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An Efficient Two-Scale Hybrid Embedded Fracture Model for Shale Gas Simulation

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Abstract

Natural and hydraulic fractures existence and state differs on a reservoir-by-reservoir or even on a well-by-well basis leading to the necessity of exploring the flow regimes variations with respect to the diverse fracture-network shapes forged. Conventional Dual-Porosity Dual-Permeability (DPDP) schemes are not adequate to model such complex fracture-network systems. To overcome this difficulty, in this paper, an iterative Hybrid Embedded multiscale (two-scale) Fracture model (HEF) is applied on a derived fit-for-purpose shale gas model. The HEF model involves splitting the fracture computations into two scales: 1) fine-scale solves for the flux exchange parameter within each grid cell; 2) coarse-scale solves for the pressure applied to the domain grid cells using the flux exchange parameter computed at each grid cell from the fine-scale. After that, the D dimensions matrix pressure and the (D-1) lower dimensional fracture pressure are solved as a system to apply the matrix-fracture coupling. HEF model combines the DPDP overlapping continua concept, the DFN lower dimensional fractures concept, the HFN hierarchical fracture concept, and the CCFD model simplicity. As for the fit-for-purpose shale gas model, various fit-for-purpose shale gas models can be derived using any set of selected properties plugged in one of the most popularly used proposed literature models as shown in the appendix. Also, this paper shows that shale extreme low permeability cause flow behavior to be dominated by the structure and magnitude of high permeability fractures.

Keywords: Shale Gas, CCFD, DPDP, two-scale, fracture network, hybrid fracture method.

1 Introduction

Rocks are aggregates of minerals and cavities mixtures. Cavities include fractures and pores. Analyzing the extent to which hydrocarbons including oil and shale gas are trapped in a formation and whether those hydrocarbons can be removed from rock cavities and produced or not has to begin with an analysis of the rocks themselves. The properties of interest are often controlled more by the rock fabric than by the bulk composition. Cavities ability to contain hydrocarbons is usually more

important than the mineral frame. Numerous attempts have been made to extract rock properties from images of rocks bulk and void spaces. These techniques often work well, but they depend significantly on many aspects that include: observation scales, representative natures of images, and internal heterogeneities. Cavities form collections of complex networks made of natural and hydraulic fractures that need to be considered when studying flows through rocks (porous mediums).

Hydrocarbon reservoirs exist in all three types of rocks: igneous, metamorphic, and sedimentary. Sedimentary rocks are considered the most common rocks associated with hydrocarbons. They also include shales. Shale exploration, development and production is getting more attention lately in oil industry due to the high gas prices and the success in shale gas production (Wang et. al. 2012, Zhang et. al. 2014). However, It is still important to know the shale gas dynamic characteristics and consider them within the modeling assumptions to improve the production estimations. This gives an indication to many needed countermeasures during the extraction and production processes.

Many theoretical, analytical, semi-analytical and empirical models as well as various properties specification techniques have been proposed over the years to characterize the behavior of shale gas systems. Also, many numerical approaches have been developed over the years to deal with fractured systems. Section A of the appendix generally discusses the most popularly used proposed literature models as well as some of the shale properties specification techniques used to design fit-for-purpose models for research or industrial interests based on a set of selected properties. Section B of the appendix derives two fit-for-purpose models based on some selected properties to explore within this paper for adsorption vs. no-adsorption, single-phase, single-component slightly compressible shale formations. Section C of the appendix derives Section B parameters assuming capillary tube shape fractures. Sections D & E of the appendix discuss some of the HEF-model numerical details.

In literature, Civan et. al. (2011) derived a fully compressible *single continuum* theoretical model to determine essential gas permeability and diffusivity within shale formations. They incorporated slip, flow transition, mass of gas adsorbed per shale volume, and free molecules flow properties in a permeability unified Hagen-Poiseuille-type equation using non-linear pressure. Moreover, Ertekin et. al. (1986) suggested a dynamic slippage model. It quantified apparent gas permeability changes with pressure and desorption. Clarkson et. al. (2012) used that dynamic slippage after modifying the desorption by a set of pseudo-variables and using analysis methods. Furthermore, Medeiros et. al. (2007) found that designing fractures to focus stress around wells is more useful than designing fractures to accomplish high conductivity. This limits effective drainage area to naturally fractured regions around wells and the hydraulic fractures within. To simplify the fit-for-purpose model tested in this paper from stress point of view, we assume slightly compressible shale causing first order porosity change approximation.

Figure 1: Dual-Continuum models (including DPDP) overlapping continua representation

Dual-Continuum (DC) model introduced by Warren and Root (1963) for fractured porous medium was one of the most popular shale gas modeling techniques. DC assumed an idealized fractured porous medium intersected by a network of interconnected fractures dividing the domain into series of disjoint blocks of porous rocks called matrices. The key aspect of DC models is to represent the

reservoir by two detached overlapping continua: fracture where fracture networks act as the major flow routes; matrices where porous rock blocks serve as the main storage as illustrated in Figure 1. Communication between the two continua is coordinated by a flux exchange parameter that can be evaluated using fracture properties within each grid cell.

Diverse derivations and modifications were added to DC assumptions over the years. For example, Li et. al. (2014) developed a *Dual-Porosity-Dual-Permeability (DPDP)* model for multistage fractured horizontal wells in shale gas reservoirs incorporating slippage corrected gas permeability and gas adsorption. In the model, gas molecules were assumed to slip along pore surfaces if the mean free path of gas molecules is larger than or equal to the average effective rock pore-throat radius. This caused a gas permeability overestimation compared to liquid fluids true absolute permeability (Clarkson et. al. 2012). Based on micro-seismic data, modeling flow through horizontal wells residing in complex network of well-developed cavities with multiple orientations and complex patterns is considered one of the main challenges. Conventional DPDP models assume pseudo steady state matrix to fracture flow while matrix porosity in shale gas reservoirs raises the matrix transient behavior importance. This gives an indication that conventional DPDP models are not adequate to model such complex systems. Since DPDP models sometimes do not effectively capture the transient behaviors in ultra tight shale matrix blocks, *Discrete Fracture Network (DFN)* models were introduced and used to deal with the geological complexity and connectivity of fractures (Cipolla et. al. 2010, Zhang et. al. 2009). Zhang et. al. (2009) used a stochastic DFN model to facilitate multiple comprehensions for quantitative measures, uncertainty and validations. However, it was found that it is neither practical nor beneficial to model dense fractures with DFN model due to the significant CPU time necessary for a single well test simulation. Though Basquet et. al. (2005) proposed a *DFN homogenization* to reduce the geometrical complexity while conserving hydraulic properties, the resulting homogenized model was almost equivalent to Warren and Root (1963) model. As a result, it also ignored the structural details effects of interest due to multiple orientations and complex patterns.

To deal with this dilemma many *Hybrid Fracture Network (HFN)* models combining several schemes were developed over the years. Moinfar et. al. (2013) combined DFN for hydraulic fractures with DPDP for natural fractures. Li et. al. (2008) proposed a Hybrid Finite Volume (HFV) model for homogenized media with short, medium and long fractures. In HFV, short and medium fractures were modeled using effective permeability and large fractures were modeled using DFN. Lee et. al. (2000, 2001) also proposed a Finite Difference (FD) model for media with short, medium and long fracture networks. Matrix permeability for medium size fractures in FD was enhanced by an analytical analysis for short fractures. To enhance effective conductivity, medium size fractures were modeled using boundary element method. Finally, large fractures were modeled explicitly as major fluid conductors like wells in conventional reservoirs. Both methods dealt with different fracture lengths in numerically different ways and distinguished fracture size classes by the ratio of fracture length to grid scale.

In this paper, we deal with this dilemma by applying an *iterative Hybrid Embedded multiscale (two-scale) Fracture (HEF)* model using Cell Centered Finite Difference (CCFD) scheme. HEF model involves splitting the fracture computations into two scales: fine and coarse. Fine scale solves for the matrix-fracture flux exchange parameter analytically within each grid cell depending on fractures properties that include their shapes, orientations, etc., allowing control over fractures properties details as illustrated in sections D & E of the appendix. Coarse scale solves for matrix and fracture pressures for the domain grid cells coupled as a system (matrix in D dimensions and fracture in (D-1) dimensions) using the flux exchange parameter computed at each grid cell from the fine scale. HEF combines the DPDP overlapping continua concept, the DFN lower dimensional fractures concept, the HFN hierarchical fracture concept, where the small-scale fractures are homogenized and represented as part of the continuum described by permeability and porosity values, and the CCFD simplicity. In this paper, various possible fracture networks are explored using HEF model within shale formations.

2 Shale Gas Model

2.1 Flow model

Gas in shale formations usually splits between matrices and fractures systems. The two systems retain shale gas as free gas between structures and gas adsorbed within structures. However, the quantities of each form differ significantly according to the system of interest. Matrices implicate much more adsorbed gas compared to free gas forming as a result the prime gas storage within shale structures. On the other hand, fractures implicate much more free gas compared to adsorbed gas forming as a result the major gas flow channels within shale formations. Gas adsorbed within shale matrices varies according to dominant pressure and organic adsorbent conditions. Conservation of mass equations derivations for both systems are illustrated in section B of the appendix. Matrices conservation equation used a unified Hagen-Poiseuille-type equation (Civan et. al. 2011) and applied Langmuir isotherm (Civan et. al. 2011, Li et. al. 2014), real gas density, slightly compressible rocks properties (LeVeque 2007), and linearized the pressure. The derivation led to equation (1) for adsorbed gas within matrices with equation (2) as the time derivative coefficient:

$$F_m(p) \frac{\partial p}{\partial t} + \frac{M_g p}{ZRT} \nabla \cdot v = Q_m \quad (1)$$

$$F_m(p) = \left\{ \frac{M_g \phi_0}{ZRT} (1 + c_\phi [2p - p_0]) + \frac{\rho_s M_g V_L}{V_{std}} \left(\frac{p \phi_0 c_\phi}{(p_L + p)} + \frac{p_L (1 - \phi_0 [1 + c_\phi (p - p_0)])}{(p_L + p)^2} \right) \right\} \quad (2)$$

$F_m(p)$ matrix gas time derivative coefficient as a function of pressure p ; M_g molecular weight; R universal gas constant; T temperature; ρ_s material density; V_L Langmuir gas volume; V_{std} molar volume of gas at standard temperature (273.15 K) and pressure (101,325 Pa); p_L Langmuir gas pressure; v velocity; ϕ_0 is porosity at pressure p_0 ; ϕ is porosity at pressure p ; c_ϕ is porosity compressibility.

Gas within fractures was derived using the standard mass conservation equation (Chen 2007), real gas density, slightly compressible porosity (LeVeque 2007), and linearized the pressure. The derivation led to equation (3) for free gas within fractures with equation (4) as the time derivative coefficient:

$$F_f(p) \frac{\partial p}{\partial t} + \frac{M_g p}{ZRT} \nabla \cdot v = Q_f \quad (3)$$

$$F_f(p) = \left\{ \frac{M_g \phi_0}{ZRT} (1 + c_\phi [2p - p_0]) \right\} \quad (4)$$

$F_f(p)$ fracture gas time derivative coefficient as a function of pressure p .

However, certain coupling mechanisms need to be introduced to demonstrate matrix-fracture interactions along with solving equations (1) and (3) simultaneously as a system. In both equations, source/sink terms Q_m and Q_f include an additional flux exchange parameter derived in section B.3 of the Appendix (Chen 2007, Chen et. al. 2006, Kazemi 1968, Warren and Root 1963):

$$Q_m = \alpha \frac{k}{\mu} (p_f - p_m) + q_{ext} \quad (5)$$

$$Q_f = \alpha \frac{k}{\mu} (p_m - p_f) + q_{ext} \quad (6)$$

q_{ext} is external source/sink term; the first term in both equations is the flux exchange parameter; Let $\alpha = \alpha^* |L_f|/h$; h is mesh size if $h_x = h_y$. $|L_f|$ is fracture absolute length within each fracture grid cell; k is permeability; μ is viscosity. The flux exchange parameter is discussed in details latter in this paper.

2.2 Model assumptions and parameters derivations:

Fluids could be classified either according to their response to external pressures (compressibility) as was illustrated in section B of the Appendix or according to shear stress effects (Chhabra et. al. 2008). Shear stress is defined as the force acting parallel to a tiny portion of the surface per unit area as illustrated by F in Figure 2. It is primarily caused by friction between fluid particles due to fluid viscosity causing it to be computed as the product of fluid viscosity and shear rate. For example, in Figure 2, the unidirectional shear stress can be computed as: $\gamma_x = \mu (-\partial v_x / \partial y)$. Shear rate: is the velocity gradient perpendicular to shear force. The slop of shear stress with respect to shear rate curve gives viscosity and can be used to classify fluids as illustrated by example in Figure 3.

Figure 2: illustrating a unidirectional shear rate as a velocity in a direction perpendicular to the shear force F.

Figure 3: fluids classification according to the slop of shear stress vs. shear rate curves.

Linear curves give constant viscosities and indicate Newtonian fluids. Newtonian fluid examples include water, air, thin motor oil, mineral oil, gasoline, alcohol, etc. Non-linear curves give varying viscosities and indicate non-Newtonian fluids (Pearson et. al. 2002).

Non-Newtonian fluid behaviors can be divided into: 1) Shear-thickening (Dilatant): fluid viscosity increase when shear is applied causing the system to act more like liquids than solids similar to quicksand, Silly putty, etc. 2) Shear-thinning (pseudoplastic): fluid viscosity decrease when shear is applied allowing them to thin and wet out surfaces evenly such as paint, asphalt, glue, etc. Non-Newtonian fluids characteristics are usually observed in structured fluids (Witten 1990). Furthermore, the structure of externally applied shear forces and the type of non-Newtonian fluid flow behavior are

correlated. This gives an indication that it is possible to synthesize a substance of desired rheological characteristics by tampering with its structure (Chhabra et. al. 2008). In addition, rheological measurements can be used to gain useful insight about the microstructure properties of such systems.

In this paper, we apply both by assuming that fractures take an approximately circular capillary tube shape with length L and radius R that coincides with the x -axis as illustrated in Figure C.1. Gas flows through the capillary tube due to pressure drop across the tube length. Relying on these assumptions, the parameters in the Equations (1-6) are analytically derived approximately.

The non-Newtonian fluid velocity v is derived using the power law, Ostwald de Waele properties (Chhabra et. al. 2008, Chen 2007), for such systems in section C.1 of the Appendix for both: slip and no-slip cases. The resulting equations are:

1) Velocity in axial direction as a function of r :

$$\text{No-Slip Case: } v_x(r) = \left(\frac{n}{n+1}\right) \left[\left(\frac{-\Delta p}{L}\right) \frac{R}{2m}\right]^{1/n} R \left[1 - \left(\frac{r}{R}\right)^{(n+1)/n}\right] \quad (7)$$

$$\text{Slip Case: } v_x(r) = \left[\left(\frac{-\Delta p}{L}\right) \frac{R}{2m}\right]^{1/n} R \left(\frac{c\lambda_{mean}}{R} + \left(\frac{n}{n+1}\right) \left[1 - \left(\frac{r}{R}\right)^{(n+1)/n}\right]\right) \quad (8)$$

2) Average velocity V integration over $[0: R]$:

$$\text{No-Slip Case: } V = \left(\frac{n}{3n+1}\right) \left[\left(\frac{-\Delta p}{mL}\right) \frac{R}{2}\right]^{1/n} R \quad (9)$$

$$\text{Slip Case: } V = \left(\frac{c\lambda_{mean}}{R} + \frac{n}{3n+1}\right) \left[\left(\frac{-\Delta p}{mL}\right) \frac{R}{2}\right]^{1/n} R \quad (10)$$

3) Average velocity $V_{avg} = v_x/V$:

$$\text{No-Slip Case: } V_{avg} = \left(\frac{3n+1}{n+1}\right) \left[1 - \left(\frac{r}{R}\right)^{(n+1)/n}\right] \quad (11)$$

$$\text{Slip Case: } V_{avg} = \left\{\frac{c\lambda_{mean}}{R} + \left(\frac{n}{n+1}\right) \left[1 - \left(\frac{r}{R}\right)^{(n+1)/n}\right]\right\} / \left\{\frac{c\lambda_{mean}}{R} + \frac{n}{3n+1}\right\} \quad (12)$$

4) Volumetric flow rate $Q = \pi R^2 V$:

$$\text{No-Slip Case: } Q = \pi \left(\frac{n}{3n+1}\right) \left[\left(\frac{-\Delta p}{2mL}\right)\right]^{1/n} R^{(3n+1)/n} \quad (13)$$

$$\text{Slip Case: } Q = \pi \left(\frac{c\lambda_{mean}}{R} + \frac{n}{3n+1}\right) \left[\left(\frac{-\Delta p}{2mL}\right)\right]^{1/n} R^{(3n+1)/n} \quad (14)$$

n is known as the flow behavior index, which is an empirical curve-fitting parameter. Newtonian fluids are actually a special case derived from the generalized non-Newtonian fluid when the index $n = 1$. For the other non-Newtonian cases: 1) Shear-thickening: $n > 1$; 2) Shear-thinning: $0 < n < 1$. As the value of n decrease, the degrees of shear thinning increase (Chhabra et. al. 2008).

The generalized non-Newtonian fluid equations are used to derive the Newtonian fluid velocity equations by substituting $n = 1$ in section C.2 of the Appendix. The resulting equations are:

$$\text{No-Slip Case: } V = - \left(\frac{R^2}{8m}\right) \left(\frac{\Delta p}{L}\right) \quad (15)$$

$$\text{Slip Case: } V = - \left(\frac{R^2}{8m}\right) \left(\frac{\Delta p}{L}\right) \left[1 + \frac{4c\lambda_{mean}}{R}\right] \quad (16)$$

The mean free path of gas molecules λ_{mean} is not negligible at low pressures compared to the average effective rock pore throat radius. It causes the slippage of gas molecules along pore surfaces (Florence and Rushing 2007). As a result, the measured gas permeability is overestimated compared to

the liquids true absolute permeability. This phenomenon was investigated since 1941 when Klinkenberg approximated linear relationship linking gas permeability and the reciprocal absolute pressure (Klinkenberg 1941). After few decades Sampath and Keighin (1981) proposed a correlation that uses the effective porosity as a third parameter. In this paper, a theoretical derivation for the gas slippage factor using the three parameters (permeability, porosity, and pressure) along with some additional thermodynamic properties and structural properties is shown in section C.3 of the Appendix. The resulting flow equations used in simulation are listed in Table 1 for non-slipping flows and in Table 2 for slipping flows:

Table 1: No-slip derived flow equations through capillary tube shaped fractures.

No-Slip conditions:

Physical quantity	Derived equation
Boundary condition velocity at $r_{max} = R$	$V = 0$ (Chen et. al. 2006)
Non-Newtonian velocity in a single tube	$v = R \left[\left(\frac{-\Delta p}{L} \right) \frac{R}{2m} \right]^{1/n} \left(\frac{n}{n+1} \right) \left[1 - \left(\frac{r}{R} \right)^{(n+1)/n} \right]$
Non-Newtonian integrated average velocity	$V = R \left(\frac{n}{3n+1} \right) \left[\left(\frac{-\Delta p}{L} \right) \frac{R}{2m} \right]^{1/n}$
Volumetric flow rate Q :	$Q = \pi R^2 V = \pi \left(\frac{n}{3n+1} \right) \left[\frac{-\Delta p}{2mL} \right]^{1/n} R^{(3n+1)/n}$
Generalized integrated non-Newtonian Darcy's law average	$V = R \left(\frac{n}{3n+1} \right) \left[-\frac{4k_{NoSlip}}{R\mu} (\nabla p - \rho g \nabla z) \right]^{1/n}$
Generalized integrated Newtonian Darcy's law average	$V_{NoSlip} = - (k_{NoSlip}/\mu) (\nabla p - \rho g \nabla z)$
Permeability	$k_{NoSlip} = (NR^2/8D)$
Radius of capillary tubes given permeability	$r = 2\sqrt{2} \sqrt{Dk_{NoSlip}/\varphi}$

A relationship between porosity and the number of identical capillary tubes within one large porous capillary tube was established in the derivation as: $\varphi = Nr^2/R^2$; N is number of identical capillary tubes (fractures) with an orientation parallel to one of the dimensions axis's, as explained by an example in Figure C.4, caused by certain permeability computed as: $N = 8Dk_{NoSlip}/R^2$; R is the largest capillary tube radius; r is the identical small capillary tubes radius (fractures) within the large capillary tube (matrix). Given permeability and porosity, small capillary tubes radius can be approximated as: $r = 2\sqrt{2} \sqrt{Dk_{NoSlip}/\varphi}$. Given any of these values, one could derive an indication to all of the other values as illustrated by examples in Table 3.

Table 2: Slip derived flow equations through capillary tube shaped fractures.

Slip conditions:

Physical quantity	Derived equation
Boundary condition velocity at $r_{max} = R$	$v_0 = -c \lambda_{mean} \partial v / \partial r$ (Kundt and Warburg 1875)
Non-Newtonian velocity in a single tube	$v = R \left[\left(\frac{-\Delta p}{L} \right) \frac{R}{2m} \right]^{1/n} \left(\frac{c\lambda_{mean}}{R} + \left(\frac{n}{n+1} \right) \right) \left[1 - \left(\frac{r}{R} \right)^{(n+1)/n} \right]$
Non-Newtonian integrated average velocity	$V = R \left(\frac{c\lambda_{mean}}{R} + \frac{n}{3n+1} \right) \left[\left(\frac{-\Delta p}{L} \right) \frac{R}{2m} \right]^{1/n}$

Volumetric flow rate Q :	$Q = \pi R^2 V = \pi \left(\frac{c\lambda_{mean}}{R} + \frac{n}{3n+1} \right) \left[\frac{-\Delta p}{2mL} \right]^{1/n} R^{(3n+1)/n}$
Generalized integrated non-Newtonian Darcy's law average	$V = R \left(\frac{c\lambda_{mean}}{R} + \frac{n}{3n+1} \right) \left[-\frac{4k_{NoSlip}}{R\mu} (\nabla p - \rho g \nabla z) \right]^{1/n}$
Generalized integrated Newtonian Darcy's law average	$V_{slip} = -(k_{slip}/\mu)(\nabla p - \rho g \nabla z)$
Permeability	$k_{slip} = (NR^2/8D)[1 + 4c\lambda_{mean}/R]$ $k_{slip} = (NR^2/8D)[1 + b_k/p]$
Radius of capillary tubes given permeability	$r = 2\sqrt{2} \sqrt{Dk_{slip}/\phi[1 + 4c\lambda_{mean}/R]}$

The mean free path λ_{mean} of gas molecules is proportional to the reciprocal mean pressure, such that the relation can be written as: $4c\lambda_{mean}/r \propto b_k/p_{mean} \rightarrow 4c\lambda_{mean}/r \equiv b_k/p_{mean}$; c is constant less than 1. Similarly, Knudsen diffusion is proportional to the reciprocal mean pressure (Javadpour et. al. 2007, Javadpour 2009). However, the proportionality coefficient b_k known as Klinkenberg gas slippage factor differs significantly between this paper derivations and Knudsen derivations due to the different pore shapes assumed at each case.

The derivation in section C.3 of the Appendix for the *slippage factor* (Klinkenberg effect) was built on the following assumptions:

- 1- Capillarities are oriented randomly through a solid material with the same radius r .
- 2- In D dimensions, flow direction is parallel to one of the dimension axes and there are N capillarities.
- 3- 3D poiseuille equation gives the non-slipping flow rate q_{NoSlip} through capillarities. So, slippage factor uses 1/3 of the total flow at each direction.

The parameters for the velocity equations in Table 1 and Table 2 are derived in section C.4 of the Appendix. Their resulting equations are presented in Table 3:

Table 3: parameters specifications.

Physical quantity	Derived equation
Initial porosity ϕ_0	Taken from Table C.2 in the appendix
Initial fracture permeability	$k_f = b^3/12$
Porosity	$\phi = Nr^2/R^2$
Mean free path of gas molecules	$\lambda_{mean} = 1/\sqrt{2} N(\pi d^2)$ $\lambda_{mean} = R_g T / \sqrt{2} PV(\pi d^2)$
Molecular diameter d	Taken from Table C.1 in the appendix
Gas slippage factor using λ_{mean}	$b_k = 4c\lambda_{mean}p_{mean}/r$ $b_k = 4cR_g T / r\sqrt{2} V(\pi d^2)$ $b_k = R_g T / V(\pi d^2) \sqrt{k/\phi}$ [Constant $c = 1$]
Number of capillary tubes	Given permeability: $N = 8Dk/R^2$ Given porosity: $N = \phi R^2/r^2$

In Table 3, R_g is gas constant; T is temperature; P is pressure; V is volume; d is molecules diameter; b is the fracture half-aperture width.

Slippage effects can be incorporated within permeability computations as illustrated in Table 1 and Table 2. Fractures velocity computations utilize the integrated average velocities over the volume of

interest especially when applied to lower dimensions (D-1). On the other hand, matrices velocities on D dimensions depend significantly on the number of fractured capillary tubes within each grid cell of interest, grain sizes, and tortuosity. A simplified assumption of N capillary tubes within each cell could estimate a matrix permeability to be used. Another more accurate approach is to use Kozeny-Carman equation relating absolute permeability, porosity, and grain size (Allen and Sun 2014, Dvorkin 2009, Steiakakis et. al. 2012).

3 Fractured Systems

Fractures properties in rock fabric can vary from fracture to fracture according to their length, conductivity, position and orientation. These fractures form complex networks of natural and hydraulic fractures that need to be taken into consideration when studying flows through porous mediums. In the HEF model, we homogenize small-scale fractures and represent them within the continuum described by matrix porosity and permeability of dimension D. For large-scale fractures, a discrete (D-1) lower dimensional simulation is applied with lower dimension porosity and permeability. In DPDP model, we detach the flux through fractures from the flux inside the matrices as two overlapping D dimension continua as illustrated in Figure 1. In both models, flow exchange parameter between matrix and fracture systems have the most significant rule.

3.1 Fractures in HEF Fine-Scale

Since the state and existence of fractures differs on a reservoir-by-reservoir or even a well-by-well basis, we use the fine-scale in the HEF model to identify complex large-scale fractures flow features within fractured reservoirs. For example: in a 2D plane, fractures shapes within a domain are defined by the x, y positions of the points specifying the fracture lines shape as illustrated in the left side of Figure 4 taking into account the possible discontinuities. Then, fracture lines segments are formed by finding the intersection points of each fracture line with the x -axis and the y -axis as well as the fracture lines intersection points with each other, as illustrated in the right side of Figure 4. Each constructed segments resides within a grid cell that contains fractures as illustrated in Figure 5.

Figure 4: (Left): defining fractures shapes using the segments x, y positions; (Right): segments enclosed by two subsequent intersection points with x -axis's, y -axis's and with other segments.

It is important to have a certain mechanism to define the sequence of points and segments in the drawn fracture lines as illustrated in Figure 5. The sequence is used to recognize the points interacting

directly with each other in a segment or in neighboring segments when applying the CCFD scheme. Values like pressure, porosity, etc., are computed at the middle of each segment. Most of these values depend on the fracture size and orientation within each grid cell (segment) enclosed by two subsequent axis's intersection points as illustrated in Figure 5. It is also important to normalize the fracture positions before applying the fine-scale flow exchange parameter computations. In other words, we define a uniform cell with size $h \times h$ if the mesh cells sizes are defined as $h_x = h_y = h$; then, we find the positions of the fracture points inside the uniform cell such that each fracture length, shape and orientation are preserved within the cell. Cells are assumed to be symmetric, so the equations derived for one side in section D of the Appendix can be applied to all sides of the cell. However, some implementation details including signs, values representations, etc., needs to be taken care of during implementation.

Fractures form boundaries with the matrices. Having a certain mechanism to distinguish if a fracture segment resides within a list of segments sequence or at the end of the sequence is important in deciding when to apply the boundary condition effects to that segment. In this paper, if the segment is at the end of the sequence we apply Dirichlet boundary conditions affected by the matrix properties of the same exact grid cell. Since as illustrated in Figure 4 example, fracture segments might also intersect with each other. Having a mechanism to find these intersection points is also important to avoid duplication of fracture computations within the same cell since each segment generated resides within a grid cell.

Figure 5: two subsequent axis's intersection points enclose segments and define their sizes and their sequence defines their interactions.

Sometimes the lines intersect with each other at the same points they intersect with the x -axis and the y -axis. In this case, each segment resides in a grid cell causing the pressure at middle of the segment used for the cell computations to be computed only once. On the other hand, if the lines intersect with each other at the middle of a grid cell, there will be two segments in a grid cell causing the existence of two values for the final cell-wise computations. A mechanism is needed to define a unique value for each grid cell either by adding the values, averaging them, or finding another way to deal with them. In this paper, all the intersection points are used in generating the segments. Then, all the values computed from segments existing in one grid cell are averaged over the cell. As a result, at each grid cell, a unique pressure is solved for in each system.

3.2 Flux Exchange Parameter

The *flux exchange parameter* is described following Warren, Root and Kazemi (Chen et. al. 2006, Warren and Root 1963, Kazemi 1968). It relates fluid mobility, differences between fractures and matrices, capillary pressures, gravities, viscous forces, influences of pressure gradients across each matrix block, and molecular diffusion rates for each component. Flux exchange parameter derivation

for this paper is discussed in section B of the Appendix. Since by definition the total flow balance between matrices and fractures is guaranteed:

$$\int Q_{mt} = - \int Q_{ft} \quad (17)$$

As a result, the final exchange parameter for matrix systems become: $Q_{mt} = \alpha(k/\mu)(p_f - p_m)$ (18).

The final exchange parameter for fracture systems become: $Q_{ft} = \alpha(k/\mu)(p_m - p_f)$ (19).

Using $\alpha = \alpha^* |L_f|/h$ (20); h is mesh size if $h_x = h_y$ as illustrated in Equation (5) and Equation (6).

The *dimensionless shape factor* α^* is defined by the fracture actual shape, length, and orientation within each grid cell. Many ways can be used to define α^* : 1) analytic solutions; 2) using Finite Element Method (FEM) or the DFN model to approximate the fracture boundary value problem solution within each grid cell; 3) using thermodynamic properties obtained from molecular simulation.

In order to reduce computational costs while maintaining the physical accuracy, we use the analytic solution approach to solve for the shape factor α^* within each grid cell in this paper. Several analytic solutions can be derived in such systems. The analytic solution we choose for 2D in this paper is built on assuming a linear matrix pressure function $P_m^L(d)$. Pressure values are defined by the vertical distances d_i between fractures and coarse grid cell edges within each cell. The theoretical derivation for the assumptions considering several possible fracture shapes within each grid cell is shown in section D of the Appendix. The resulting shape factors functions according to the formed fracture shapes when intersecting with the grid cell edges are listed in Table 4:

Table 4: α^* equations in 2D according to fracture shapes within each grid cell (derivations are illustrated in section D of the Appendix)

Fracture Shape	Distance d_i	α^*
Triangle shape fractures	$d_i = ((y_d - y_0)(x_d - x_0))/ \gamma \rightarrow$ if both fracture points are on grid cell corners. $d_i = (1/\gamma) * [(y_d - y_0)(x - x_0) - (x_d - x_0)(y - y_0)] \rightarrow$ if one fracture point is on a grid cell corner and the other is on a grid cell edge.	One side: $\alpha^* = h/d_{avg}^i$ Both sides: $\alpha^* = 2h/d_{avg}^i$
Vertical shape fractures	One side: $d_i = \Delta x_i$. Both sides: $d_i = (\Delta x_1 + \Delta x_2)/2$.	One side: $\alpha^* = 2h/ \gamma d_{avg}^i$ Both sides: $\alpha^* = 4h/ \gamma d_{avg}^i$
Horizontal shape fractures	One side: $d_i = \Delta y_i$. Both sides: $d_i = (\Delta y_1 + \Delta y_2)/2$.	One side: $\alpha^* = 2h/ \gamma d_{avg}^i$ Both sides: $\alpha^* = 4h/ \gamma d_{avg}^i$
Vertical Tetragonal shape fractures	$d_i = [(x_1 - x_2) * (y_1 - y_2)]/ \gamma \rightarrow$ If selected distances forms a triangle with both fracture points on the grid cell corners. $d_i = (1/\gamma) * [(y_2 - y_1)(x - x_1) - (x_2 - x_1)(y - y_1)] \rightarrow$ If selected distances forms a triangle with one fracture point on a grid cell corner and the other on a grid cell edge. $d_i = \sqrt{(y - y_1)^2 + (x - x_0)^2} \rightarrow$ General case. Since $b^2 = (x^2 + y^2) - (x_1 + x_0)x - (2y_1)y + (x_1x_0 + y_1^2) = 0$; $d_i = \sqrt{b^2}$ (Not linear).	One side: $\alpha^* = h/d_{avg}^i$ Both sides: $\alpha^* = 2h/d_{avg}^i$

(One side): indicates that distances d_i are computed from one vertical side of the grid cell fracture;

(Both sides): indicates that distances d_i are computed from both vertical sides of the grid cell fracture.

DPDP shape factor parameter α^* can be similarly derived by assuming that fractures reside at the middle of each grid cell in both x and y directions, as was illustrated in section D of the Appendix. Using an approach similar to the approach applied for vertical and horizontal fractures forming rectangle shapes with grid cell edges, the resulting shape factors are listed in Table 5:

Table 5: DPDP framework α^* equations (derivations in section D of the Appendix)

	Fracture length	Fracture sides considered	α^*	Substituting fracture length
Considering both orientations	$ \gamma = h + h = 2h$	One side	$\alpha^* = 8/ \gamma $	$\alpha^* = 4/h$
		Both sides	$\alpha^* = 16/ \gamma $	$\alpha^* = 8/h$
Averaging over the whole cell	$ \gamma = h$	One side	$\alpha^* = 8/ \gamma $	$\alpha^* = 8/h$
		Both sides	$\alpha^* = 16/ \gamma $	$\alpha^* = 16/h$

This analytic approach solution is applicable in 3D cases too by applying a slight change in the assumptions. The analytic solution we choose for 3D in this paper is built on assuming a square matrix pressure function $P_m^S(d_i, z)$ indicating the area A_i formed by cutting the cubic grid cell with a plane perpendicular to one of its faces forming a shape on that face and also perpendicular to the fracture slice. The theoretical derivation for the assumptions considering several possible fracture shapes within each cubic grid cell is shown in section E of the Appendix. The resulting shape factors functions according to the formed fracture shapes when intersecting with the grid cell edges are listed in Table 6:

Table 6: α^* equations in 3D according to fracture slices shapes within each grid cell (derivations are illustrated in section E of the Appendix)

Shapes formed by fracture slicing vertically	Area A_i	α^*
Fracture slice forming vertical triangular shapes	$A_i = (z/ \gamma) * [(y_d - y_0)(x - x_0) - (x_d - x_0)(y - y_0)]$	One side: $\alpha^* = h/d_{avg}^i$ Both sides: $\alpha^* = 2h/d_{avg}^i$
Fracture slice forming vertical rectangular shapes	$A_i = [(y_d - y) + (y - y_0)]\gamma$	Both sides: $\alpha^* = h/d_{avg}^i$
Fracture slice forming vertical tetragonal	$A_i = (z/\gamma) * [(y_2 - y_1)(x - x_1) - (x_2 - x_1)(y - y_1)] \rightarrow$ Simplified case. $A_i = z\sqrt{(y - y_1)^2 + (x - x_0)^2} \rightarrow$ General case. Since $b^2 = (x^2 + y^2) - (x_1 + x_0)x - (2y_1)y + (x_1x_0 + y_1^2) = 0;$ $A_i = z\sqrt{b^2}$ (Not linear).	One side: $\alpha^* = h/d_{avg}^i$ Both sides: $\alpha^* = 2h/d_{avg}^i$

In 3D, it is important to have a certain mechanism to check each fracture depth span within z -direction. In other words, at each layer in z -direction we need to check fractures dead ends, fractures spanning from previous layers, and fractures spanning to next layers to consider within computations. All cells that a certain fracture spans in direction z have identical shapes and properties in xy -plane according to the vertical grid slicing fractures assumptions used within this paper. Figure 6 illustrates an example of 15 cubic grid cells chosen from a bigger 3D system. Each fracture slice forms a vertical rectangle with depth z . At each grid cell, we deal only with the portion of the fracture that resides within that cell for fine-scale computations. Then, the results are plugged-in into the coarse-scale computations and a system is solved for pressure in both scales.

Figure 6: 3D Example illustrating that fractures could span different z depths.

4 Discretization and Numerical Results

As discussed in previous sections, DPDP framework deals with two overlapping similar D dimension continua and HEF framework splits the fracture computations into two scales: 1) fine scale to get the flux exchange parameter; 2) coarse scale solves for matrix pressure in D dimension and fracture pressure in $(D-1)$ dimension. Coarse-scale matrix discretization is similar in both cases. However, fine-scale fracture discretization is slightly different between both cases. To simplify the discretization notations, Let:

- 1) The superscript β refers either to m superscript for matrix or f superscript for fractures. So, $F_\beta(p^k)$ is taken either from Equation (2) for the first or Equation (4) for the second; k indicates that the function is evaluated at the previous time step and computed as a constant.
- 2) Ignore gravity.
- 3) $c_2 = M_g p / ZRT = \rho_g$; M_g molecular weight; p previous step pressure; Z real gas deviation factor; R universal gas constant; T temperature.
- 4) $\alpha_t = \alpha^* |L_f| k / h\mu$; α^* is the dimensionless flux exchange parameter discussed in section 3.2; $|L_f|$ is fracture length within each grid cell; k permeability; μ viscosity.

The derivations in this paper are built on: 1) cutting the cubic grid cells with planes perpendicular to fracture slices and at the same time perpendicular to the xy -face forming different shapes within each cell; 2) each fracture depth is rounded up to the closest grid cell depth in z -direction; 3) the fine-scale treats each cell separately. As a result, the main property affecting the fracture flow in this assumption would be the different shapes formed cutting through the xy -direction. So, in order to observe the additional behavioral information we can get using the HEF scheme in the different shapes formed cutting through the xy -direction, in this paper we focus our testing examples on 2D xy -layers.

Gas adsorbed on bedrock surfaces and fracture walls referred to as matrices in the fractured systems is estimated to account to about 20%~85% of the total shale gas (Guo et. al. 2014). In this paper, we assume gas flows through matrix blocks into corresponding fracture blocks. Also, we assume injection through a well at the southwest corner and production through a well at the northeast corner of the continuum.

DPDP

Injection well is assumed at southwest corner and production well is assumed at northeast corner of the matrix continuum. The matrix continuum flow affects the fracture continuum through the flux exchange parameter causing it to behave similarly. Discretizing Equation (1) and Equation (2) within DPDP framework gives:

$$F_m(p_m) \frac{\partial p_m^{k+1}}{\partial t} + c_2 \nabla \cdot u_m = \alpha (p_f^{k+1} - p_m^{k+1}) + q_{ext} \text{ on } \Omega^m \subset \mathbb{R}^n \quad (21)$$

$$F_f(p_f) \frac{\partial p_f^{k+1}}{\partial t} + c_2 \nabla \cdot u_f = \alpha (p_m^{k+1} - p_f^{k+1}) \text{ on } \Omega^f \subset \mathbb{R}^n \quad (22)$$

Utilizing Table F.1 in Equation (21) and Equation (22) discretization along with the Newtonian Darcy's law for velocity in 2D leads to the numerical results illustrated in the Figures (7-9).

Figure 7: The flux-exchange-parameter in the DPDP framework causes the fracture velocity (right) to behave similar to matrix velocity (left) as observed from streamline figures.

Figure 8: Figures (a) and (b) illustrate the pressure conversion rates in the DPDP matrix and fractures measured in Pa; also Figures (c) and (d) illustrate the porosity conversion rates due to the assumed slight compressibility caused by pressure changes.

Matrix blocks in DPDP serve as a major storage sources and fracture networks act as the major flow channels. Traditional dual porosity models assume pseudo steady state matrix to fracture flux. However, the matrix porosity in shale gas reservoirs raises the matrix transient behavior importance. Flow within matrix blocks affects the flow through corresponding fracture blocks due to the flux exchange parameter. In other words, in our example, injection and production wells both exist in the matrix continuum but their effect extends to the fracture continuum. This effect causes a similar behavior in both continuums as illustrated in Figure 7 (matrix (left) and fracture (right)).

Figure 8 illustrates the changes in matrix and fracture porosity caused by an assumed slight compressibility due to pressure change through domain. Figures (a-d) illustrate that the flux exchange parameter causes a similar behavior in both continuums for each corresponding block. However, the matrix continuum transient behavior is considered more significant than the fracture continuum due to the adsorbed gas and the porosity effect. In fracture continuum, the changes in values start appearing after approximately 10 decimal digits in this example.

Figure 9 illustrates various DPDP plots of cumulative rates resulting from different fracture aperture sizes tests. It can be established that as the fracture aperture magnitude increase, the reservoir reaches its depletion faster and vice versa. As the fracture aperture magnitude decrease, the reservoir takes more time to reach its depletion.

However, DPDP approximations do not effectively capture the flow transient behavior in tight shale matrix blocks. So, we use a multiscale CCFD Hybrid Embedded Fracture (HEF) model to explore the transient behavior within fractures in this paper.

Figure 9: DPDP cumulative rate for different fracture aperture sizes show faster depletion with larger aperture sizes

Multiscale CCFD Hybrid Embedded Fracture Model (HEF)

In our HEF framework, reservoir models are represented by two overlapping continua Ω^m on D dimensions and Ω^f on (D-1) dimensions. In other words, coarse-scale solves for matrix pressure in D dimension and fracture pressure in (D-1) dimension as a system to apply the matrix-fracture coupling. Coarse-scale solution depends enormously on the fine-scale solution for the flow exchange parameter computed within each grid cell as illustrated in section 3 derivations. Discretizing Equation (1) and Equation (2) in the HEF framework gives:

$$F_m(p_m) \frac{\partial p_m^{k+1}}{\partial t} + c_2 \nabla \cdot u_m = \alpha (p_f^{k+1} - p_m^{k+1}) + q_{ext} \text{ on } \Omega^m \subset \mathbb{R}^n \quad (23)$$

$$F_f(p_f) \frac{\partial p_f^{k+1}}{\partial t} + c_2 \nabla \cdot u_f = \alpha (p_m^{k+1} - p_f^{k+1}) \text{ on } \Omega^f \subset \mathbb{R}^{n-1} \quad (24)$$

Utilizing Table F.2 in Equation (23) and Equation (24) discretization along with the Newtonian Darcy's law for velocity in 2D leads to the numerical results illustrated in the Figures (10-14).

Figure 10: In the HEF model, fluids flow from matrix into fractures and the flux behavior is dominated by the magnitude and structure of fractures within fracture networks.

(a)

(b)

(c)

(d)

Figure 11: Fracture flow pressures in HEF measured in Pa during the first few steps in Example (c) of Figure 10. (Right-side) show fractures pressures with respect to matrices without showing matrices pressures in the figure. (Left-side) show matrices pressure with respect to fractures without showing matrices pressures.

Matrix blocks serve as the major storage sources and fracture networks act as the main flow channels here too. However, the effect of the matrix flow on the fractures flow due to the flux exchange parameters is larger in the HEF scheme than the influence of the fracture flow on the matrix flow. This causes the fluids to flux from matrices into fractures and the flux behavior is dominated by the magnitude and structure of fractures as illustrated in Figure 10.

Pressure decay over time in any system is affected by production wells and fractures. Fracture segments existing in any grid cell effect the pressure of the whole cell as illustrated in Figures 11-14. Figure 11 explores pressure variations due to flow in fracture in Example (c) of Figure 10 during the first few steps for fractures with respect to matrices (right) and matrices with respect to fractures (left). In the HEF model, Fractures are assumed to have Dirichlet boundary conditions and are influenced by the matrices around them. Flow from the matrices propagates through the fractures as illustrated in Figure 11 (right). Figure 12 illustrates fractures pressure changes within fractures with respect to the surrounding matrices pressures by combing both. In general, matrix pressures are higher around the fractures as illustrated in Figure 11 (left) and the combination in Figure 12. This causes fluids to flow into fractures as illustrated in Figure 10.

Figure 12: fractures pressure changes within fractures with respect to the surrounding matrices pressures in the HEF model.

Figure 13: Matrix porosity affected by the assumed slight compressibility due to pressure changes.

Figure 13 illustrates the change in HEF matrix porosity caused by the assumed slight compressibility due to pressure changes. The matrix continuum transient behavior is considered more significant than the fracture continuum due to the adsorbed gas and the porosity effect. Figure 14 illustrates different cumulative rate plots resulting from different fracture aperture sizes. As the fracture aperture size increase, the reservoir reaches its depletion faster and vice versa which is a behavior similar to what was encountered in Figure 9 for the DPDP model.

Figure 14: HEF cumulative rate for different fracture aperture sizes.

5 Conclusion

Flux behavior in fractured tight shale systems is dominated by the magnitudes and structures of fractures. In this paper an iterative multiscale (two-scale) Cell Centered Finite Difference (CCFD)

Hybrid Embedded Fracture (HEF) model is presented to solve for the flow within fractured tight shale systems. The HEF model involves splitting the fracture computations into two scales: 1) fine scale solves for the flux exchange parameter within each grid cell; 2) coarse scale solves for the matrix pressure in D dimensions and the fracture pressure in (D-1) dimensions as a system to apply the matrix-fracture coupling. The coarse scale solution is applied to the domain grid cells (cell-wise) using the flux exchange parameter computed at each grid cell from the fine scale. HEF model combines the DPDP overlapping continua concept, the DFN lower dimensional fractures concept, the HFN hierarchical fracture concept, and the CCFD model simplicity.

Linear fracture functions are introduced to grasp fractures properties for the flux exchange parameter in fine scale. At each coarse scale grid cell there exist only one fracture pressure function that the system solves for including grid cells contain intersecting fracture segments since we presented a mechanism to deal with such cases.

The accuracy of the proposed scheme was assessed for tight shale gas systems and the corresponding results are documented in this paper. The results demonstrate that the proposed HEF model expands our understanding of the flow behavior in fractured systems, where such an understanding may eventually be used to estimate reservoir properties.

The characterization of shale gas systems behavior through the derivation of fit-for-purpose models based on a set of selected properties is also explored. An example of a fit-for-purpose model is utilized in a numerical simulation to examine different tight shale gas systems scenarios. Visualizations of flow using velocity quivers, plots of rate, as well as maps of pressure and porosity distributions are used to detect and demonstrate the diverse flow regimes progressively occurring over time.

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7 Appendix

Part 1 of the appendix shows some of the possible options to design a fit-for-purpose shale gas model based on any set of selected properties. Part 2 focuses on the derivation of certain fit-for-purpose model that will be used heavily in this paper. Part 3 show the parameters assuming fractures take an approximately circular capillary tube shape. Part 4 derives the analytic solutions for the dimensionless parameter α^* in 2D while Part 5 derives them in 3D. Part 6 displays the initial parameters used in the testing examples.

A. Part 1: Designing fit-for-purpose models for shale gas

- To consider different flux properties according to a unified Hagen-Poiseuille-type equation with fully compressible shale gas properties forming a non-linear pressure equation, the mass conservation and momentum conservation laws used are (Civan et. al. 2011):

$$\frac{\partial}{\partial t}(\rho\varphi) + \frac{\partial}{\partial t}[(1-\varphi)q] = -\nabla \cdot (\rho u)$$

ρ density; u volumetric flux vector; φ porosity.

- Incorporating continuum, slip, transition, and free molecular flow properties in one K equation using Knudsen number K_n for the permeability (Civan et. al. 2011, Li et. al. 2014):

$$K = K_\infty f(K_n); f(K_n) = \left(1 + \alpha K_n\right) \left(1 + \frac{4K_n}{1 - bK_n}\right); K_n = \frac{\lambda}{R_h}; \lambda = \frac{\mu}{p} \sqrt{\frac{\pi R_g T}{2M_g}}; R_h = 2\sqrt{2\tau_h} \sqrt{\frac{K_\infty}{\varphi}}$$

$f(K_n)$ flow condition function of Knudsen number K_n that expresses the free path of molecules as a mean hydraulic radius of flow paths R_h ration to the mean free path of molecules λ ; K_∞ intrinsic permeability; α dimensionless rarefaction coefficient; b empirical slip coefficient; τ_h tortuosity; φ porosity; μ viscosity; R_g universal gas constant; T temperature; M_g molecular mass

- Incorporating the adsorbed portion of gas that remained within shale due to the dominant pressure and organic adsorbent conditions using Langmuir isotherm estimation (Langmuir 1915, 1932 and 1933, Civan et. al. 2011, Li et. al. 2014):

$$q_a = \frac{V_L p}{p_L + p}; q = \frac{\rho_s M_g}{V_{std}} q_a \rightarrow q = \frac{\rho_s M_g}{V_{std}} \left(\frac{V_L p}{p_L + p} \right)$$

ρ_s material density; q gas mass adsorbed per solid volume; q_a standard volume of gas mass adsorbed per solid volume; V_L Langmuir gas volume (ultimate adsorption amount); V_{std} molar volume of gas at standard temperature (273.15 K) and pressure (101,325 Pa); p_L Langmuir gas pressure (pressure when adsorption reaches half the adsorption amount); M_g molecular weight.

- Incorporating the dynamic slippage factor (Ertekin et. al. 1986) into the transport equation governed by Fick's law (Clarkson et. al. 2012):

$$\frac{\partial}{\partial x} \left(B_c \frac{A_x k_{\infty, x}}{\mu_g B_g k_g} \frac{\partial P}{\partial x} \right) \Delta x + \frac{\partial}{\partial y} \left(B_c \frac{A_y k_{\infty, y}}{\mu_g B_g k_g} \frac{\partial P}{\partial y} \right) \Delta y + q_{sc} + q_m = \frac{V_b \varphi}{\alpha_{c1}} \frac{\partial}{\partial t} \left(\frac{1}{B_g} \right)$$

This model assumes that the shale gas matrix consists of two parts (organic + inorganic). A_x grid block cross sectional area; B_c transmissibility conversion factor; q_{sc} well injection/production rate; q_m gas flow from micro-porosity; α_{c1} unit conversion factor; k_g apparent gas permeability change caused by slippage effect.

- Incorporating slippage effect using the pressure driven Darcy's law flow and the concentration driven Fick's law flow to find b_a (Clarkson et. al. 2012):

$$k_g = \left(1 + \frac{b_a}{p}\right)^{-1}; \frac{b}{p} = \frac{4k_l \lambda}{r_c}; b_a = \frac{p C_g \mu_g D}{\alpha_{c2} k_\infty}$$

C_g gas compressibility; μ_g gas viscosity; D diffusion coefficient; α_{c2} unit conversion factor; λ mean free path; r_c pore radius.

- Incorporating slippage corrected gas permeability and gas adsorption in a multistage DPDP model for horizontally fractured shale gas wells (Li et. al. 2014):

$$\begin{aligned} \nabla \cdot \left[\frac{k_m}{\mu_g B_g} (\nabla p_m - \gamma_g \nabla z) \right] - \alpha k_m \frac{p_m - p_f}{\mu_g B_g} &= \frac{\partial}{\partial t} \left(\frac{\varphi_m}{B_g} \right) + \frac{\partial}{\partial t} (\rho_s V_{ads}) + q_m \\ \nabla \cdot \left[\frac{k_f}{\mu_g B_g} (\nabla p_f - \gamma_g \nabla z) \right] - \alpha k_m \frac{p_m - p_f}{\mu_g B_g} &= \frac{\partial}{\partial t} \left(\frac{\varphi_f}{B_g} \right) + q_f \end{aligned}$$

The model takes the adsorbed gas as an accumulation term and uses the apparent permeability to describe the slip flow. m matrix; f fracture; ρ_s shale density; B_g formation volume factor of gas in fraction; ϕ porosity; V_{ads} adsorption; q is volume flow rates.

- Incorporating Kazemi's (1968) shape factor (Li et. al. 2014), where L_D are natural fracture spacing: $\alpha = 4 (1/L_x^2 + 1/L_y^2 + 1/L_z^2)$.

B. Part 2: Deriving a fit-for-purpose model for this paper (considering adsorption and free gas models)

Adsorbed gas molecules become attached to coal or organic material surfaces in shale. Most of the gas accumulated by adsorption exists in a compressed near liquid state on the surfaces, like a magnet attached to a metal surface. Organic matters like Kerogen are better in adsorbing. Adsorption is a reversible process since it implicates weak attraction forces. It is distinctive from absorption in which one substance becomes stuck inside another, like a sponge soaking up water. The amount of adsorbed gas depends on adsorbent pressure and conditions. Usually shale reservoirs can pile far more gas in the adsorbed state than conventional reservoirs can hold by compression. Since in reality the volume of fractures is small compared to the volume of the reservoir, free gas in fractures accounts only for a small portion of the gas stored in shale. As a result, the pressure volume relationship is better described by describing isotherms (Tongwei et. al. 2012). The releasing process of the adsorbed gas can be either simulated using molecular simulation (Sharma et. al. 2014) or described by a pressure relationships like Langmuir Isotherm (Langmuir 1915, 1932 and 1933). In this part of the appendix, a fit-for-purpose model is derived based on a set of selected properties from section A of the appendix for experimental and analytical objectives of this paper. Since gas in shale formations usually splits into free gas within fractures and adsorbed gas within matrices, an equation is derived for each case here using Langmuir Isotherm for adsorption.

B.1. Matrices gas flow

The conservation of flowing gas mass considering the loss due to adsorption per unit volume of porous medium and per unit time is computed using the unified Hagen-Poiseuille-type equation (Civan et. al. 2011):

$$\frac{\partial}{\partial t}(\rho\phi) + \nabla \cdot (\rho u) = -\frac{\partial}{\partial t}[(1-\phi)q_m] + Q \rightarrow \frac{\partial}{\partial t}[\rho\phi + (1-\phi)q_m] + \nabla \cdot (\rho u) = Q; \quad (B.1)$$

For real gas, the density is given by (B.2). The adsorbed gas is estimated using Langmuir isotherm (Langmuir 1915, 1932 and 1933, Civan et. al. 2011, Li et. al. 2014). Langmuir isotherm assumes single layers of gas attach to shale surfaces.

$$\rho = \frac{M_g p}{ZRT} \quad (B.2); \quad q_m = \frac{\rho_s M_g}{V_{std}} \left(\frac{V_L p}{p_L + p} \right) \quad (B.3)$$

$q_a = V_L p / (p_L + p)$ is standard volume of gas adsorbed per solid mass; q_m is the matrix mass of gas adsorbed per solid volume; ρ_s shale density; M_g gas molecular weight; V_{std} molar volume of gas at standard temperature $273.15K^o$ and pressure $101,325Pa$. Substituting (B.2) and (B.3) in (B.1) and using the thermodynamic properties, we end up with:

$$\left\{ \frac{M_g}{ZRT} \left(\phi + p \frac{\partial \phi}{\partial p} \right) + \frac{\rho_s M_g V_L}{V_{std}} \left(\frac{p}{p_L + p} \frac{\partial \phi}{\partial p} + \frac{(1-\phi)p_L}{(p_L + p)^2} \right) \right\} \frac{\partial p}{\partial t} + \nabla \cdot (\rho u) = Q \quad (B.4)$$

Assuming slightly compressible rocks (Chen et. al. 2006), porosity compressibility is computed by integration and finding the exponent of the value (LeVeque 2007) as illustrated here:

$$c_\varphi = \frac{1}{\varphi} \frac{\partial \varphi}{\partial p} = \frac{\partial \ln \varphi}{\partial p} \rightarrow c_\varphi dp = d \ln \varphi \rightarrow \text{integrate} \rightarrow \int_{p_0}^p c_\varphi dp = \ln \varphi$$

$$\text{exponent} \rightarrow \exp \left(\int_{p_0}^p c_\varphi dp \right) = \varphi \rightarrow \varphi = \varphi_0 \exp \left(\int_{p_0}^p c_\varphi dp \right) \rightarrow \varphi = \varphi_0 \exp[c_\varphi \Delta p]$$

The first order Taylor approximation of φ is:

$$\varphi \approx \varphi_0(1 + c_\varphi \Delta p) \rightarrow \varphi \approx \varphi_0(1 + c_\varphi [p - p_0]) \rightarrow \frac{\partial \varphi}{\partial p} = \varphi_0 c_\varphi \quad (B.5)$$

c_φ is porosity compressibility (function or constant); φ_0 is porosity at pressure p_0 ; φ is porosity at pressure p . Substituting from (B.5) into (B.4), we get:

$$\left\{ \frac{M_g \varphi_0}{ZRT} (1 + c_\varphi [2p - p_0]) + \frac{\rho_s M_g V_L}{V_{std}} \left(\frac{p \varphi_0 c_\varphi}{(p_L + p)} + \frac{p_L (1 - \varphi_0 [1 + c_\varphi (p - p_0)])}{(p_L + p)^2} \right) \right\} \frac{\partial p}{\partial t} + \nabla \cdot (\rho u) = Q$$

Let:

$$F_m(p) = \left\{ \frac{M_g \varphi_0}{ZRT} (1 + c_\varphi [2p - p_0]) + \frac{\rho_s M_g V_L}{V_{std}} \left(\frac{p \varphi_0 c_\varphi}{(p_L + p)} + \frac{p_L (1 - \varphi_0 [1 + c_\varphi (p - p_0)])}{(p_L + p)^2} \right) \right\} \quad (B.6)$$

To avoid the non-linearity, we deal with it as a function of the previous time step pressure and the equation is written as:

$$F_m(p) \frac{\partial p}{\partial t} + \nabla \cdot (\rho u) = Q \quad (B.7); \text{ Let } G = \nabla \cdot (\rho u) = \rho \nabla \cdot u + u \nabla \cdot \rho = \frac{M_g}{ZRT} (p \nabla \cdot u + u \nabla \cdot p) \quad (B.8)$$

This would result a non-linear equation that needs to be linearized first. For simplification sake, in this paper we assume the density to be computed from the previous time step. So, we can deal with it as a constant and (B.8) becomes:

$$G = \nabla \cdot (\rho u) = \rho \nabla \cdot u + u \nabla \cdot \rho = \frac{M_g p}{ZRT} \nabla \cdot u \quad (B.9)$$

This means that (B.7) in adsorption case becomes:

$$F_m(p) \frac{\partial p}{\partial t} + \frac{M_g p}{ZRT} \nabla \cdot u = Q \quad (B.10)$$

B.2. Fractures gas flow

The conservation of mass (Chen 2007) of the flowing gas within fractures is computed by:

$$\frac{\partial}{\partial t} (\rho \varphi) + \nabla \cdot (\rho u) = Q \quad (B.11)$$

Using the real gas density from (B.2), Taylor series porosity from (B.5) and the simplification from (B.9):

$$\left\{ \frac{M_g}{ZRT} \left(\varphi + p \frac{\partial \varphi}{\partial p} \right) \right\} \frac{\partial p}{\partial t} + \nabla \cdot (\rho u) = Q \rightarrow \left\{ \frac{M_g \varphi_0}{ZRT} (1 + c_\varphi [2p - p_0]) \right\} \frac{\partial p}{\partial t} + \frac{M_g p}{ZRT} \nabla \cdot u = Q \quad (B.12)$$

$$F_f(p) = \left\{ \frac{M_g \varphi_0}{ZRT} (1 + c_\varphi [2p - p_0]) \right\} \quad (B.13)$$

$$F_f(p) \frac{\partial p}{\partial t} + \frac{M_g p}{ZRT} \nabla \cdot u = Q \quad (B.14)$$

B.3. Flux Exchange Parameters

The flow exchange parameter including influences of capillary pressures, gravities, viscous forces and the influences of pressure gradients across each matrix block are (Chen 2007, Chen et. al. 2006, Kazemi 1968, Warren and Root 1963):

$$Q_m = \alpha \frac{k_m}{\mu} (\varphi_m - \varphi_f + L_c \Delta p_m) \quad (B.15)$$

L_c characteristic length for matrix-fracture flux; $\varphi_\gamma, \gamma = m \text{ or } f$ is phase potential. $\varphi_\gamma = p_\gamma - \rho_\gamma g z$; pressure gradient effect $\Delta p_\gamma = g |\rho_f - \rho_\gamma|$; g is gravity. Ignoring gravity:

$$Q_{mt} = -\alpha \frac{k_m}{\mu} (p_m - p_f) = \alpha \frac{k}{\mu} (p_f - p_m) \quad (B.16)$$

$$Q_{ft} = -Q_{mt} = \alpha \frac{k}{\mu} (p_m - p_f) \quad (B.17)$$

Let $\alpha = \alpha^* |L_f|/h$; h is the mesh size if $h_x = h_y$. $|L_f|$ is the fracture absolute length within each fracture grid cell.

C. Part 3: Parameters derivations according to assumptions

Assuming fractures take an approximately circular capillary tube shape with length L and radius R coincide with the x -axis as illustrated in Figure C.1. Gas flows through the tube due to pressure drop across the tube span. This indicates that velocity v is a function of radius distance r in the x -axis only.

Figure C.1: Assuming fractures have an approximately circular capillary tube shape with length L and radius R coinciding with the x -axis such that the velocity v is a function of radius distance r in the x -axis.

C.1. Non-Newtonian slip vs. no-slip flow derivation in capillary tube shaped fractures using the power law

Considering a laminar, steady, incompressible flow going through the previously assumed fracture tube of Figure C.1 without any angular velocity. Force balance through x -direction for fluid elements located at distance r is written as: $p(\pi r^2) - (p + \Delta p)\pi r^2 = \gamma_{rx}(2\pi r)L$. This result: $\gamma_{rx} = -(\Delta p/L)(r/2)$ (C.1); where γ_{rx} is shear stress. Equation (C.1) gives a linear shear stress distribution through the tube cross-section depending on r as illustrated in Figure C.2. At the axis, $\gamma_{rx} = 0$.

Figure C.2: The linear shear stress distribution through the tube cross-section from Equation (C.1) that depends on the distance r from the x -axis.

Applying the power law for fluids in tubes, the shear stress is related to shear rate by (Chhabra et. al. 2008): $\gamma_{rx} = m(-\partial v_x / \partial r)^n$ (C.2); n is flow behavior index; m is a consistency coefficient; v_x is velocity in axial directions at radius r . Relating (C.1) and (C.2) then integrating with respect to r to get the velocity distribution:

$$\left(\frac{-\Delta p}{L}\right) \frac{r}{2} = m \left(-\frac{\partial v_x}{\partial r}\right)^n \rightarrow \left[\left(\frac{-\Delta p}{L}\right) \frac{r}{2m}\right]^{1/n} = -\frac{\partial v_x}{\partial r} \rightarrow \int \partial v_x = - \int \left[\left(\frac{-\Delta p}{L}\right) \frac{1}{2m}\right]^{1/n} r^{1/n} \partial r \quad (C.3)$$

$$v_x = -\left(\frac{n}{n+1}\right) \left[\left(\frac{-\Delta p}{L}\right) \frac{1}{2m}\right]^{1/n} r^{(n+1)/n} + \text{constant} \quad (C.4)$$

Equation (C.4) is then used to derive the slip and no-slip flow velocity using the standard velocity profile illustrated in Figure C.3.

No-slip condition:

In Figure C.3, to satisfy the *no-slip* condition, the velocity $v_x = 0$ at $r_{max} = R$. Substituting in Equation (C.4) the constant becomes:

$$\text{constant} = \left(\frac{n}{n+1}\right) \left[\left(\frac{-\Delta p}{L}\right) \frac{1}{2m}\right]^{1/n} r^{(n+1)/n} \quad (C.5)$$

Arranging the substitution of Equation (C.5) into Equation (C.4) and multiplying and dividing by $R^{(n+1)/n}$: $v_x = \left(\frac{n}{n+1}\right) \left[\left(\frac{-\Delta p}{mL}\right) \frac{R}{2}\right]^{1/n} R \left[1 - \left(\frac{r}{R}\right)^{(n+1)/n}\right]$ (C.6)

Figure C.3: The standard velocity profile distribution across the tube cross-section depending on the distance r from the x -axis.

The velocity profile is also expressed in terms of average velocity V as (Chhabra 2008, Chen 2007):

$$V = \frac{Q}{\pi R^2} = \frac{1}{\pi R^2} \int_0^R 2\pi r v_x dr \quad (C.7)$$

Q is liquid volumetric flow rate; substituting v_x from Equation (C.6) into Equation (C.7) and simplifying:

$$V = 2 \left(\frac{n}{n+1}\right) \left[\left(\frac{-\Delta p}{L}\right) \frac{R}{2m}\right]^{1/n} R \int_0^R \left(\frac{r}{R}\right) \left[1 - \left(\frac{r}{R}\right)^{(n+1)/n}\right] d\left(\frac{r}{R}\right) \quad (C.8)$$

$$\int_0^R \left(\frac{r}{R}\right) \left[1 - \left(\frac{r}{R}\right)^{(n+1)/n}\right] d\left(\frac{r}{R}\right) = \left[\frac{1}{2} \left(\frac{r}{R}\right)^2 - \left(\frac{n}{3n+1}\right) \left(\frac{r}{R}\right)^{(3n+1)/n}\right]_0^R = \left[\frac{1}{2} - \left(\frac{n}{3n+1}\right)\right] \quad (C.9)$$

Substituting Equation (C.9) in Equation (C.8) and since $(1 - 2n/(3n+1)) * (n/n+1) = n/(3n+1)$, Equation (C.8) becomes:

$$V = \left(\frac{n}{3n+1}\right) \left[\left(\frac{-\Delta p}{mL}\right) \frac{R}{2}\right]^{1/n} R \quad (C.10)$$

Using Equation (C.6) and Equation (C.10), the average velocity becomes:

$$V_{avg} = \frac{v_x}{V} = \left(\frac{3n+1}{n+1} \right) \left[1 - \left(\frac{r}{R} \right)^{(n+1)/n} \right] \quad (C.11)$$

Since according to velocity profiles illustrated by example in Figure C.3, the maximum velocity is reached at $r = 0$, Using Equation (C.11): $V_{avg}^{max} = (3n+1)/(n+1)$. Equation (C.10) can be further simplified collecting the R's and multiplying by πR^2 to give the volumetric flow rate Q :

$$Q = \pi R^2 V = \pi \left(\frac{n}{3n+1} \right) \left[\frac{-\Delta p}{2mL} \right]^{1/n} R^{(3n+1)/n} \quad (C.12)$$

Slip condition:

On the other hand, to apply the *slip* conditions at $r_{max} = R$, walls velocities were found to be proportional to velocity gradients according to Kundt and Warburg (1875) findings. As a result, we use the equation $v_x = -c\lambda_{mean} (\partial v / \partial r)$ (C.13) for the velocity in Equation (C.4); c is a constant less than 1; λ_{mean} is the mean-pressure mean free path of gas molecules. Substituting Equation (C.13) into Equation (C.4) and finding $v_x(R)$ the constant becomes:

$$constant = \left(\frac{n}{n+1} \right) \left[\left(\frac{-\Delta p}{L} \right) \frac{1}{2m} \right]^{1/n} R^{(n+1)/n} - c\lambda_{mean} \left(\frac{\partial v_x}{\partial r} \right) \quad (C.14)$$

Using Equation (C.3) and substituting into (C.14), the constant becomes:

$$constant = \left(\frac{n}{n+1} \right) \left[\left(\frac{-\Delta p}{L} \right) \frac{1}{2m} \right]^{1/n} R^{(n+1)/n} + c\lambda_{mean} \left[\left(\frac{-\Delta p}{L} \right) \frac{r}{2m} \right]^{1/n} \quad (C.15)$$

Substituting Equation (C.15) into Equation (C.4) and multiplying and dividing by $R^{(n+1)/n}$, the velocity is:

$$v_x = \left[\left(\frac{-\Delta p}{L} \right) \frac{R}{2m} \right]^{1/n} R \left(\frac{c\lambda_{mean}}{R} + \left(\frac{n}{n+1} \right) \left[1 - \left(\frac{r}{R} \right)^{(n+1)/n} \right] \right) \quad (C.16)$$

Replacing the velocity in the average velocity V in (C.7) by (C.16) and simplifying:

$$V = 2 \left[\left(\frac{-\Delta p}{L} \right) \frac{R}{2m} \right]^{1/n} R \int_0^R \left\{ \frac{c\lambda_{mean}}{R} \left(\frac{r}{R} \right) + \left(\frac{n}{n+1} \right) \left[1 - \left(\frac{r}{R} \right)^{(n+1)/n} \right] \right\} d \left(\frac{r}{R} \right) \quad (C.17)$$

Solving the integration part:

$$\begin{aligned} & \int_0^R \left\{ \frac{c\lambda_{mean}}{R} \left(\frac{r}{R} \right) + \left(\frac{n}{n+1} \right) \left[1 - \left(\frac{r}{R} \right)^{(n+1)/n} \right] \right\} d \left(\frac{r}{R} \right) \\ &= \left[\frac{c\lambda_{mean}}{2R} \left(\frac{r}{R} \right)^2 + \left(\frac{n}{n+1} \right) \left(\frac{1}{2} \left(\frac{r}{R} \right)^2 - \left(\frac{n}{3n+1} \right) \left(\frac{r}{R} \right)^{(3n+1)/n} \right) \right]_0^R = \left[\frac{c\lambda_{mean}}{2R} + \frac{1}{2} \left(\frac{n}{3n+1} \right) \right] \quad (C.18) \end{aligned}$$

Substituting Equation (C.18) in Equation (C.17):

$$V = \left(\frac{c\lambda_{mean}}{R} + \frac{n}{3n+1} \right) \left[\left(\frac{-\Delta p}{mL} \right) \frac{R}{2} \right]^{1/n} R \quad (C.19)$$

Using Equation (C.16) and Equation (C.19), the average velocity becomes:

$$V_{avg} = \frac{v_x}{V} = \left\{ \frac{c\lambda_{mean}}{R} + \left(\frac{n}{n+1} \right) \left[1 - \left(\frac{r}{R} \right)^{(n+1)/n} \right] \right\} / \left\{ \frac{c\lambda_{mean}}{R} + \frac{n}{3n+1} \right\} \quad (C.20)$$

Equation (C.19) can be further simplified collecting the R's and multiplying by πR^2 to give the volumetric flow rate Q :

$$Q = \pi R^2 V = \pi \left(\frac{c\lambda_{mean}}{R} + \frac{n}{3n+1} \right) \left[\frac{-\Delta p}{2mL} \right]^{1/n} R^{(3n+1)/n} \quad (C.21)$$

n is known as the flow behavior index, which is an empirical curve-fitting parameter. In Newtonian fluids: $n = 1$; Shear-thickening: $n > 1$; Shear-thinning: $0 < n < 1$.

C.2. Deriving Newtonian flow velocity equation from the non-Newtonian power law generalized equations

The Average velocity V integration over $[0: R]$ was:

$$\text{No-Slip Case: } V = R \left(\frac{n}{3n+1} \right) \left[\left(\frac{-\Delta p}{mL} \right) \frac{R}{2} \right]^{1/n} \quad (\text{C.10});$$

$$\text{Slip Case: } V = R \left(\frac{c\lambda_{mean}}{R} + \frac{n}{3n+1} \right) \left[\left(\frac{-\Delta p}{mL} \right) \frac{R}{2} \right]^{1/n} \quad (\text{C.19})$$

Since in Newtonian fluids: $n = 1$, the equations become:

$$\text{No-Slip Case: } V = \left(\frac{R^2}{8m} \right) \left(\frac{-\Delta p}{L} \right) \quad (\text{C.22})$$

$$\text{Slip Case: } V = \left(\frac{c\lambda_{mean}}{R} + \frac{1}{4} \right) \left(\frac{-\Delta p}{2mL} \right) R^2 \rightarrow V = \left(\frac{-c\lambda_{mean}\Delta p R^2}{2mLR} + \frac{-\Delta p R^2}{8mL} \right) \text{ multiplying and dividing by 4}$$

and simplifying, the equation becomes:

$$V = - \left(\frac{R^2}{8m} \right) \left(\frac{\Delta p}{L} \right) \left(\frac{4c\lambda_{mean}}{R} + \frac{4}{4} \right) \rightarrow V = - \left(\frac{R^2}{8m} \right) \left(\frac{\Delta p}{L} \right) \left(\frac{4c\lambda_{mean}}{R} + 1 \right) \quad (\text{C.23})$$

Comparing Equation (C.67) with Equation (C.65) ignoring (N/D) and Equation (C.22) gives the relation: $(k_{NoSlip}/\mu) = (R^2/8\mu) = (R^2/8m)$ (C.74). Since both Equations (C.10) and (C.19) use $(R/2)$, we can find a value for it using Equation (C.74) as: $R/2 = 4mk_{NoSlip}/R\mu$ (C.75). Substituting Equation (C.75) in Equations (C.10) and (C.19) we get:

$$\text{No-Slip: } V_{NoSlip} = R \left(\frac{n}{3n+1} \right) \left[\left(\frac{-\Delta p}{mL} \right) \frac{R}{2} \right]^{1/n} \quad (\text{C.10}) \rightarrow V_{NoSlip} = R \left(\frac{n}{3n+1} \right) \left[\left(\frac{-\Delta p}{L} \right) \frac{4k_{NoSlip}}{R\mu} \right]^{1/n} \quad (\text{C.76})$$

$$\text{Slip: } V_{slip} = R \left(\frac{c\lambda_{mean}}{R} + \frac{n}{3n+1} \right) \left[\left(\frac{-\Delta p}{mL} \right) \frac{R}{2} \right]^{1/n} \quad (\text{C.19}) \rightarrow V_{slip} = R \left(\frac{c\lambda_{mean}}{R} + \frac{n}{3n+1} \right) \left[\left(\frac{-\Delta p}{L} \right) \frac{4k_{NoSlip}}{R\mu} \right]^{1/n} \quad (\text{C.77})$$

Equations (C.76) and (C.77) can be used to derive a generalized non-Newtonian Darcy's flow equation as:

$$\text{No-Slip: } V_{NoSlip} = R \left(\frac{n}{3n+1} \right) \left[- (4k_{NoSlip}/R\mu) (\nabla p - \rho g \nabla z) \right]^{1/n} \quad (\text{C.78})$$

$$\text{Slip: } V_{slip} = R \left(\frac{c\lambda_{mean}}{R} + \frac{n}{3n+1} \right) \left[- (4k_{NoSlip}/R\mu) (\nabla p - \rho g \nabla z) \right]^{1/n} \quad (\text{C.79})$$

To derive the generalized Newtonian flow equation by substituting $n = 1$ in (C.78) and (C.79):

$$\text{No-Slip: } V_{NoSlip} = - (k_{NoSlip}/\mu) (\nabla p - \rho g \nabla z) \quad (\text{C.80})$$

$$\text{Slip: } V_{slip} = - (k_{NoSlip}/\mu) (\nabla p - \rho g \nabla z) \left(1 + \frac{4c\lambda_{mean}}{R} \right) \rightarrow V_{slip} = - (k_{NoSlip}/\mu) (\nabla p - \rho g \nabla z) \left(1 + \frac{bk}{p} \right) \quad (\text{C.81})$$

Equations (C.78) and (C.79) are the generalized non-Newtonian flow equations and Equations (C.80) and (C.81) are the generalized Newtonian flow equations.

C.3. Newtonian slip vs. no-slip properties derivation in capillary tube shaped fractures using permeability, porosity, and pressure.

Considering a cylinder shell of radii $R + dr$ in a capillary pipe with length L and radius R coinciding with the x -axis as shown in Figure C.1. The force F_1 influencing the shell cross section is the pressure caused by the flowing gas. The pressure is defined as the force influencing a unit area. If the pressure p acts on a small area dA , the force exerted on this area will be: $F_1(p) = p dA = p (2\pi r) dr \rightarrow dF_1 = 2\pi r \Delta p dr$. If we assumed a steady-state flux motion, the force F_1 is neutralized due to viscous drag applied by gas flow through inner and outer surfaces along the x -axis of the cylindrical shell. Viscous drag is described by: $F_\mu = \mu s (dv/dr)$; μ gas viscosity; s surface

considered (Chen 2007). Gas velocity approaches maximum in the cylinder axis when $r = 0$ and decreases radially toward the wall until it reaches zero velocity as was illustrated in Figure C.3 according to the velocity standard profile. Therefore, (dv/dr) is negative. According to this the viscous drag applied on the inner surface become: $dF_{inner} = -2\pi r L \mu (dv/dr)$ (C.24) and on the outer surface: $dF_{outer} = -2\pi (r + dr) L \mu (d/dr (v + (dv/dr)dr))$ (C.25). This means that on the entire shell: $dF_2 = dF_{outer} - dF_{inner}$ and at equilibrium: $dF_1 = dF_2$.

Multiplying $(d/dr (v + (dv/dr)dr))$ from Equation (C.25) by $(r + dr)$ and neglecting the small $dr^2(d^2v/dr^2)$; then simplifying: $r\Delta p = -\mu L (r (\partial^2 v/\partial r^2) + (dv/dr))$; dividing by $-r\mu L$ we get: $(\partial^2 v/\partial r^2) + (1/r) (dv/dr) = -(\Delta p/\mu L)$ (C.26); we find the solution of Equation (C.26) as a particular solution for a non-linear equation like this: let (C.26) become $v'' + 1/r v' = -(\Delta p/\mu L)$ (C.27). Multiplying by $r^2 \rightarrow r^2 v'' + r v' = -(\Delta p/\mu L) r^2$ (C.28); it correspond to the homogenous function: $r^2 v'' + r v' = 0$ (C.29) in the form: $ay'' + by' + cy = f(r)$; a, b, c are constants. $a \neq 0$; $f(r) \neq 0$.

The solution can be expressed as: $y = y_c + \lambda$ (C.30); y_c is the general complementary solution of the corresponding homogenous function; λ is the particular solution that specifies the non-homogenous equation. The characteristic function from Equation (C.29) is: $r^2 y'' + r y' = 0$; it gives the complementary solution: $y_c = c_1 e^{-1/r}$ (C.31). The non-homogenous equation from (C.28) has: $f(r) = -(\Delta p/\mu L) r^2$ (C.29); this gives: $v = -A (\Delta p/\mu L) r^2$; $v' = -2A (\Delta p/\mu L) r$; $v'' = -2A (\Delta p/\mu L)$; Substituting them to the original equation (C.28), gives $A = 1/4$. From (C.29), $f(r) = \lambda = -(\Delta p/4\mu L) r^2$ (C.32). Finally, using (C.31 and C.32), (C.30) becomes: $v = y_c + \lambda = c_1 e^{-1/r} - (\Delta p/4\mu L) r^2$ (C.33); where $A_0 = c_1 e^{-1/r}$ is a constant stabilized by r value and the boundary conditions. Equation (C.33) is then used to derive the no-slip and slip boundary conditions using the standard velocity profile illustrated in Figure C.3.

Slip condition:

In Figure C.3, to satisfy the *slip* condition, the velocity $v = v_0 \neq 0$ at $r_{max} = R$. Using the proportionality relation found by Kundt and Warburg (1875) between wall velocity and velocity gradient with respect to radius r written as: $v_0 = -c \lambda_{mean} \partial v/\partial r$ (C.34); c constant less than one; λ_{mean} mean free path of gas molecules at mean pressure. Substituting in Equation (C.33) gives: $v = A_0 - (\Delta p/4\mu L) r^2$ (C.35). It's derivative is: $\partial v/\partial r = -(\Delta p/2\mu L) r$ (C.36). Substituting both in (C.34) gives: $v_0 = c \lambda_{mean} (\Delta p/2\mu L) R$ (C.36) at the wall $r = R$. From (C.35): $A_0 = v_0 + (\Delta p/4\mu L) R^2$ (C.37) at the wall. Substituting from (C.36): $A_0 = (\Delta p/4\mu L) \{2c\lambda_{mean}R + R^2\}$ (C.38). Substituting in (C.35): $v = (\Delta p/4\mu L) \{2c\lambda_{mean}R + R^2 - r^2\}$ (C.39).

The *slip flow rate* is obtained by integrating over the considered cylinder shell area $= (2\pi r dr)v$ over a cross section of the capillary tube $0 \rightarrow R$ that gives the poiseuille equation:

$$q_{slip} = \pi (\Delta p/2\mu L) \int_0^R [2c\lambda_{mean}Rr + R^2r - r^3] dr \quad (C.40). \quad q_{slip} = (\pi \Delta p/8\mu L) R^4 [1 + 4c\lambda_{mean}/R] \quad (C.41).$$

$v_{slip} = q_{slip}/A_{tube} = (N/D)(R^2/8)(\Delta p/\mu L)[1 + 4c\lambda_{mean}/R]$ (C.42). $k_{NoSlip} = (NR^2/8D)$ (C.43) in no-slip case as will be shown latter, $v_{slip} = q_{slip}/A_{cube} = (k_{slip}/\mu)(\Delta p/L)$ (C.44); N is number of tubes as illustrated in Figure C.4; D is number of dimensions. $v_{slip} = k_{slip} (NR^2/8D)(\Delta p/\mu L)[1 + 4c\lambda_{mean}/R]$ (C.45). Darcy law: $v_{slip} = (k_{slip}/\mu)(\Delta p/L)$ (C.46); $k_{slip} = (NR^2/8D)[1 + 4c\lambda_{mean}/R]$ (C.47); k_{slip} apparent permeability. Assuming 2N in 2D makes Equation (C.47) become: $k_{slip} = (R^2/8)[1 + 4c\lambda_{mean}/R]$ (C.48). Equating (C.43) give: $N = 8Dk_{NoSlip}/R^2$ (C.49). Equation (C.49) can be used to approximately get the number of identical parallel fractures caused by certain permeability. Since the mean free path λ_{mean} of gas molecules is proportional to the reciprocal mean pressure, the relation can be written as: $4c\lambda_{mean}/r \propto$

$b_k/p_{mean} \rightarrow 4c\lambda_{mean}/r \equiv b_k/p_{mean}$ (C.50). b_k is a proportionality coefficient known as Klinkenberg gas slippage factor. As a result of applying the relation (C.50), k_{slip} from Equation (C.48) can be written as: $k_{slip} = (R^2/8)[1 + b_k/p]$ (C.51); $k_{slip} = k_{\infty}$ in our definition, which is Klinkenberg corrected permeability. The total velocity through N capillary tubes is calculated by multiplying (C.41) by (N/D) : $q_{total} = (N/D)(\pi\Delta p/8\mu L) r^4[1 + 4c\lambda_{mean}/R]$ (C.52). The total area velocity using Darcy's law from Equation (C.46) would be: $q_{total} = A_{area} (k_{slip}/\mu)(\Delta p/L)$ (C.53). Equating (C.52) and (C.53), we get: $Nr^4[1 + 4c\lambda_{mean}/R]/8D = R^2 k_{slip}$ (C.54). This leads to: $N = 8DR^2 k_{slip}/(r^4[1 + 4c\lambda_{mean}/R])$ (C.55). $k_{slip} = Nr^4[1 + 4c\lambda_{mean}/R]/(8DR^2)$. At maximum r : $k_{slip} = NR^2[1 + 4c\lambda_{mean}/R]/(8D)$.

Since *porosity* is found by assuming N identical capillary tubes as illustrated by example in Figure C.4. The assumed capillary tubes have an orientation parallel to one of the dimensions axis's (x -axis in this example). By definition, the porosity is:

$$\phi = \frac{\text{void volume}}{\text{bulk volume}} = \frac{N\pi r^2 L}{\pi R^2 L} = \frac{Nr^2}{R^2} \quad (\text{C.56})$$

This leads to: $N = \phi R^2/r^2$ (C.57). Equating (C.55) and (C.57) results:

$$r = 2\sqrt{2} \sqrt{Dk_{slip}/\phi[1 + 4c\lambda_{mean}/R]} \quad (\text{C.58}). \text{ Equation (C.58) can be used to get the capillary tubes radius given } k_{slip} \text{ and } \phi.$$

Figure C.4: a capillary tube example with porosity $N = 3$ identical void capillary tubes.

Equation (C.42) can be rewritten as: $v_{slip} = (R^2/8\mu)(\Delta p/L)[1 + 4c\lambda_{mean}/R]$ (C.59). Comparing Equation (C.59) with Equation (C.23) gives an indication that in Newtonian fluids $m = \mu$.

No-Slip condition:

To satisfy the *no-slip* condition in Figure C.3, the velocity $v = 0$ at $r_{max} = R$. Substituting in Equation (C.33) gives: $A_0 = (\Delta p/4\mu L) R^2$ (C.60) and the velocity becomes: $v = (\Delta p/4\mu L) [R^2 - r^2]$ (C.61). The liquid flow rate is obtained similar to the slip condition by integrating over the considered cylinder shell area $= (2\pi r dr)v$ over the cross section of the capillary tube $0 \rightarrow R$ that gives the poiseuille equation: $q_{NoSlip} = 2\pi (\Delta p/4\mu L) \int_0^R [rR^2 - r^3] dr$ (C.62). Integrating: $q_{NoSlip} = (\pi\Delta p/8\mu L) R^4$ (C.63). The no-slip flow rate going through all capillary tubes of each dimension: $q_{NoSlip} = (N/D)(\pi\Delta p/8\mu L) R^4$ (C.64). N is number of tubes; D is number of dimensions. $v_{NoSlip} = q_{NoSlip}/A_{tube} = (N/D)(\Delta p/8\mu L) R^2$ (C.65). On the other hand, Darcy law for a cubic rock sample area applied on surfaces $A_{cube} = L^2$ gives: $q_{NoSlip} = L^2 (k_{NoSlip}/\mu)(\Delta p/L)$ (C.66). $v_{NoSlip} = q_{NoSlip}/A_{cube} = (k_{NoSlip}/\mu)(\Delta p/L)$ (C.67). Assuming 100% saturation $\rightarrow$ single-phase. Equating (C.65) and (C.67) gives: $k_{NoSlip} = (NR^2/8D)$ (C.68). Equation (C.68) proves Equation (C.43) used as a simplified approximation. The total velocity through N capillary cubes is calculated by multiplying (C.63) by N : $q_{total} = (N\pi r^4 \Delta p/8\mu L)$ (C.69). The total area velocity using Darcy's law from Equation (C.66) would be: $q_{total} = A_{rea} (k_{NoSlip}/\mu)(\Delta p/L)$ (C.70). The cross section area of cylinder is: $A_{rea} = \pi R^2$ (C.71). Equating (C.69) and (C.70), we get: $8k_{NoSlip} = N r^4/R^2$ (C.72).

$\rightarrow N = 8R^2 k_{NoSlip}/r^4$ (C.72). Equating (C.72) and (C.57) results: $r = 2\sqrt{2} \sqrt{k_{NoSlip}/\varphi}$ (C.73). It can be noticed here that Equation (C.73) is identical to the no-slip case Equation (C.58).

C.4. Velocity parameters derivations

The mean free path of gas molecules λ_{mean} is dealt with in many ways in literature either as rigid hard spheres, or as soft shape changing molecules, or using ideal gas derivations, etc. 1) Dealing with soft shape-changing molecules depends on using Lennard-Jones parameters as diameter and compute the results using probabilistic ensembles (Hirschfelder et. al. 1954). 2) Assuming hard sphere molecules model based on gas viscosity depending on temperature and pressure as well as molecular diameters (Nave 2016). 3) Smooth rigid elastic spherical molecules not surrounded by force fields could also be assumed. 4) The distance molecules travel while colliding with other molecules could also be averaged (Tong 2012). 5) The mean free path of gas molecules can also be computed simply as a sum of all different paths traversed by M molecules divided by the number of molecules, that is M . To simplify the argument in this paper, we consider λ_{mean} to be the average distance traversed by a molecule between collisions (Tong 2012). Assuming gas molecules to be spheres of diameter d and speed v . The radius is $d/2$ and the particle appears as a disc with area $\pi (d/2)^2$ with an effective particle cross-sectional area πd^2 . A single molecule collides with any molecule that comes within a distance d between the centers. In time Δt , molecules will travel a distance $(v\Delta t)$ sweeping a volume of $(\pi d^2)(v\Delta t)$. Assuming N molecules per unit volume, the collisions will become $N (\pi d^2)(v\Delta t)$ in Δt time. This means that the time between two sequential collisions on average is: $\tau = 1/N(\pi d^2)v$. The average distance between two sequential collisions is called the mean free path: $\lambda_{mean} = \tau v = 1/N(\pi d^2)$, where N/v is particle density. This derivation is built on assuming that all the molecules are stationary except for one that moves. Assuming that identical particles velocities have a Maxwell distribution and carrying out the computations we get the mean free path: $\lambda_{mean} = 1/\sqrt{2} N(\pi d^2)$. To get the number of molecules, molecular simulation can be used fixing some other system parameters like in PVT ensembles, μ VT ensembles, etc. To simplify the argument in this paper, we use the ideal gas law to get the number of molecules, $N = PV/R_g T$. This gives the mean free path: $\lambda_{mean} = R_g T/\sqrt{2} PV(\pi d^2)$; R_g gas constant; T temperature; P pressure; V volume; d molecules diameter.

Molecules diameter d can be computed using Lennard-Jones parameter too. However, in this paper, since we need the average molecule diameters according to the species we are simulating, we use the average values provided from literature as a result of many experiments from Table C.1.

Table C.1: Effective size and effective length of molecules (Mao and Sinnott 2001).

Molecule Name	Effective diameter d ($\text{\AA}$)	Effective length f ($\text{\AA}$)	f/d
CH_4 Methane	3.988	3.988	1
C_2H_6 Ethane	3.988	4.755	1.2
$n - C_4H_{10}$ (n-butane)	4.150	8.240	1.99
$n - C_4H_{10}$ (iso-butane)	6.352	6.352	1

Two parameters are used in Table C.1 to analyze the size of molecules: d (the effective diameter of the molecule) and f (the effective length of molecules). The difference between the two is illustrated in Figure C.5. Notes: 1) As either d or f increases, the size of the molecules increase, 2) As the ratio f/d increases, the linearity of the molecules increases.

Initial porosity ϕ_0 can be specified using experimental porosity measurements. For example, Table C.2 shows some descriptions and measurements for different rock types found by Edward (1963).

Initial fracture permeability is computed as a cubic function of the fracture half-aperture width b : $k_f = b^3/12$. For example, if a fracture aperture width is 0.1 mm (Freeman and Moridis 2009): $0.0001 \text{ m} \rightarrow k_f = (0.0001/2)^3/12 = 1.042 \times 10^{-14} \text{ m}^2$.

Figure C.5: illustrating the difference between the effective diameter and effective length in Table C.1

Table C.2: experimental porosity measurements found by Edward (1963).

Rock type	Description	Porosity%
Sandstone	Sand-sized quartz and/or feldspar composing classic sedimentary rocks	5.0 - 25.0
Shale	Chips Mix of clay minerals and tiny fragments of other minerals, like quartz and calcite composing Fine-grained sedimentary rocks.	6.0 - 30.0
Limestone	Calcite and aragonite compose sedimentary rocks	5.0 - 20.0
Dolomite	Calcium magnesium carbonate $CaMg(CO_3)_2$ compose Anhydrous carbonate mineral.	1.0 - 5.0
Quartzite	Hard, non-foliated metamorphic rock. Originally pure quartz sandstone but converted to quartz due to heat and pressure.	1.0 – 25.0

D. Part 4: 2D Analytic solutions for the dimensionless parameter α^*

Assuming a linear matrix pressure function $P_m^L(d)$, where pressure values are defined as the vertical distances between the fracture and the coarse grid cell edges d within each coarse grid cell; the matrix and fracture pressure at the fracture is zero and greater than zero elsewhere; matrix pressure gradient is 1 which is d in this assumption; d is vertical on the fracture γ . The previous assumptions can be written as:

$P_m^L(d): E \rightarrow \mathbb{R}$ such that:

$$P_m^L(0) = 0$$

$$\nabla P_m^L \perp \text{Fracture } \gamma$$

$$|\nabla P_m^L| = 1$$

$$P_m^L \geq 0$$

$$P_f = 0$$

The assumption could constructs different structural shape fractures at each grid cell:

Figure D.1: distances d_i vertical on the fracture γ and forms a triangle at the left-bottom corner of the cell

A) If the fracture forms a triangle intersecting with the grid cell edges as illustrated in Figure D.1. Each vertical d_i on γ forms a right triangle. Focusing on one of these triangles in Figure D.2, we get the triangle ABD includes d between AE and the triangle ABC includes γ between BC. Assuming ABC has a height $h = d_i$, its area is $A = d_i\gamma/2$ (D.1). Triangle area can also be computed as: $A = xy/2$ (D.2). Combining the Equations (D.1) and (D.2), we get: $d_i\gamma/2 = xy/2 \rightarrow d_i = xy/\gamma$ (D.3). Applying it to the triangle ABC with h , it becomes: $d_i = ((y_d - y_0)(x_d - x_0))/|\gamma|$ (D.4). Since d_i could indicate a vertical line on both sides of the fracture, the distance considering both directions becomes: $d_i = (1/\gamma) * [(y_d - y_0)(x - x_0) - (x_d - x_0)(y - y_0)]$ (D.5)

This gives an indication that in a triangle forming fracture as illustrated in Figure D.1 each distance d_i is computed using two triangles, one includes the fracture and the other includes the distance d_i to be computed. For example, in Figure D.2, the two triangles are ABC and ABD for the fracture BC. ABC is used in specifying $(y_d - y_0)(x - x_0)$ and ABD is used in specifying $(x_d - x_0)(y - y_0)$. Assuming:

$P_m = d_i \rightarrow P_m = (1/\gamma) * [(y_d - y_0)(x - x_0) - (x_d - x_0)(y - y_0)]$ (D.6). This means that the derivative becomes:

$\nabla P_m = \partial P_m / \partial x + \partial P_m / \partial y = \{(y_d - y_0) - (x_d - x_0)\} / \gamma = 1$ (D.7) proving the assumption $|\nabla P_m^L| = 1$. This means that we can write ∇P_m as:

$$\nabla P_m = \{(y_d - y_0) - (x_d - x_0)\} = |\gamma| \text{ (D.8).}$$

Figure D.2: distance d_i computation details in right triangle forming fracture case

This indicates that at the boundaries, the velocity can be computed in two ways:

- 1) Using Darcy velocity: $\int u_m \cdot n = \int |(-\lambda_m \nabla P_m)| \cdot n = \lambda_m$; $\lambda_m = k_m / \mu$ (D.9)
- 2) Using ∇P_m derivation result: $\int u_m \cdot n = \int |(-\lambda_m |\gamma|)| \cdot n = |\gamma| \lambda_m$ (D.10)

Assuming the source/sink term between the matrix and the fracture is computed by:

$$Q = (\alpha^* |\gamma| \lambda_m (P_{avg}^m - P_{avg}^f)) / h \text{ (D.11). } P_f = 0 \text{ in fractures: } Q = (\alpha^* |\gamma| \lambda_m (P_{avg}^m)) / h \text{ (D.12).}$$

Since fractures define matrix boundaries, Equation (D.10) can be used: $(\alpha^* |\gamma| \lambda_m (P_{avg}^m)) / h = |\gamma| \lambda_m$ (D.13). Equation (D.13) gives: $\alpha^* = \{|\gamma| \lambda_m h\} / \{|\gamma| \lambda_m (P_{avg}^m)\} = h / P_{avg}^m$ (D.14). Since the derivation assumed that $P_{avg}^m = d_{avg}^i$, it can be concluded that $\alpha^* = h / d_{avg}^i$ (D.15) considering a single side distance between a fracture and the coarse grid cell edges. To consider the two side distances of a fracture to the coarse grid cell edges: $\alpha^* = 2h / d_{avg}^i$ (D.16). Similar techniques can be used to deal with all other cases of right triangle forming fractures (Figure D.3) considering fracture corner, chosen point within fracture, distance between the two points intersecting with the grid cell, sign of distance used.

Figure D.3: Examples of possible fractures shapes forming triangles with grid cell edges.

B) If the fracture forms a rectangle intersecting with the grid cell edges vertically as illustrated in Figure D.4. Each vertical d on γ can be computed by: $Area = d_i \gamma$ (D.17). Substituting the area, we get: $d_i = (x_d - x)$ for one side. Considering the distance from both sides: $Area = d_1 \gamma + d_2 \gamma \rightarrow ((x_d - x) + (x - x_0)) = Area / \gamma$ (D.18). Dividing area by γ gives the distance d_i : $d_i = ((x_d - x) + (x - x_0))$.

The average is: $d_i = ((x_d - x) + (x - x_0)) / 2$ (D.19).

Similarly in case the fracture forms a rectangle intersecting with the grid cell edges horizontally:

From one side of the fracture: $d_i = (y_d - y)$. From both sides of the fracture: $d_i = ((y_d - y) + (y - y_0)) / 2$ (D.20).

The average distance considering both horizontal and vertical fractures is: From one side: $d_{avg}^i = \{(x_d - x) + (y_d - y)\} / 2$ (D.21). For both sides:

$$d_{avg}^i = \{((x_d - x) + (x - x_0)) + ((y_d - y) + (y - y_0))\} / 4 \text{ (D.22).}$$

C) If the fracture forms a tetragonal intersecting with the grid cell edges vertically as illustrated in Figure D.5. Some of the distances d_i can be computed using the two points defining the fracture as was illustrated in the fractures forming triangles intersecting with grid cell edges:

$$d_i = [| (x_1 - x_2) | * | (y_1 - y_2) |] / | \gamma | \quad (D.23).$$

$$d_i = (1/\gamma) * [(y_2 - y_1)(x - x_1) - (x_2 - x_1)(y - y_1)] \quad (D.24)$$

This value is simple to compute but it is not necessarily close to the average d_i . Other d_i values are computed using the two points defining the fracture intersecting with the grid cell, the right triangles forming 90-degree angles located on the fracture $|\gamma|$ as illustrated in Figure D.6 and the right triangle altitude theorem. For example, if we take the right triangle ABC illustrated in Figure D.7, we have the points: (x_0, y_1) , (x_1, y_1) fixed and (x, y) is any point on the fracture. Using the right triangle altitude theorem and the distance formula:

$$c = \sqrt{(x_1 - x_0)^2 + (y_1 - y_1)^2} \rightarrow c^2 = (x_1 - x_0)^2 \quad (D.25)$$

$$\text{Similarly: } a^2 = (y_1 - y)^2 + (x_1 - x)^2 \quad (D.26)$$

$$b^2 = (y - y_1)^2 + (x - x_0)^2 \quad (D.27); \quad b^2 = c^2 - a^2 \quad (D.28)$$

Edge b in Figure D.7 is d_1 the length we want to compute in Figure D.6. Equating Equations (D.27) and (D.28), we end up with the equation:

$$b^2 = (x^2 + y^2) - (x_1 + x_0)x - (2y_1)y + (x_1x_0 + y_1^2) = 0 \quad (D.29).$$

Figure D.4: distances d_i from fracture forming rectangle.

Figure D.5: Simplified distances d_i from fracture forming tetragonal.

Figure D.6: Other distances d_i from fracture forming tetragonal

Figure D.7: right triangle formed from Figure D.6

This equation allows us to get the distance $d_i = b$ for any selected point (x, y) on the fracture. However, it doesn't apply the linear function assumption that we enforced before. In this paper, we used the simplified function illustrated in Figure D.5. Similar cases with different orientations can be treated similarly.

It is possible to compute distances d_i mean that indicates P_m in the *Cell fractures that form rectangles intersecting with grid cell edges vertically or horizontally* as discussed in (B). It is also possible to find α^* from it considering distances d_i from both sides of the fracture by assuming that:

$$P_m = d_i \rightarrow P_m = \{((x_d - x) + (x - x_0)) + ((y_d - y) + (y - y_0))\}/4 \quad (D.30).$$

Finding P_m derivative gives: $|\nabla P_m| = |\partial P_m / \partial x| + |\partial P_m / \partial y| = |\{(1 + 1) + (1 + 1)\}/4| = 1 \quad (D.31).$

Equation (D.31) proves the assumption $|\nabla P_m^L| = 1$. From (D.31), we can write: $\nabla P_m = |\{(1 + 1) + (1 + 1)\}| = 4 \quad (D.32)$. This indicates that at the boundaries, the velocity can be computed in two ways:

- 1) Darcy's velocity: $\int u_m \cdot n = \int |(-\lambda_m \nabla P_m)| \cdot n = \lambda_m$; $\lambda_m = k_m / \mu \quad (D.33)$ [Since $\nabla P_m = 1$ in (D.31)]
- 2) Using ∇P_m from (D.32): $\int u_m \cdot n = \int |(-\lambda_m 4)| \cdot n = 4 \lambda_m \quad (D.34)$

The derived source/sink term between matrix and fractures in Equation (5) and Equation (6) is:

$$Q = (\alpha^* |\gamma| \lambda_m (P_{avg}^m - P_{avg}^f)) / h \quad (D.35). \quad P_f \text{ is assumed to be zero in fractures leading to the equation:}$$

$$Q = (\alpha^* |\gamma| \lambda_m (P_{avg}^m)) / h \quad (D.36). \quad \text{Fractures define some matrix boundaries, so using Equation (D.34):}$$

$$(\alpha^* |\gamma| \lambda_m (P_{avg}^m)) / h = 4 \lambda_m \quad (D.37). \quad \text{As a result: } \alpha^* = 4h / |\gamma| P_{avg}^m \rightarrow \alpha^* = 4h / |\gamma| d_{avg}^i \quad (D.38).$$

Equation (D.38) gives α^* considering both sides of a fracture horizontally or vertically intersecting with the grid cell edges. However, α^* considering one side of a fracture horizontally or vertically intersecting with the grid cell edges can be computed in a similar but more simplified way:

$$P_m = d_i \rightarrow P_m = \{\Delta x_i + \Delta y_i\} / 2 \rightarrow \nabla P_m = \partial P_m / \partial x + \partial P_m / \partial y = |(1 + 1) / 2| = 1 \quad (D.39).$$

Equation (D.39) also Proves $|\nabla P_m^L| = 1$. From Equation (D.39), we can write: $\nabla P_m = |(1 + 1)| = 2 \quad (D.40)$. This indicates that at the boundaries, the velocity can be computed in two ways:

- 1) Using Darcy's velocity: $\int u_m \cdot n = \int |(-\lambda_m \nabla P_m)| \cdot n = \lambda_m$; $\lambda_m = k_m / \mu \quad (D.41).$
- 2) Using ∇P_m derivation from (D.40): $\int u_m \cdot n = \int |(-\lambda_m 2)| \cdot n = 2 \lambda_m \quad (D.42).$

So, using (D.42) with Equation (D.36), the source/sink term in the matrix-fractures systems becomes:

$$(\alpha^* |\gamma| \lambda_m (P_{avg}^m)) / h = 2 \lambda_m \quad (D.43). \quad \text{As a result: } \alpha^* = 2h / |\gamma| P_{avg}^m \rightarrow \alpha^* = 2h / |\gamma| d_{avg}^i \quad (D.44).$$

Equation (D.44) gives α^* considering one side of a fracture that is horizontally or vertically intersecting with the grid cell edges.

D) Applying a similar concept for DPDP model α^* by assuming that fractures reside at the middle of each grid cell in both x and y directions. Then, using an approach similar to the approach applied for

vertical and horizontal fractures to estimate α^* within the DPDP framework. An illustration example is shown in Figure D.8. Since the derived flow exchange parameter in matrix-fracture systems from Equation (5) and Equation (6) was: $Q_{mt} = \alpha(k/\mu)(p_f - p_m)$ and $Q_{ft} = \alpha(k/\mu)(p_m - p_f)$ (D.45).

The assumed parameter α was: $\alpha = \alpha^* |L_f|/h$ (D.46).

The derived α^* considering the distance from one side of a fracture was: $\alpha^* = 2h/|\gamma|d_{avg}^i$ (D.47).

Considering the distance from the two sides of the fracture: $\alpha^* = 4h/|\gamma|d_{avg}^i$ (D.48). Adding the effect of both horizontal and vertical fractures going through each grid cell as illustrated in Figure (D.8):

From one side:

$$\alpha^* = (2h/|\gamma|d_{avg}^{horizontal} + 2h/|\gamma|d_{avg}^{vertical}) \quad (D.49).$$

From both sides:

$$\alpha^* = (4h/|\gamma|d_{avg}^{horizontal} + 4h/|\gamma|d_{avg}^{vertical}) \quad (D.50).$$

Figure D.8: illustrating the flow exchange on a DPDP grid Example

Assuming similar mesh sizes in x and y directions = h according to DPDP assumptions. This gives an indication that: $d_{avg}^{horizontal} = d_{avg}^{vertical} = avg(h/2 + h/2) = h/2$ (D.51). The total fracture size within each DPDP grid cell considering both orientations is: $|\gamma| = |h + h| = 2h$ (D.52). Averaging over the whole cell: $|\gamma| = h$ (D.53). Substituting Equation (D.51) in Equation (D.49) for one side: $\alpha^* = (4h/|\gamma|h + 4h/|\gamma|h)$ (D.54). Substituting Equation (D.51) in Equation (D.50) for both sides: $\alpha^* = (8h/|\gamma|h + 8h/|\gamma|h)$ (D.55). Equation (D.54) leads to $\alpha^* = 8/|\gamma|$ (D.56) and Equation (D.55) leads to $\alpha^* = 16/|\gamma|$ (D.57). Considering both orientations from Equation (D.52): for one side: $\alpha^* = 4/h$ (D.58) and for both sides: $\alpha^* = 8/h$ (D.59). Averaging over the whole cell from Equation (D.53): for one side: $\alpha^* = 8/h$ (D.60) and for both sides: $\alpha^* = 16/h$ (D.61).

As a result, we can conclude that distance and α equations change according to the fracture shape within each grid cell as illustrated in this section derivations details.

E. Part 5: 3D Analytic solutions for the dimensionless parameter α^*

A slight change in the assumptions makes the analytic approach solution of α^* applicable to 3D cases. In 3D, we assume a square matrix pressure function $P_m^S(d, Z)$ indicating the area A_i formed by cutting the cubic grid cell with a plane perpendicular to one of its faces forming a shape on that face and also perpendicular to the fracture slice. For example in Figure E.1, cutting the illustration cubic grid cells with a plane perpendicular to the xy -face forming a triangular shape in the first pair of figures and a vertical rectangular shape in the second pair and perpendicular to the fracture slice (fracture $\gamma \times z$) forms area A_i . Each fracture depth in this assumption is rounded up to the closest grid cell depth Cz .

Figure E.1: Area A_i is formed by a cut of the cubic grid cell that is perpendicular to one of its faces (here the xy -plane face) and it is also perpendicular to the fracture slice $\gamma \times z$. First pair explains triangular shapes while second pair explains vertically rectangular shapes intersecting with the cubic grid cell edges.

The matrix and fracture pressure at the fracture is zero and greater than zero elsewhere. Matrix pressure gradient is 1, which is A_i in this assumption; A_i is a slice vertical to fracture γ . The previous assumptions in 3D are written as:

$$\begin{aligned} P_m^s(d, z): E &\rightarrow \mathbb{R}^2 \text{ such that:} \\ P_m^s(0, 0: Z) &= 0 \\ \nabla P_m^s &\perp \text{Fracture slice } (\gamma \times (0: Z)) \\ |\nabla P_m^s| &= 1 \\ P_m^s &\geq 0 \\ P_f &= 0 \end{aligned}$$

The assumptions in 3D construct a wider variety of structural shape fractures at each grid cell compared to 2D. The 3D derivations additionally assume: 1) rounding fractures depths to the closest grid cell depth multiple Cz ; 2) fracture shapes are defined on the xy -plane; 3) fracture shapes are identical at each layer on the z -direction.

Figure E.2: Area $A_i = zd_i$ in the fractures slicing the grid cell vertically forming tetragonal too.

C is a natural number indicating how many cells in depth Z the fracture spans. Two examples are illustrated in Figure E.1 and one in Figure E.2:

A) If the fracture slices the grid cell vertically forming triangular shapes as illustrated in Figure E.1 first pair of figures. Since the distances d_i is vertical on γ forming right triangles similar to what was illustrated in section D of the appendix, we use Equation (D.4) and Equation (D.5) for d_i :

$$\text{Assuming } P_m^s = A_i = zd_i \rightarrow P_m^s = (z/|\gamma|) * [(y_d - y_0)(x - x_0) - (x_d - x_0)(y - y_0)] \quad (\text{E.1}).$$

The derivative becomes:

$$\nabla P_m^s = \partial P_m^s / \partial x + \partial P_m^s / \partial y + \partial P_m^s / \partial z = (z/|\gamma|) [((y_d - y_0) - (x_d - x_0))] = 1 \quad (\text{E.2}) \text{ since } z \text{ is constant and using the assumption } |\nabla P_m^s| = 1. \text{ This means that we can write } \nabla P_m^s \text{ as: } \nabla P_m^s =$$

$$z [((y_d - y_0) - (x_d - x_0))] = |\gamma| \quad (\text{E.3}) \quad [z [((y_d - y_0) - (x_d - x_0))] \rightarrow \text{is constant. This indicates that at the boundaries, the velocity can be computed in two ways:}$$

1) Darcy's velocity: $\int u_m \cdot (n \times m) = \int |(-\lambda_m \nabla P_m^s)| \cdot (n \times m) = \lambda_m$; $\lambda_m = k_m/\mu$ (E.4) [Used (E.2)].

2) ∇P_m^s derivation result: $\int u_m \cdot (n \times m) = \int |(-\lambda_m |\gamma|)| \cdot (n \times m) = |\gamma| \lambda_m$ (E.5) [Used (E.3)].

Since the source/sink term between matrix-fracture systems act on surfaces here, Equation (D.11) becomes:

$$Q = (\alpha^* |\gamma| \lambda_m (P_{avg}^m - P_{avg}^f))/h^2 \text{ (E.6). } P_f = 0 \text{ in fractures: } Q = (\alpha^* |\gamma| \lambda_m (P_{avg}^m))/h^2 \text{ (E.7).}$$

Since fractures define matrix boundaries, Equation (E.5) can be used: $(\alpha^* |\gamma| \lambda_m (P_{avg}^m))/h^2 = |\gamma| \lambda_m$ (E.8). Equation (E.8) gives: $\alpha^* = \{|\gamma| \lambda_m h^2\}/\{|\gamma| \lambda_m (P_{avg}^m)\}$. Since the derivation assumed that $[P_{avg}^m = z d_i]$ and $[z = h]$ in the cubic grid cell assumption: $\alpha^* = h^2/P_{avg}^m$ (E.9), it can be concluded that $\alpha^* = h/d_{avg}^i$ (E.10) considering a single side distance between a fracture and the coarse grid cell face edges. To consider the two side distances of a fracture to the coarse grid cell face edges: $\alpha^* = 2h/d_{avg}^i$ (E.11). Similar techniques can be used to deal with all other cases of different orientation right triangles while considering fracture corners, chosen points within fracture, area between the two points intersecting with the grid cell, signs of distances used, etc.

B) If the fracture slices the grid cell vertically forming rectangular shapes as illustrated in Figure E.1 second pair of figures. The cube volume is found by: $Volume = d_i \gamma z$ (E.12). Substituting d_i from one side: $d_i = \Delta y_i \rightarrow A_i = (y_d - y) \gamma$ (E.13). Considering the distance from both sides, Equation (E.12) become: $Volume = [(y_d - y) + (y - y_0)] \gamma z$ (E.14). From Equation (E.14), we can say that: $[(y_d - y) + (y - y_0)] \gamma = Volume/z = Area \rightarrow A_i = [(y_d - y) + (y - y_0)] \gamma$ (E.16). The average becomes: $A_i = [(y_d - y) + (y - y_0)] \gamma/2$ (E.17).

Similarly in case *the fracture forms a rectangle intersecting with the grid cell edges horizontally*:

From one side of the fracture: $A_i = (x_d - x) \gamma$ (E.18).

From both sides of the fracture: $A_i = [(x_d - x) + (x - x_0)] \gamma/2$ (E.19).

The average area considering both horizontal and vertical fractures is:

From one side: $A_{avg}^i = \{(x_d - x) + (y_d - y)\} \gamma/2$ (E.20).

For both sides: $A_{avg}^i = \{[(x_d - x) + (x - x_0)] + [(y_d - y) + (y - y_0)]\} \gamma/4$ (E.21).

Considering A_i from both sides of the fracture from Equation (E.21):

$$P_m^s = A_i \rightarrow P_m^s = \{[(x_d - x) + (x - x_0)] + [(y_d - y) + (y - y_0)]\} \gamma/4 \text{ (E.22).}$$

Its derivative is: $|\nabla P_m^s| = |\partial P_m^s/\partial x| + |\partial P_m^s/\partial y| = (2 + 2)|\gamma|/4 = |\gamma|$ (E.23) and $|\nabla P_m^s| = 1$ (E.24) is assumed. This indicates that at the boundaries, the velocity can be computed in two ways:

1) Darcy's velocity: $\int u_m \cdot (n \times m) = \int |(-\lambda_m \nabla P_m^s)| \cdot (n \times m) = \lambda_m$; $\lambda_m = k_m/\mu$ (E.25) [Used (E.24)].

2) Using ∇P_m^s derivation result: $\int u_m \cdot (n \times m) = \int |(-\lambda_m |\gamma|)| \cdot (n \times m) = |\gamma| \lambda_m$ (E.26) [Used (E.23)].

Using the source/sink surfaces derivations version of Equation (D.11) introduced in Equation (E.6):

$$Q = (\alpha^* |\gamma| \lambda_m (P_{avg}^m - P_{avg}^f))/h^2 \text{ (E.6). } P_f = 0 \text{ in fractures: } Q = (\alpha^* |\gamma| \lambda_m (P_{avg}^m))/h^2 \text{ (E.27).}$$

Since fractures define matrix boundaries, Equation (E.26) can be used: $(\alpha^* |\gamma| \lambda_m (P_{avg}^m))/h^2 = \lambda_m |\gamma|$ (E.28) $\rightarrow \alpha^* = \{\lambda_m h^2 |\gamma|\}/\{|\gamma| (P_{avg}^m) \lambda_m\} = h^2/P_{avg}^m$ (E.29). Since the derivation assumed that $[P_{avg}^m = d_{avg}^i z]$ and $[z = h]$, Equation (E.29) becomes $\alpha^* = h/d_{avg}^i$ (E.30) considering A_i from both sides of the fracture. Similar approach depending on the area $A_i = z d_i$ can be applied for fractures slicing the grid cell vertically forming tetragonal as illustrated in Figure E.2 or any other possible shape.

F. Part 6: Testing examples initial parameters

DPDP framework parameters:

Table F.1: DPDP scheme initial parameters

Variable	Value	Unit
Domain size ($L_x \times L_y$)	22×22	$m \times m$
Fracture aperture	$0.002 \mid 0.003 \mid 0.001$ $\rightarrow 0.002 \text{ m} = 0.2 \text{ cm}$	m
Fracture initial permeability	$K_{fi} =$ $(\text{Fracture aperture}/2)^3/12$	m^2
Matrix initial permeability	$K_{mi} = K_{fi} * 1E - 11$	m^2
Fracture initial porosity	$\phi_{fi} = 0.001$	Dimensionless
Matrix initial porosity	$\phi_{mi} = 0.05$	Dimensionless
Initial Pressure	$Pressure_{init} = 7.863275E6$	Pa
Bottom whole pressure	$P_w = 2.447E6$	Pa
Shale rock density	$\rho_s = 2550$	kg/m^3
Shale gas density	$\rho_g = 0.656$	kg/m^3
Molecule effective diameter	$d_{eff} = 3.988E - 10$ $M_g = 0.01604$	m kg/mole
Gas constant	$R = 8.314$	$(Pa * m^3)/(mol * K)$
Temperature	$T = 430.372$	K
Initial viscosity	$\mu = 0.0000184$	Pa.s
Fracture initial porosity compressibility	$6.5267E - 8$	Pa^{-1}
Matrix initial porosity compressibility	$3.6259E - 8$	Pa^{-1}
Fracture and Matrix gas compressibility factors	$Z_f = 1$ $Z_m = 1$	
Langmuir pressure	$P_L = 2.07 * 10E6$	Pa
Langmuir volume	$V_L = 2.83E - 03$	m^3/kg
Standard gas volume	$V_{std} = 0.0224$	m^3/mol
Injection rate	$-(2E - 6 * L_x * L_y)/(24 * 3600)$	m^2/s

HEF framework parameters:

Table F.2: Figure 10 HEF scheme initial parameters

Variable	Value	Unit
Domain size ($L_x \times L_y$)	$22 \times 22^{(a,b)} \mid 88 \times 88^{(c,d)}$	m $\times$ m
Fracture aperture	0.0017 0.0019 0.002 0.0024	m
Fracture initial permeability	$K_{fi} = (Fracture\ aperture)^3 / 12$	m ²
Matrix initial permeability	$K_{mi} = K_{fi} * 1E - 11$	m ²
Fracture initial porosity	$\varphi_{fi} = 0.001$	Dimensionless
Matrix initial porosity	$\varphi_{mi} = 0.05$	Dimensionless
Initial Pressure	$Pressure_{init} = 7.863275E6$	Pa
Bottom whole pressure	$P_w = 6.447E6$	Pa
Shale rock density	$\rho_s = 2550$	kg/m ³
Shale gas density	$\rho_g = 0.656$	kg/m ³
Molecule effective diameter	$d_{eff} = 3.988E - 10$	m
	$Mg = 0.01604$	kg/mole
Gas constant	$R = 8.314$	(Pa * m ³)/(mol * K)
Temperature	$T = 430.372$	K
Initial viscosity	$\mu = 0.0000184$	Pa.s
Fracture initial porosity compressibility	$6.5267E - 8$	Pa ⁻¹
Matrix initial porosity compressibility	$3.6259E - 8$	Pa ⁻¹
Fracture and matrix gas compressibility factor	$Z_f = 1$ $Z_m = 1$	
Langmuir pressure	$P_L = 2.07 * 10E6$	Pa
Langmuir volume	$V_L = 2.83E - 03$	m ³ /kg
Standard gas volume	$V_{std} = 0.0224$	m ³ /mol
Injection rate	$-(2E - 6 * L_x * L_y) / (24 * 3600)$	m ² /s

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Highlights

- An iterative Hybrid Embedded multiscale (two-scale) Fracture model (HEF) is proposed.
- HEF splits computations into two scales: fine scale solves analytically for flux exchange parameter; and coarse scale solves for matrix and fracture pressures coupled as a system.
- HEF allows control over almost all of the fractures properties details.
- HEF model combines the DPDP overlapping continua concept, the DFN lower dimensional fractures concept, the HFN hierarchical fracture concept and the CCFD simplicity.
- A fit-for-purpose shale gas model is utilized to examine various fractures scenarios.