(\gamma, \frac{2k\pi}{M}). \quad (21)$$

Replacing (19) in (21), Corollary 4.1 is obtained. ■

Similar to the MQAM scheme, the SER of MPSK is expressed in terms of the BMGF, which imposes high computational complexity. Hence, a simplifying approximation for the SER in MPSK modulation is given in the following proposition.

Proposition 2 The conditional downlink SER in 3NT with Laplacian inter-user interference for the MPSK modulation can be approximated by

$$P_{MPSK}^A(e|\gamma) = e^{-2\delta\sqrt{\gamma}} + \frac{2 \tan \theta}{M(1 + \tan \theta)} e^{-2\sqrt{\gamma}} + \frac{1}{M} \sum_{k=1}^{\frac{M}{4}-1} \frac{\sin 2\theta}{\cos 2\theta + \sin \frac{4k\pi}{M}} e^{-2\sqrt{2}\gamma \sin(\frac{2k\pi}{M} + \frac{\pi}{4})}. \quad (22)$$

Proof: The exact expressions for $\Pr[Y_1 \geq 0]$ and $\Pr[Y_2 \leq 0]$ are already in the exponential form, and hence, no approximation is required for these terms. On the other hand, the BMGF appears in $\Pr[Y_1 \geq 0, Y_2 \leq 0]$, which can be approximated assuming independent interference components as in [28, Eq. (30)]. It is worth noting that (22) is based on approximating only the term $\Pr[Y_1 \geq 0, Y_2 \leq 0]$ with a similar expression to [28, Eq. (30)], while the two other terms are derived for the first time in this paper using the interference described in (3). ■

TABLE I: SER Modulation-Specific Parameters n , a_k , and b_k

Modulation Scheme	Modulation Specific Parameters			
	n	k	a_k	b_k
BPSK	1	1	$\frac{1}{2}$	1
PAM	1	1	$\rho(M)$	$2\sqrt{\frac{3}{M^2-1}}$
Approximated Square-QAM	2	1	$2\rho(\sqrt{M})$	$\sqrt{\frac{6}{M-1}}$
		2	$-\rho(\sqrt{M})^2$	$2\sqrt{\frac{6}{M-1}}$
Approximated Rectangular-QAM	3	1	$\rho(M_I)$	$2\frac{\sqrt{3}}{d}$
		2	$\rho(M_Q)$	$2\tau\frac{\sqrt{3}}{d}$
		3	$\rho(M_I)\rho(M_Q)$	$2(1+\tau)\frac{\sqrt{3}}{d}$
Approximated MPSK	$\frac{M}{4} + 1$	$1 \leq k \leq n-2$	$\frac{\sin 2\theta}{M(\cos 2\theta + \sin \frac{4k\pi}{M})}$	$2\sqrt{2} \sin(\frac{2k\pi}{M} + \frac{\pi}{4})$
		$n-1$	$\frac{2 \tan \theta}{M(1 + \tan \theta)}$	2
		n	1	2δ

C. Unified Expression of the SER

By virtue of the approximations given in Proposition 1 and Proposition 2, we can obtain a unified expression for the error rates in all considered modulation schemes. Particularly, the error rates derived in Theorem 2, Corollary 2.1, Proposition 1, and Proposition 2, can be expressed using the following unified expression

$$SER = \sum_{k=1}^n a_k e^{-b_k \sqrt{\gamma}}, \quad (23)$$

where n , a_k , and b_k are related to the used modulation. Their possible values are summarized in Table I.

V. AVERAGE SER OVER EGK FADING CHANNEL

Using the conditional error rates obtained in the previous section, the average SER ($\overline{SER}$) is derived by averaging over the EGK fading distribution on the downlink channel, where the EGK distribution is given in [15, Eq. (2)]. The EGK distribution presents a versatile fading envelope model that generalizes many commonly used statistical models that describe signal fluctuations in the received SIR due to multipath, shadowing, or a mixture of such processes in the environment [15]. Moreover, the EGK distribution captures many known fading distributions as special cases, like Rayleigh fading, Nakagami- m fading, K distribution and many other distributions summarized in [36]. In the present model, assuming that $\mathbb{E}[|h|^2] = 1$, the average SIR is given by $\bar{\gamma} = \mathbb{E}[\gamma] = \frac{P_d A_d}{P_u A_u}$, $\mathbb{E}[\cdot]$ represented the expected value a random variable. The PDF of γ can be expressed as follows

$$p_\gamma(\gamma) = \frac{\xi}{\Gamma(m_s)\Gamma(m)} \left(\frac{\beta_s \beta}{\gamma}\right)^{m\xi} \gamma^{m\xi-1} \times \Gamma\left(m_s - m\frac{\xi}{\xi_s}, 0, \left(\frac{\beta_s \beta}{\gamma}\right)^{m\xi} \gamma^{m\xi}, \frac{\xi}{\xi_s}\right), \quad (24)$$

which is defined over $\gamma \in (0, \infty)$ and the parameters m ($0.5 \leq m < \infty$) and ξ ($0 \leq \xi < \infty$) represent the fading figure (diversity severity/order) and the fading

shaping factor, respectively, while m_s ($0.5 \leq m_s < \infty$) and ξ_s ($0 \leq \xi_s < \infty$) represent the shadowing severity and the shadowing shaping factor (inhomogeneity), respectively. In addition, we define the parameters $\beta = \Gamma(m+1/\xi)/\Gamma(m)$ and $\beta_s = \Gamma(m_s+1/\xi_s)/\Gamma(m_s)$. In (24), $\Gamma(\cdot, \cdot, \cdot, \cdot)$ is the extended incomplete Gamma function defined in [37, Eq. (6.2)] as

$$\Gamma(a, x, b, \beta) = \int_x^\infty r^{a-1} \exp(-r - br^{-\beta}) dr, \quad (25)$$

where $a, \beta, b \in \mathbb{C}$ and $x \in \mathbb{R}^+$.

To simplify the computation, the identities [37, Eq. (6.22)] and [38, Eqs. (2.1.4), (2.1.5) and (2.1.11)] are used to express (24) in terms of the Fox's H function, as

$$p_\gamma(\gamma) = \frac{1}{\Gamma(m_s)\Gamma(m)\gamma} H_{0,2}^{2,0} \left[\frac{\beta_s \beta}{\gamma} \gamma \left| (m_s, \frac{1}{\xi_s}), (m, \frac{1}{\xi}) \right. \right], \quad (26)$$

where $H_{p,q}^{m,n}[\cdot]$ denotes the Fox's H function (FHF) whose definition is given by [38, Eq. (1.1.1)] and [39] as

$$H_{p,q}^{m,n} \left[z \left| \begin{matrix} (a_i, \alpha_i)_{1,p} \\ (b_j, \beta_j)_{1,q} \end{matrix} \right. \right] = \frac{1}{2\pi i} \int_{\mathcal{C}} \frac{\prod_{j=1}^m \Gamma(b_j + \beta_j s) \prod_{i=1}^n \Gamma(1 - a_i - \alpha_i s)}{\prod_{j=n+1}^p \Gamma(a_i + \alpha_i s) \prod_{j=m+1}^q \Gamma(1 - b_j - \beta_j s)} z^{-s} ds, \quad (27)$$

where $\mathcal{C}$ denotes the Mellin-Barnes contour. Note that the FHF is recently used in wireless communication to provide closed form expressions for several system performance metrics [40]. Furthermore, it can be efficiently computed using MATHEMATICA® [41, Appendix].

Using the EGK distribution, the average symbol error probability is given in the following sections.

A. Unified Average SER

Theorem 5. *The average unified SER for BPSK, PAM, approximated QAM, and approximated MPSK in 3NT with EGK fading on the downlink, Laplacian inter-user interference, and maximum likelihood detector is given by*

$$\overline{SER} = \sum_{k=1}^n \frac{2a_k}{\Gamma(m)\Gamma(m_s)} H_{1,2}^{2,1} \left[\frac{\beta \beta_s}{\gamma b_k^2} \left| (1, 2) \right. (m, \frac{1}{\xi})(m_s, \frac{1}{\xi_s}) \right], \quad (30)$$

where a_k and b_k are defined in Table I.

Proof: From (23), the average SER can be determined by solving the integral $\mathcal{I}(b_k) = \int_0^\infty p_\gamma(\gamma) e^{-b_k \sqrt{\gamma}} d\gamma$, where b_k are defined in Table I. Using (26), and the expression of the exponential in terms of the FHF [38, Eq. (2.9.4)], we obtain an integral of the product of two FHF, which is solved using the identity [38, Eq. (2.8.4)]. ■

B. Exact Average SER for Rectangular QAM

The exact average SER of the QAM constellation is obtained by averaging (9) over the SIR distribution (26). The result is described in Theorem 6.

Theorem 6. *The average SER of rectangular QAM in 3NT with EGK fading on the downlink, Laplacian inter-user interference, and maximum likelihood detector is described in (28).*

In (28), $H_{2,2}^{2,2}[\cdot, \cdot]$ is the bivariate Fox's H function (BFHF) that is defined and studied in [34]. A pseudo MATLAB® code for the BFHF is outlined in [42].

Proof: From (9), the average SER can be determined by solving an integral in the form $\int_0^\infty p_\gamma(\gamma) P_{QAM}(e|\gamma) d\gamma$. The first two terms are similar to (30) and expressed in terms of the FHF. While the last term is solved using the Mellin representation of the BMGF [34]. ■

C. Exact Average SER for MPSK

The average SER of MPSK modulation is characterized in the following theorem.

Theorem 7. *The average SER of MPSK in 3NT with EGK fading on the downlink, Laplacian inter-user interference, and maximum likelihood detector is described in (29).*

Proof: Using a similar proof to Theorem 6, the average SER of MPSK modulation can be obtained by averaging (20) over the PDF of γ . The first term will give a result similar to (30), while the last term can be expressed in terms of the BFHF using the Mellin representation of the BMGF. ■

D. Special Cases of Fading and Simplification of $\overline{SER}$

In what follows we investigate several special cases for the EGK distribution where the average SER reduces to more simplified expression. Note that we only consider the special cases for the unified average SER given in Theorem 5, which is given in terms of a_k and b_k defined in Table I. On the other hand, the exact average SER of the rectangular QAM and MPSK cannot be further simplified because they are expressed in terms of the BFHF. Hence, only the approximations of the SER of QAM and MPSK (that are included in the unified formula in Theorem 5) are investigated in the following special cases.

Special Case 1 (Generalized- $\mathcal{K}$ fading) is obtained by setting $\xi = \xi_s = 1$ in (30). Using the identity [39, Eq. (8.3.22)] that relates the FHF to the MGF for rational parameters, the average SER can be simplified to

$$\overline{SER} = \sum_{k=1}^n \frac{a_k}{\sqrt{\pi} \Gamma(m) \Gamma(m_s)} G_{2,2}^{2,2} \left[\frac{4mm_s}{\gamma b_k^2} \left| 1, \frac{1}{2} \right. m, m_s \right]. \quad (31)$$

This expression is in terms of the MGF which is less complex in terms of computation than the FHF. ■

Special Case 2 ($\mathcal{K}$ fading distribution) is a special case of the generalized- $\mathcal{K}$ fading distribution, that is obtained by setting $\xi_s = \xi = 1$ and $m_s = 1$. Hence the average SER can be simplified to

$$\overline{SER} = \sum_{k=1}^n \frac{a_k}{\sqrt{\pi} \Gamma(m)} G_{2,2}^{2,2} \left[\frac{4m}{\gamma b_k^2} \left| 1, \frac{1}{2} \right. 1, m \right]. \quad (32)$$

■

$$\begin{aligned} \overline{SER}_{QAM} = & \frac{2\rho(M_I)}{\Gamma(m)\Gamma(m_s)} H_{1,2}^{2,1} \left[\frac{\beta\beta_s d^2}{12\bar{\gamma}} \middle| \begin{matrix} (1, 2) \\ (m, \frac{1}{\xi})(m_s, \frac{1}{\xi_s}) \end{matrix} \right] + \frac{2\rho(M_Q)}{\Gamma(m)\Gamma(m_s)} H_{1,2}^{2,1} \left[\frac{\beta\beta_s d^2}{12\tau^2\bar{\gamma}} \middle| \begin{matrix} (1, 2) \\ (m, \frac{1}{\xi})(m_s, \frac{1}{\xi_s}) \end{matrix} \right] \\ & - \frac{\rho(M_I)\rho(M_Q)}{\pi\Gamma(m)\Gamma(m_s)} H_{2,1,1,2,1,2}^{0,2,2,0,2,1} \left[\tau^2, \frac{\beta\beta_s d^2}{3\bar{\gamma}} \middle| \begin{matrix} (\frac{1}{2}, 1, 1), (1, 1, 1) \\ (0, 1, 1) \end{matrix} \middle| \begin{matrix} (1, 1) \\ (\frac{1}{2}, 1), (0, 1) \end{matrix} \middle| \begin{matrix} (0, 1) \\ (m, \frac{1}{\xi}), (m_s, \frac{1}{\xi_s}) \end{matrix} \right]. \end{aligned} \quad (28)$$

$$\begin{aligned} \overline{SER}_{MPSK} = & \frac{2}{\Gamma(m)\Gamma(m_s)} H_{1,2}^{2,1} \left[\frac{\beta\beta_s}{4\delta^2\bar{\gamma}} \middle| \begin{matrix} (1, 2) \\ (m, \frac{1}{\xi})(m_s, \frac{1}{\xi_s}) \end{matrix} \right] \\ & - \frac{2\delta}{M\sqrt{\pi}} \frac{1}{\Gamma(m)\Gamma(m_s)} \sum_{k=0}^{\frac{M}{4}-1} H_{1,1,1,1,2,2}^{0,1,1,0,2,2} \left[\delta^2, \frac{4\beta\beta_s\delta^2(1-\delta^2)}{\bar{\gamma}(\cos(\frac{2k\pi}{M})-\delta^2)} \middle| \begin{matrix} (\frac{1}{2}, 1, 1) \\ (-\frac{1}{2}, 1, 1) \end{matrix} \middle| \begin{matrix} (\frac{1}{2}, 1) \\ (0, 1) \end{matrix} \middle| \begin{matrix} (1, 1), (0, 1) \\ (m, \frac{1}{\xi}), (m_s, \frac{1}{\xi_s}) \end{matrix} \right]. \end{aligned} \quad (29)$$

Special Case 3 (Generalized Nakagami- m GNM) is obtained by setting $\xi_s = 1$ and taking the limit $m_s \rightarrow +\infty$ in the EGK distribution, which makes the SER converges to

$$\overline{SER} = \sum_{k=1}^n \frac{2a_k\xi}{\Gamma(m)} \left(\frac{\beta}{b_k^2\bar{\gamma}} \right)^{m\xi} \Gamma \left(2m\xi, 0, \left(\frac{\beta}{b_k^2\bar{\gamma}} \right)^\xi, -2\xi \right). \quad (33)$$

Such expression can be deduced from the inverse Mellin transform of the extended incomplete Gamma function [37, Eq. (6.29)]. ■

Special Case 4 (Nakagami- m) appears as a special case of the GNM when $\xi = 1$. Using the identity [37, Eq. (6.44)] that reduces $\Gamma(\cdot, \cdot, \cdot, \cdot)$ to the Tricomi confluent hypergeometric function $U(\cdot, \cdot, \cdot)$ [27, Chapter 13], the average SER further simplifies to

$$\overline{SER} = \sum_{k=1}^n \frac{2a_k\Gamma(2m)}{4^m\Gamma(m)} U \left(m, \frac{1}{2}, \frac{b_k^2\bar{\gamma}}{4} \right). \quad (34)$$

Special Case 5 (Rayleigh) is the last and simplest special case, which is a special case of the Nakagami- m fading for $m = 1$. In this case, the SER is obtained in its simplest form as

$$\overline{SER} = \sum_{k=1}^n a_k \left(1 - b_k \sqrt{\pi\bar{\gamma}} e^{\frac{b_k^2\bar{\gamma}}{4}} Q \left(b_k \sqrt{\frac{\bar{\gamma}}{2}} \right) \right), \quad (35)$$

where $Q(\cdot)$ is the Gaussian Q function [20, Eq. (A.1)]. ■

Although the average SER over any special case of fading of the EGK can be deduced from (30) by setting the quadruplet (m, m_s, ξ, ξ_s) to the desired values, the previous studied special cases are highlighted because they are the most commonly used distribution to characterize the flat fading channels. Moreover, the results obtained in this section are given in their simplest forms, which require several manipulations after substituting (m, m_s, ξ, ξ_s) by their desired values.

VI. NUMERICAL RESULTS

This section shows some selected numerical results supported by Monte Carlo MATLAB® simulations to validate the developed mathematical framework and obtain insights for NLOS user pairing in 3NT FD operation. Particularly, we validate the Gaussian signaling assumption for the interfering

symbol in (1), the dominance of the inter-user interference when compared to the noise in (2), and the MQAM and MPSK error rates approximations in Proposition 1 and Proposition 2, respectively.

All results are plotted against the average SIR $\bar{\gamma} = \frac{P_d A_d}{P_u A_u}$, which is varied from 0 dB to 50 dB to provide insights for modulation scheme selection based on the average SIR values of the paired devices. When multiple devices are present, the BS should pair all NLOS devices such that the lowest SIR among the paired devices is achieved. Then, using the presented results, the BS can assign the appropriate modulation scheme for each of the paired devices based on the estimated average SIR on the interference link.

Fig. 3: SER of BPSK and QPSK schemes with and without Rayleigh fading.

Fig. 3 shows the SER of BPSK and 4-QAM (or equivalently QPSK). In Fig. 3 the analysis is validated via two simulation scenarios, namely the “Noisy Simulation” and “Non-Gaussian Interfering Symbols”. In the *Noisy Simulation*, the received signal is perturbed by a Gaussian noise in addition to the Laplacian interference, which perfectly matches the analysis and validates the derived SER expressions. The interfering UE in *Non-Gaussian Interfering Symbols* scenario randomly

and uniformly alternates between BPSK and M-QAM for $M \in \{4, 16, 64\}$. Hence, “Non-Gaussian Interfering Symbols” scenario validates the Gaussian signaling assumption of the interfering UEs. The figure shows that the accuracy of the Gaussian signaling assumption is better for faded downlink scenarios. This is because fading on the downlink increases the level of uncertainties in the detection process at the test UE, which puts less significance on the interfering symbol distribution from the interfering UE and vice versa. Nevertheless, the Gaussian signaling approximation always shows good accuracy in the low SIR regime, which is a critical region of operation to avoid throughput degradation.

Fig. 4: SER of QPSK and 16-QAM schemes over Nakagami- m fading for $m = 1, 2$, and 10 and negligible noise. The stars denote the simulation results and the squares denote the approximation SER.

Fig. 3 and Fig. 4 show the cost of inter-user interference in terms of error probability for different modulation schemes and fading model on the downlink. Generally, the figures show that inter-user interference severely degrade the downlink performance at low SIR and that the effect of inter-user interference diminishes for high $\bar{\gamma}$ values specially for good downlink channel condition. Accordingly, based on the results of Fig. 3 and Fig. 4 along with the downlink quality of service requirement, the BS can decide the pairing SIR threshold for NLOS users. Fig. 4 also shows that the gap between the constellations decreases by increasing the fading severity (from 5dB to 2dB for $\text{SER}=10^{-5}$), which is due to the dominance of the fading conditions on the error probability.

Fig. 5 is plotted to investigate the Laplacian interference dominance assumption in (2) and to show the model accuracy when varying the interference-to-noise-power-ratio (INR). The figure plots the SER of 16PSK scheme in two types of fading channels, namely, Nakagami-4 channel and Rayleigh channel. The SER is plotted versus the signal to interference plus noise ratio (SINR) (where σ_n^2 represents the noise power) for fixed noise-plus-interference power but different values for the INR. Particularly, three values of INR are investigated, (i)

Fig. 5: SER of 16PSK scheme through Rayleigh fading and Nakagami-4 channels with different values of INR.

interference limited $\text{INR} = 20 \text{ dB}$, (ii) less interference limited $\text{INR} = 10 \text{ dB}$, and (iii) equal ratio between interference and thermal noise $\text{INR} = 0 \text{ dB}$. At first, the figure shows a close match between the theoretical results and simulations in all fading scenarios and INR values at low SINR regime. However, at high SINR regime, the model accuracy depends on the fading scheme. In the Rayleigh fading scenario, the noise effect is negligible and the theoretical results exactly match all INR cases in the high SINR regime. However, in Nakagami-4 scenario (i.e., less fading severity) the theoretical results represent an upper-bound for the SER at high SINR regime due to the heavy tailed distribution of the Laplacian perturbation. Note that the gap between the theoretical results and simulations diminishes at severe fading (i.e., Rayleigh) and/or low SINR regime because the effect of the noise/interference tail distributions on the SER performance is not significant. That is, the error may occur at low values of interference plus noise, and hence, the tail distribution does not have a prominent effect. On the other hand, at high SINR regime and/or less severe fading, the error occurs at high interference plus noise power. Hence, the tail distribution effect becomes prominent and the gap is more noticeable.

The error rate analysis shows only the negative effect of the NLOS user pairing in 3NT FD cellular networks. This is because the error rate for a given modulation scheme is a function of the SIR only which is degraded by the inter-user interference. In order to see the overall effect of the proposed NLOS user pairing criterion, we look at the throughput which is a function of both the degraded SIR as well as the improved BW. The throughput is defined as

$$\mathcal{T} = \frac{\log_2(M) (1 - \overline{\text{SER}})}{t_s}, \quad (36)$$

where $\log_2(M)$ is the number of bits per symbol and t_s is the symbol duration. For the HD case, we assume error free

Fig. 6: Throughput Gain of BPSK and QPSK over Rayleigh and Nakagami-0.5.

transmission and channel BW of B . Consequently, the HD throughput is given by

$$\mathcal{T}_{HD} = B \log_2(M). \quad (37)$$

For the FD case, the 3NT increases the channel BW to $2B$ on the expense of inter-user interference that imposes detection errors. Consequently, the FD throughput is given by

$$\mathcal{T}_{FD} = 2B \log_2(M) (1 - \overline{SER}). \quad (38)$$

For each modulation scheme, we define the throughput gain imposed by the NLOS user pairing via 3NT FD communication as

$$\mathcal{G} = \frac{\mathcal{T}_{FD}}{\mathcal{T}_{HD}} = 2 (1 - \overline{SER}). \quad (39)$$

Fig. 6 shows the complete picture of the inter-user interference effect imposed by the 3NT FD by looking at the throughput gain with respect to the interference free HD scenario. Interestingly, this figure shows that the throughput gain is always greater than unity for BPSK regardless of the SIR severity. This is because the FD doubles the transmission rate while the worst case error probability for BPSK is one half. However, for higher modulation schemes, which are typical for downlink transmission, the inter-user interference can significantly degrade the effective throughput. For instance, an 80% throughput degradation occurs for 64-QAM modulation at 0 dB SIR. On the other hand, at high SIR regime, the proposed NLOS user pairing via 3NT doubles the effective throughput. The throughput gain, which relay on the developed error rate model, can be used to maximize the transmission rate via adaptive modulation based on the experienced SIR and fading severity. The results in Fig. 6 can be used by the BS to assign the appropriate downlink modulation scheme for each paired HD device based on the average SIR.

VII. CONCLUSION

This paper develops a mathematical framework to study downlink error rate and throughput for half-duplex users when served by FD BSs via the 3NT. The developed model is used to motivate long-term user pairing for user with NLOS interfering links. A comprehensive performance assessment is presented for such pairing scheme and insights into the downlink modulation scheme selection are provided. Particularly, approximating the interfering user symbols by Gaussian signals, the base-band NLOS inter-user interference is shown to be Laplacian with dependent real and imaginary parts. The paper then derives closed form expressions for the average symbol error rate for BPSK, PAM, QAM, and MPSK modulations schemes for an unfaded and then an EGK faded downlink channel. To this end, we derive the effective throughput gain imposed by NLOS user pairing via 3NT FD communication when compared to an idealized interference and noise free HD scenario. The results can be used to determine SIR pairing threshold based on the downlink quality of service requirements as well as to adaptively change the modulation scheme according the SIR and downlink channel condition to maximize the downlink throughput. All mathematical findings of the paper are verified via independent Monte Carlo simulations.

APPENDIX A PROOF OF LEMMA 1

From the definition of the CHF, we can write

$$\varphi_Z(t_1, t_2) = \mathbb{E} \left[e^{j(t_1 \Re(Z) + t_2 \Im(Z))} \right]. \quad (A.1)$$

Replacing the real and imaginary parts of Z as follows in (A.1), we have

$$\begin{aligned} \Re(Z) &= g_r x_r - g_i x_i \\ \Im(Z) &= g_r x_i + g_i x_r, \end{aligned} \quad (A.2)$$

to obtain the following new expression of the CHF

$$\begin{aligned} \varphi_Z(t_1, t_2) &= \mathbb{E} [\exp(jx_r(t_1 g_r + t_2 g_i)) \\ &\quad \times \exp(jx_i(t_2 g_r - t_1 g_i))]. \end{aligned} \quad (A.3)$$

By conditioning with respect to g_r and g_i (which are independent Gaussian random variables), the CHF of Z can be written as the product of the CHF of x_r and x_i , since x_r and x_i are independent among themselves and independent to g_r and g_i . Now the CHF of Z can be expressed as the mean over g_r and g_i of the conditional CHF

$$\begin{aligned} \varphi_Z(t_1, t_2) &= \mathbb{E}_{g_i, g_r} [\varphi_{x_r|g_i, g_r}(t_1 g_r + t_2 g_i) \\ &\quad \times \varphi_{x_i|g_i, g_r}(t_2 g_r - t_1 g_i)]. \end{aligned} \quad (A.4)$$

Note that if X follows a Gaussian distribution with zero mean and variance σ^2 , its CHF is expressed as $\varphi_X(x) = e^{-\frac{1}{2}\sigma^2 x^2}$. Using this fact in (A.4) will give us the expression of $\varphi_Z(\cdot, \cdot)$ as

$$\begin{aligned} \varphi_Z(t_1, t_2) &= \int_{\mathbb{R}} \int_{\mathbb{R}} f_{g_r}(g_r) f_{g_i}(g_i) \\ &\quad \times e^{-\frac{1}{4}(t_1 g_r + t_2 g_i)^2} e^{-\frac{1}{4}(t_2 g_r - t_1 g_i)^2} dg_r dg_i \\ &= \left(\frac{1}{\sqrt{\pi}} \int_{\mathbb{R}} e^{-\frac{1}{4}g^2(4+t_1^2+t_2^2)} dg \right)^2. \end{aligned} \quad (A.5)$$

Finally, by solving the remaining integral, the CHF gets its final expression as

$$\varphi_Z(t_1, t_2) = \frac{4}{4 + t_1^2 + t_2^2} = \frac{1}{1 + \frac{|\mathbf{t}|^2}{4}}, \quad (\text{A.6})$$

where the vector $\mathbf{t} = (t_1, t_2)$. We can conclude at this point that the expression of the CHF of Z is the same as the CHF of a bivariate Laplace distribution [23].

APPENDIX B PROOF OF THEOREM 1

The CDF is defined as the primitive of the inverse Fourier transform of the CHF (primitive of the PDF that vanishes at $(-\infty, -\infty)$), defined also as

$$F_Z(z_r, z_i) = F_{Z_r}(z_r) + F_{Z_i}(z_i) + c + \frac{1}{4\pi^2} \iint_{z_r, z_i} \iint_{t_1, t_2} e^{j(z_r t_1 + z_i t_2)} \varphi_Z(t_1, t_2) dt_1 dt_2 dz_r dz_i,$$

where $F_{Z_r}(\cdot)$ and $F_{Z_i}(\cdot)$ are the CDFs of Z_r and Z_i respectively, and c is a real constant to be determined. In the following analysis we are focusing on the last part (four integrals), namely $\bar{F}_Z(z_r, z_i)$, which is the cumulative CDF (CCDF).

Looking at the CHF, one notice that it is an even function on both variables, so the Fourier transform becomes a cosine transform

$$\begin{aligned} \bar{F}_Z(z_r, z_i) &= \frac{1}{\pi^2} \iint_{z_r, z_i} \int_0^\infty \int_0^\infty \cos(z_r t_1) \cos(z_i t_2) \\ &\quad \times \varphi_Z(t_1, t_2) dt_1 dt_2 dz_r dz_i \\ &= \frac{4}{\pi^2} \int_0^\infty \int_0^\infty \frac{\sin(z_r t_1) \sin(z_i t_2)}{t_1 t_2 (4 + t_1^2 + t_2^2)} dt_1 dt_2. \end{aligned} \quad (\text{B.1})$$

Using the representation of the sine function in terms of the Meijer's G function (MGF) [38, Eq. (2.9.7)] and its Mellin-Barnes representation [38, Eq. (1.1.2)], yields

$$\begin{aligned} \sin(x) &= \sqrt{\pi} G_{0,2}^{1,0} \left[\frac{x^2}{4} \middle| \frac{1}{2}, 0 \right] \\ &= \sqrt{\pi} \frac{1}{2\pi j} \int_{\mathcal{C}} \frac{\Gamma(\frac{1}{2} + s)}{\Gamma(1 - s)} \left(\frac{x}{2} \right)^{-2s} ds. \end{aligned} \quad (\text{B.2})$$

Substituting the expressions of the sine function (B.2) in (B.1), we get another expression of the CCDF as

$$\begin{aligned} \bar{F}_Z(z_r, z_i) &= \frac{4}{\pi (2\pi j)^2} \int_{\mathcal{L}_1} \int_{\mathcal{L}_2} \frac{\Gamma(\frac{1}{2} + s) \Gamma(\frac{1}{2} + t)}{\Gamma(1 - s) \Gamma(1 - t)} \\ &\quad \left(\frac{z_r}{2} \right)^{-2s} \left(\frac{z_i}{2} \right)^{-2t} \int_0^\infty \int_0^\infty \frac{t_1^{-2s-1} t_2^{-2t-1}}{4 + t_1^2 + t_2^2} dt_1 dt_2 ds dt. \end{aligned} \quad (\text{B.3})$$

The last double integral has a closed form solution which is equal to

$$\begin{aligned} \int_0^\infty \int_0^\infty \frac{t_1^{-2s-1} t_2^{-2t-1}}{4 + t_1^2 + t_2^2} dt_1 dt_2 &= 2^{-4-2s-2t} \Gamma(1 + s + t) \\ &\quad \times \frac{\Gamma(1 - s) \Gamma(1 - t) \Gamma(s) \Gamma(t)}{\Gamma(1 + s) \Gamma(1 + t)}. \end{aligned} \quad (\text{B.4})$$

This result is valid for $s \in \mathcal{L}_1$ and $t \in \mathcal{L}_2$. Hence (B.3) can be re-written as

$$\begin{aligned} \bar{F}_Z(z_r, z_i) &= \frac{1}{4\pi} \frac{1}{(2\pi j)^2} \int_{\mathcal{L}_1} \int_{\mathcal{L}_2} \Gamma(1 + s + t) \Gamma(s) \Gamma(t) \\ &\quad \times \frac{\Gamma(\frac{1}{2} + s) \Gamma(\frac{1}{2} + t)}{\Gamma(1 + s) \Gamma(1 + t)} z_r^{-2s} z_i^{-2t} ds dt \\ &= \frac{1}{4\pi} G_{1,0,2,1,2,1}^{0,1,0,2,0,2} \left[\frac{1}{z_r^2}, \frac{1}{z_i^2} \middle| \begin{matrix} 0 \\ 0 \end{matrix} \middle| \begin{matrix} \frac{1}{2}, 1 \\ \frac{1}{2}, 1 \end{matrix} \right]. \end{aligned} \quad (\text{B.5})$$

Since Z_r and Z_i have marginal Laplace distribution, their CDFs have the same form that is given by

$$\begin{cases} F_{Z_r}(z_r) &= 1 - \frac{1}{2} e^{-2z_r} \\ F_{Z_i}(z_i) &= 1 - \frac{1}{2} e^{-2z_i}. \end{cases} \quad (\text{B.6})$$

Finally it is easy to proof that $c = -1$, and thus end of the proof.

APPENDIX C PROOF OF COROLLARY 1.1

The PDF is the derivative of the CDF with respect to z_r and z_i , $f_Z(z_r, z_i) = \frac{\partial^2 F_Z(z_r, z_i)}{\partial z_r \partial z_i}$. While both marginal CDF components vanish after the second derivation, only the CCDF component is remaining. Therefore, from (B.5) and with the help of a change of variable, the PDF of Z can be written as

$$\begin{aligned} f_Z(z_r, z_i) &= \frac{1}{\pi} \frac{1}{(2\pi j)^2} \int_{\mathcal{L}_1} \int_{\mathcal{L}_2} \Gamma(1 + s + t) \\ &\quad \Gamma(\frac{1}{2} + s) \Gamma(\frac{1}{2} + t) z_r^{-2s-1} z_i^{-2t-1} ds dt \\ &= \frac{1}{\pi} \frac{1}{(2\pi j)^2} \int_{\mathcal{L}_1} \Gamma(s) z_i^{-2s} \\ &\quad \int_{\mathcal{L}_2} \Gamma(s - t) \Gamma(t) \left(\frac{z_r}{z_i} \right)^{-2t} dt ds. \end{aligned} \quad (\text{C.1})$$

Using the identity [38, Eq. (2.9.5)] to solve the second integral, we can write

$$\begin{aligned} \frac{1}{2\pi j} \int_{\mathcal{L}_2} \Gamma(s - t) \Gamma(t) \left(\frac{z_r}{z_i} \right)^{-2t} dt &= G_{1,1}^{1,1} \left[\frac{z_r^2}{z_i^2} \middle| \begin{matrix} 1 - s \\ 0 \end{matrix} \right] \\ &= \Gamma(s) \left(1 + \frac{z_r^2}{z_i^2} \right)^{-s}. \end{aligned}$$

Finally, the PDF is simplified to the desired final results

$$\begin{aligned} f_Z(z_r, z_i) &= \frac{1}{\pi} \frac{1}{2\pi j} \int_{\mathcal{L}_1} \Gamma(s) \Gamma(s) (z_r^2 + z_i^2)^{-s} ds \\ &= \frac{1}{\pi} G_{0,2}^{2,0} \left[z_r^2 + z_i^2 \middle| \begin{matrix} 0, 0 \end{matrix} \right] \\ &= \frac{2}{\pi} K_0 \left(2\sqrt{z_r^2 + z_i^2} \right). \end{aligned} \quad (\text{C.2})$$

The last equality is obtained using the identities [38, Eq. (2.9.31) & Eq. (2.9.39)].

APPENDIX D PROOF OF THEOREM 3

A rectangular QAM constellation is formed by $M = M_I M_Q$ symbols. These symbols can be classified as follows: 4 corner symbols, $2(M_I - 2)$ edge symbols with decision distance d_I , $2(M_Q - 2)$ edge symbols with decision distance d_Q , and $(M_I - 2)(M_Q - 2)$ inner symbols. To compute the conditional error probability, we need to get the probability of error of each symbol and then the SER is obtained by averaging these probabilities over the probability of occurrence of each symbol. Let us begin with the corner symbols. Actually the probability of error of the corner symbol can be given as

$$\begin{aligned} P_e^{cor} &= 1 - \int_{-\frac{d_I}{2}}^{\frac{d_I}{2}} \int_{-\frac{d_Q}{2}}^{\frac{d_Q}{2}} f_Z(x, y) dx dy = 1 - F_Z\left(\frac{d_I}{2}, \frac{d_Q}{2}\right) \\ &= \frac{1}{2} (e^{-d_I} + e^{-d_Q}) - \bar{F}_Z\left(\frac{d_I}{2}, \frac{d_Q}{2}\right), \end{aligned} \quad (D.1)$$

where $\bar{F}_Z(\cdot, \cdot)$ is defined in (B.5).

The calculation of the probability of error of the edge points is similar for the in-phase point and the quadrature-phase points, hence we are focusing on the in-phase point. Furthermore, the probability of error detection of an in-phase edge symbol is obtained as

$$\begin{aligned} P_e^{edg, I} &= 1 - \int_{-\frac{d_I}{2}}^{\frac{d_I}{2}} \int_{-\frac{d_Q}{2}}^{\frac{d_Q}{2}} f_Z(x, y) dx dy \\ &= 2 - \frac{1}{2} e^{-d_I} - 2F_Z\left(\frac{d_I}{2}, \frac{d_Q}{2}\right) \\ &= \frac{1}{2} (e^{-d_I} + 2e^{-d_Q}) - 2\bar{F}_Z\left(\frac{d_I}{2}, \frac{d_Q}{2}\right). \end{aligned} \quad (D.2)$$

Per consequence, the probability of error detection of a quadrature phase edge symbol is given by

$$\begin{aligned} P_e^{edg, Q} &= \frac{1}{2} (e^{-d_Q} + 2e^{-d_I}) - 2\bar{F}_Z\left(\frac{d_Q}{2}, \frac{d_I}{2}\right) \\ &= \frac{1}{2} (e^{-d_Q} + 2e^{-d_I}) - 2\bar{F}_Z\left(\frac{d_I}{2}, \frac{d_Q}{2}\right). \end{aligned} \quad (D.3)$$

Finally, the probability of error detection of an inner symbol is obtained as

$$\begin{aligned} P_e^{inn} &= 1 - \int_{-\frac{d_I}{2}}^{\frac{d_I}{2}} \int_{-\frac{d_Q}{2}}^{\frac{d_Q}{2}} f_Z(x, y) dx dy \\ &= e^{-d_I} + e^{-d_Q} - 4\bar{F}_Z\left(\frac{d_I}{2}, \frac{d_Q}{2}\right). \end{aligned} \quad (D.4)$$

Hence, assuming equiprobable symbols, the SER of rectangular QAM is derived as

$$\begin{aligned} P_{QAM}(e|\gamma) &= \frac{1}{M_I M_Q} \left(4P_e^{cor} + 2(M_Q - 2)P_e^{edg, Q} \right. \\ &\quad \left. + 2(M_I - 2)P_e^{edg, I} + (M_Q - 2)(M_I - 2)P_e^{inn} \right). \end{aligned} \quad (D.5)$$

Now by replacing the decision distances by their values $d_I = \frac{2\sqrt{3}\gamma}{d}$, $d_Q = \tau d_I$, using the parameter d from (9), and after some algebraic manipulation, the result in Theorem 3 is obtained easily.

APPENDIX E PROOF OF THEOREM 4

First, the probability that $Y_1 > 0$ is equivalent to $\Pr[\tan(\varphi - \theta)I - Q \leq 0]$, because $\varphi \in [0, \frac{\pi}{4} - \frac{2\pi}{M}]$ and $\theta = \frac{\pi}{M}$. Now let's call $X = \tan(\varphi - \theta)I - Q$, then the PDF of X can be written as

$$f_X(x) = \int_{\mathbb{R}} f_{I, Q}(u, \tan(\varphi - \theta)u - x) du. \quad (E.1)$$

Which gives the following expression of the first term of (17) as

$$\begin{aligned} \Pr[Y_1 \geq 0] &= \Pr[X \leq 0] = \Pr[-X \geq 0] \\ &= \int_0^\infty \int_{\mathbb{R}} f_{I, Q}(u, \tan(\varphi - \theta)u + x) du dx \\ &= \frac{2\gamma}{\pi} \int_0^\infty \int_{\mathbb{R}} K_0\left(2\sqrt{\gamma A(u, x)}\right) du dx, \end{aligned} \quad (E.2)$$

where $A(u, x) = (u + \cos \varphi)^2 + (\tan(\varphi - \theta)u + x + \sin \varphi)^2$. Using a change of variable and the fact that the marginal of (6) is Laplacian (i.e. $\int_{\mathbb{R}} \frac{2}{\pi} K_0(2\sqrt{u^2 + v^2}) du = e^{-2|v|}$), (E.2) can be further simplified to

$$\Pr[Y_1 \geq 0] = \frac{1}{2} e^{-2 \sin \theta \sqrt{\gamma}}. \quad (E.3)$$

The same analysis can be used to compute the second term of (17), and a similar result can be easily obtained

$$\Pr[Y_2 \leq 0] = \frac{1}{2} e^{-2 \sin \theta \sqrt{\gamma}}. \quad (E.4)$$

Finally, to compute the last term in (17) (i.e. $\Pr[Y_1 \geq 0, Y_2 \leq 0]$), we define $W_1 = Q - \tan(\varphi - \theta)I$ and $W_2 = \tan(\varphi + \theta)I - Q$, then we have $\Pr[Y_1 \geq 0, Y_2 \leq 0] = \Pr[W_1 \geq 0, W_2 \leq 0]$. The PDF of the couple (W_1, W_2) can be written, using the PDF of (I, Q) , as

$$\begin{aligned} f_{W_1, W_2}(w_1, w_2) &= \frac{2\gamma \cos \varphi^2 - \sin \theta^2}{\pi \sin 2\theta} \\ &\quad K_0\left(2\sqrt{\gamma \frac{\cos \varphi^2 - \sin \theta^2}{\sin 2\theta}} \sqrt{A_1(w_1, w_2)}\right), \end{aligned} \quad (E.5)$$

where $A_1(w_1, w_2) = (w_1 + w_2 + \cos \varphi)^2 + (\tan(\varphi + \theta)w_1 + \tan(\varphi - \theta)w_2 + \sin \varphi)^2$. The desired probability of error can be written as

$$\begin{aligned} \Pr[Y_1 \geq 0, Y_2 \leq 0] &= \Pr[W_1 \geq 0, W_2 \leq 0] \\ &= \int_0^\infty \int_0^\infty f_{W_1, W_2}(w_1, w_2) dw_1 dw_2. \end{aligned} \quad (E.6)$$

By developing the squares in $A_1(w_1, w_2)$ and using a change of variable, (E.6) can be expressed as

$$\begin{aligned} \Pr[Y_1 \geq 0, Y_2 \leq 0] &= \frac{1}{2\pi \sin 2\theta} \\ &\quad \times \int_B^\infty \int_B^\infty K_0\left(\frac{1}{\sin 2\theta} \sqrt{u^2 + v^2 + 2uv \cos 2\theta}\right) du dv, \end{aligned} \quad (E.7)$$

where $B = \sqrt{\gamma \frac{\cos \varphi^2 - \sin \theta^2}{\cos \theta}}$.

An integral representation of the Bessel function, available in [43, Eq. (2.13)], $K_0(x) = \frac{1}{2} \int_0^\infty \frac{1}{t} e^{-t - \frac{x^2}{4t}} dt$, can be used in (E.7). Hence we obtain a double incomplete integral of an exponential with square argument, such term can be solved

using the two dimensional Gaussian Q function $Q(\cdot, \cdot, \cdot)$ [44, Eq. (4.3)]. Consequently, (E.7) can be reduced to one integral as follows

$$\Pr[Y_1 \geq 0, Y_2 \leq 0] = \int_0^\infty e^{-t} Q\left(\frac{B}{\sqrt{2t}}, \frac{B}{\sqrt{2t}}, -\cos 2\theta\right) dt. \quad (\text{E.8})$$

On the other hand, from [44, Eq. (4.7)], the two dimensional Q function can be reduced to only one integral if its first two arguments are equal, $Q(x, x, \alpha) = \frac{1}{\pi} \int_0^{\frac{\pi}{4}} \frac{\sqrt{1-\alpha^2}}{1-\alpha \sin 2\Phi} e^{-\frac{x^2}{2} \frac{1-\alpha \sin 2\Phi}{(1-\alpha^2) \sin^2 \Phi}} d\Phi$, which is the case in (E.8). Using a suitable change of variable, and the integral representation of $K_1(\cdot)$, (E.8) can be re-written as

$$\Pr[Y_1 \geq 0, Y_2 \leq 0] = \frac{B}{2\pi} \int_1^\infty \frac{1}{\sqrt{u^2 - u \sin^2 \theta}} K_1\left(\frac{B\sqrt{u}}{\sin \theta}\right) du. \quad (\text{E.9})$$

An alternative expression of the Bessel function and the first term that contains the square root in terms of the Meijer's G function (MGF) is available in [38, Eq. (2.9.19)] and [38, Eq. (2.9.6)], respectively

$$K_1(x) = \frac{1}{x} G_{0,2}^{2,0} \left[\frac{x^2}{4} \middle| - \right]_{0,1} \\ \frac{1}{\sqrt{1-x}} = \sqrt{\pi} G_{1,1}^{1,0} \left[x \middle| \frac{1}{2} \right]_0. \quad (\text{E.10})$$

These alternative expressions transform the integral in (E.9) as an incomplete integral of the product of two MGFs, which can be transformed also to an integral of 3 MGFs

$$\Pr[Y_1 \geq 0, Y_2 \leq 0] = \frac{\sin \theta}{2\sqrt{\pi}} \int_0^\infty u^{-3/2} G_{1,1}^{0,1} \left[x \middle| 1 \right]_0 \\ \times G_{1,1}^{0,1} \left[\frac{x}{\sin^2 \theta} \middle| \frac{1}{2} \right] G_{0,2}^{2,0} \left[x \frac{B^2}{4 \sin^2 \theta} \middle| - \right]_{0,1} dx. \quad (\text{E.11})$$

Such integral can be easily solved using [34, Eq. (2.3)] to get the results of Theorem 4 and finish the proof.

APPENDIX F

SPECIAL CASES OF THE EGK DISTRIBUTION

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TABLE II: Some Special Cases of the EGK Distribution

Envelope Distribution	m	ξ	m_s	ξ_s
Rayleigh	1	1	∞	1
	∞	1	1	1
Maxwell	3/2	1	∞	1
	∞	1	3/2	1
Half-Normal	1/2	1	∞	1
	∞	1	1/2	1
Exponential	1	1/2	∞	1
	∞	1	1	1/2
Weibull	1	ξ	∞	1
	∞	1	1	ξ_s
Nakagami- m	m	1	∞	1
	∞	1	m_s	1
Generalized Nakagami	m	ξ	∞	1
	∞	1	m_s	ξ_s
Gamma	m	1/2	∞	1
	∞	1	m_s	1/2
Generalized Gamma	m	$\xi/2$	∞	1
	∞	1	m_s	$\xi_s/2$
Half-Normal-Exponential	1/2	1	1	1
	1	1	1/2	1
Half-Normal-Gamma	1/2	1	m_s	1
	m	1	1/2	1
GNM-Lognormal	∞	0	m_s	ξ_s
	m	ξ	∞	0
Suzuki	∞	0	1	1
	1	1	∞	0
Rayleigh-Exponential	1	1	1	1
Maxwell-Lognormal	∞	0	3/2	1
	3/2	1	∞	0
Maxwell-Exponential	1	1	3/2	1
	3/2	1	1	1
Maxwell-Gamma	m	1	3/2	1
	3/2	1	m_s	1
Weibull-Lognormal	∞	0	1	ξ_s
	1	ξ	∞	0
Weibull-Exponential	1	1	1	ξ_s
	1	ξ	1	1
Weibull-Weibull	1	ξ	1	ξ_s
Weibull-Gamma	m	1	1	ξ_s
	1	ξ	m_s	1
Nakagami-Lognormal	∞	0	m_s	1
	m	1	∞	0
K-Distribution	m	1	1	1
	1	1	m_s	1
Generalized-K	m	1	m_s	1
GNM-Exponential	1	1	m_s	ξ_s
	m	ξ	1	1
GNM-Weibull	m	ξ	1	ξ_s
	1	ξ	m_s	ξ_s
GNM-Gamma	m	1	m_s	ξ_s
	m	ξ	m_s	1

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