Case History

Wavefront picking for 3D tomography and full-waveform inversion

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ABSTRACT

We have developed an efficient approach for picking first-break wavefronts on coarsely sampled time slices of 3D shot gathers. Our objective was to compute a smooth initial velocity model for multiscale full-waveform inversion (FWI). Using interactive software, first-break wavefronts were geometrically modeled on time slices with a minimal number of picks. We picked sparse time slices, performed traveltimes tomography, and then compared the predicted traveltimes with the data in-between the picked slices. The picking interval was refined with iterations until the errors in travel-time predictions fell within the limits necessary to avoid cycle skipping in early arrivals FWI. This approach was applied to a 3D ocean-bottom-station data set. Our results indicate that wavefront picking has 28% fewer data slices to pick compared with picking traveltimes in shot gathers. In addition, by using sparse time samples for picking, data storage is reduced by 88%, and therefore allows for a faster visualization and quality control of the picks. Our final travel-time tomogram is sufficient as a starting model for early arrival FWI.

INTRODUCTION

Subsurface velocity inversion methods such as full-waveform inversion (FWI) (Tarantola, 1984; Virieux and Operto, 2009) require a good initial velocity model to (1) avoid falling into a local minimum and (2) minimize the number of computationally intensive iterations. Traveltime inversion is the traditional method for generating accurate starting models (Bording et al., 1987; Dessa et al., 2004; Woodward et al., 2008; Prieux et al., 2013), whereas first-

break picking is the first step toward shallow tomographic inversion (Zhu et al., 1992; Zelt and Barton, 1998).

As the seismic industry moves toward denser source and receiver coverage, high-density data are becoming a heavy burden for visualization and picking of early arrivals for tomography, especially in the presence of irregularities in the acquisition geometry and complex geology. Many automatic picking algorithms and approaches are proposed (Peraldi and Clement, 1972; Gelchinsky and Shtivelman, 1983; Coppens, 1985; McCormack et al., 1993; Boschetti et al., 1996; Sabbione and Velis, 2010; Blias, 2012), with varying degrees of success. Some of the picking algorithms require some manual picks as a training set. Therefore, some human picking is still needed to provide reliable picks. In addition, the number of erroneous picks increases with a lower signal-to-noise ratio, the presence of spurious events before the first arrivals, and the parameters needed by the autopicker. Several authors try to analyze the different picks for a given trace to obtain a more reliable pick (Saragiotis et al., 2013; Yalcinoglu, 2014). We empirically find that it is invalid to assume that a set of automatic picks are representative of picking uncertainty, as several autopicks might be clustered around the wrong traveltimes giving a false measure of confidence. In the end, someone has to validate the picks before inverting them to ensure consistency of the picks and the validity of the final tomogram. Consequently, the data are subjected to increasing rates of picking errors and hence, demand greater efforts in quality control (QC).

We now propose a new iterative approach that minimizes the effort of manually picking traveltimes in dense 3D data. In this approach, the coordinates of the first-arrival wavefront are picked on a time slice of a shot gather, in which the time-slice interval for picking can be very coarse in the first iteration. Traveltime tomography is then applied to the picked wavefronts to compute a velocity tomogram. Using the tomogram, new early arrival traveltimes (wavefronts) are predicted in between the picked time slices and compared with the observed data. When there are significant errors in the prediction, the time-slice interval is reduced in the next pick-

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ing iteration. This procedure is repeated for the refined picks until the prediction error is less than half the period of the lowest frequency band that will be used in FWI.

A wavefront is parameterized by few picks, in which the wavefront position between the picks is interpolated by a piecewise smooth function to form a wavefront contour. Using contours minimizes the number of picks needed to represent curved wavefronts. This picking approach is an alternative to the traditional first-break traveltimes picking done on each trace of a common shot gather (CSG). The main advantage of our approach is that it determines the coordinates of the traveltime contours without being overwhelmed by the sheer number of traces within the survey. This is feasible for ocean-bottom-station (OBS)-like surveys with a relatively small number of gathers and dense wide-aperture data coverage. However, the approach can be extended to surveys with a large number of gathers if picking is automated on time slices, where the picks are used as seeds for automated picking. The wavefront contours are used as input to traveltime tomography to provide an initial model for FWI.

We apply the approach to OBS data, and the results illustrate its effectiveness. The acquisition geometry of the survey is shown in Figure 1, where there are 234 OBS stations, spaced approximately 400 m apart in a 26×9 grid with varying water depths with an average of 45 m. There are 124 sail lines with 50 m spacing. Each sail line covers 18 km of distance with a shot spacing of 50 m. The data contain low frequencies as low as 2 Hz with a minimum reliable frequency of 4 Hz. To avoid working with a small number of traces per shot gather, we use common receiver gathers (CRGs) for traveltime picking, traveltime tomography, and FWI.

The paper is organized such that concepts and procedures are demonstrated using the field data example of the OBS survey. First, we describe the picking methodology and highlight the merits and the demerits of the proposed approach. Later, we show the results from tomography based on the picks, and finally, show the FWI tomogram that uses the starting velocity model from traveltime tomography. We finally conclude with some remarks and possible future directions.

METHODOLOGY

The key idea is to pick first-arrival traveltimes from time slices of recorded CRGs instead of the traditional approach of picking traveltimes from each trace of the gathers as depicted in Figure 2. For

Figure 1. The OBS survey geometry. The red dots represent the location of OBS stations and the green lines represent air-gun sail lines.

wavefront picking on time slices, the survey geometry does not have to be regular, but the wave sampling has to be dense enough to pick events on the time slice. In such dense surveys, conventional picking is difficult to ensure areal consistency of the picks. In addition, generating common-offset or common-azimuth gathers for irregular 3D data sets requires regularization of the data (Fomel, 2000). On the other hand, areal displays of wavefronts are more convenient for picking the first-arrival wavefronts, as shown in Figure 3.

First-arrival wavefronts are almost circular in the early time slices and quickly deform into irregular shapes with complex patterns in the later time slices. Such wavefront shapes can be optimally parameterized by piecewise smooth functions.

The number of picks that can accurately represent a wavefront on a time slice is based on real-time interpolation during the picking process. An interactive application, using a graphical user interface, is developed that uses a pointer position to compute the wavefront before making the pick. Therefore, the interpreter (i.e., the user who picks the wavefronts) can optimize the number of picks by moving the pointer as far as possible from the last pick as long as the interpolated picks follow the tracked wavefront. Then, we discuss pick-interpolation algorithms that can run in real time for faster plotting and picking.

Pick interpolation

There are several options for interpolating the curve between picks (or the last pick and the pointer position)

$$\mathbf{c}(u) = (c_x(u), c_y(u)), \quad (1)$$

where the scalar $u \in [0, 1]$ is the interpolation variable and the points $\mathbf{c}(0)$ and $\mathbf{c}(1)$ are at two consecutive picks. The simplest interpolation is a linear interpolation between consecutive picks at the Cartesian positions $\mathbf{x}_i = (x_i, y_i)$ and $\mathbf{x}_{i+1} = (x_{i+1}, y_{i+1})$, where

$$c_x(u) = (x_{i+1} - x_i)u + x_i, \quad (2)$$

Figure 2. A schematic plot of the traditional picking approach, in which first arrivals are picked on every trace in sparse seismic inlines. The red mark represents the source position, the green triangles represent receivers, and the yellow dots are first-arrival picks.

$$c_y(u) = (y_{i+1} - y_i)u + y_i. \quad (3)$$

Curved wavefronts, however, require a large number of picks to be characterized using linear segments.

Splines represent an attractive alternative for interpolating curved functions between picks, which in turn reduce the number of picks needed to track curved wavefronts. Quadratic and cubic Bézier spline curves (Farin, 1997; Gallier, 2000), in particular, are flexible and inexpensive to compute, but they require additional picks to characterize the curvature between picks. Appendix A discusses further the details on optimized picking using Bézier splines.

For most time slices, a radial line extending from the source toward any azimuth $\theta \in [0, 2\pi]$, crosses the first-arrival wavefront only once. For such cases, we can use a simple interpolation scheme in polar coordinates as

$$r(u) = r_i + (r_{i+1} - r_i)u, \quad (4)$$

$$\theta(u) = \theta_i + (\theta_{i+1} - \theta_i)u, \quad (5)$$

where the coordinates can be transformed into Cartesian coordinates using

$$c_x(u) = x_{\text{src}} + r(u) \times \sin(\theta(u)), \quad (6)$$

$$c_y(u) = y_{\text{src}} + r(u) \times \cos(\theta(u)), \quad (7)$$

where x_{src} and y_{src} are the source Cartesian coordinates, (r_i, θ_i) are the polar coordinates of the i th pick, r is the radial distance from the source to the wavefront contour, and θ is the azimuthal angle. Figure 4 illustrates this interpolation procedure. Even though this interpolation is not for general

cases, it is optimal for early times where wavefronts are nearly circular, which can be represented using few picks.

Figure 5 depicts a time slice from the OBS data. To avoid aliasing, we high-cut filter the data, such that the minimum apparent wavelength is at least twice as large as the receiver interval (the wavelength is estimated for a horizontally propagating wave in the water layer). Plotting the time slices gives an impression of continuous sampling of the wavefield. Picks could be made interactively on such plots to track the wavefronts.

At the early time slice in Figure 5, the wavefront tends to be nearly circular, and therefore, we try to pick it using the interpolation

Figure 4. Interpolation between picks in polar coordinates to track curved wavefronts.

Figure 5. An early time slice with its wavefront picks (yellow dots) and the interpolated wavefront polygon (red) using, from left to right, linear interpolation, Bézier quadratic splines, and polar interpolation (i.e., linear interpolation in the polar coordinates). Gray-scale images represent the recorded waveforms at the active time slice of the CRG associated with the OBS station at the star. Zones without data coverage are indicated by the dark blue color.

Figure 3. A schematic plot of the proposed picking method, in which first arrivals are picked on time slices. Colored contours represent early arrival wavefronts, in which the colors indicate the traveltimes associated with the picked wavefront.

algorithm discussed above. With linear interpolation, a large number of picks is required to accurately represent the curved wavefront. Spline interpolation requires fewer picks, whereas polar interpolation requires the minimum number of picks to track the circular wavefront. Later, time slices require more picks as in Figure 6. Different interpolation methods can be used for different wavefront shapes to minimize the picking time. For the OBS data, we mainly pick with polar interpolation between picks.

A strategy for selecting time-slice intervals

The time-slice interval for picking is selected according to the goal of early arrival tomography. As mentioned above, our aim is to generate a starting model for multiscale FWI. The bandwidth for the first FWI run governs the time-slice interval used for picking. To avoid cycle skipping, data calculated from the initial model should be within half a period of the observed data. To satisfy this condition, the time-slice interval should be within half the period of the dominant frequency used in the first iteration of FWI. For the OBS data, FWI is applied to 2–4 Hz data, and therefore, the time-slice interval should be less than 125 ms.

Choosing the picking interval based on the cycle-skipping criteria is rather pessimistic. We can minimize the amount of picking significantly by using the redundant information in the first arrivals. For the OBS data, we start picking with a 800 ms time-slice interval (about eight times the required interval), and then we invert the traveltimes from the picked wavefront by traveltime tomography to get a velocity tomogram. The resulting tomogram is used to predict the waveforms and traveltimes for the time-slice interval of 400 ms. If the difference between the observed and the predicted traveltimes meets the criterion for avoiding cycle skipping, then we stop picking. Otherwise, the picks are refined at the 400 ms interval. This procedure is repeated at a 200 ms picking time-slice interval, in which the prediction errors satisfy the criterion for avoiding cycle skipping.

Quality control

The consistency of traveltime picks is crucial for computing a high-quality tomogram. Therefore, we develop QC tools to monitor the consistency of the picks made throughout the survey. Because wavefronts expand away from the source point with time, the picked wavefront contours from different time slices must not cross. Therefore, for every picked gather, picked wavefronts from different time slices are displayed as in Figure 7 to ensure that they do not cross one another.

Figure 6. A late-time slice with its picks.

Picks from different gathers must also be consistent. Figure 8 shows the time slices at 2.4 s for different gathers. If the wavefronts of all the gathers are picked, we can generate an apparent velocity

Figure 7. Picks within a single gather are checked and adjusted, such that the picked time contours must not cross each other. The red mark indicates the OBS position and the colored contour lines indicate wavefront picks, where the color indicates the time at the picked time slice.

Figure 8. Estimating apparent velocity from three different shot gathers for a given time slice and azimuth. The arrows indicate the radial distance from the source to the wavefront for a given azimuth.

plot as a function of OBS position $\mathbf{x}_s$, where the apparent velocity is defined as

$$v_a(\mathbf{x}_s, \theta, t) = \frac{r(\mathbf{x}_s, \theta, t)}{t}, \quad (8)$$

where $r(\mathbf{x}_s, \theta, t)$ is the radial distance from the OBS station to the wavefront at time t as indicated by the arrows in Figure 8 (if multiple first-arrival wavefronts are encountered for a given azimuth, the maximum apparent velocity is plotted. This scenario, however, was not encountered for the OBS data set). Figure 9 depicts the apparent velocity as a function of shot position for a given time and azimuth. A color-coded block is plotted on top of each source position, in which the color coding represents the apparent velocity. If a pick in a given gather does not follow the areal trend, it is reviewed and adjusted to follow the regional trend. This QC exercise is repeated for every picked time slice for several azimuths before moving to the next step.

Figure 9. Panels for QC'ing azimuth-dependent apparent velocity for the 2.4 s time slice. The arrow indicates the azimuth for the given apparent velocity. This plot is used to detect anomalous picks that do not follow the trend of surrounding gathers, as the one indicated as mispick. Mispicks are corrected to follow the regional trend.

Table 1. A workload comparison between the conventional picking, the optimized conventional picking (i.e., picking a subset of the sail lines), and the proposed picking approach.

Picking approach	Slices to pick	Relative cost
Conventional	124 sail line/gath.	100%
Opt. conv.	42 sail line/gath.	34%
Proposed	30 time slice/gath.	24%

Table 2. A comparison between the data sizes for the proposed picking approach, conventional picking, and optimized conventional picking (i.e., picking a subset of the sail lines).

	Gather size	Full size (234 gath.)	Ratio
Conventional	$360 \times 124 \text{ traces} \times (750 \text{ samples/trace}) \times 4 \text{ bytes} = 127 \text{ MB}$	30 GB	100%
Opt. conv.	$360 \times 42 \text{ traces} \times (750 \text{ samples/trace}) \times 4 \text{ bytes} = 43 \text{ MB}$	10 GB	33%
Proposed	$360 \times 124 \text{ traces} \times (30 \text{ samples/trace}) \times 4 \text{ bytes} = 5 \text{ MB}$	1.2 GB	4%

Relative cost to conventional picking

For the time-slice interval of 100 ms, the memory requirement for storing the data is significantly reduced because the intermediate time slices do not need to be visualized during the picking process. The time-sampling interval is 4 ms for the OBS data, and the associated size of the data containing the first arrivals (3 s) is more than 30 GB. For the conventional approach, this size requires a high-end workstation to hold the volume in memory and an efficient algorithm to minimize delays and cache misses related to frequent jumping between slices and gathers. On the other hand, with time-slice picking, the sparse slices require approximately 1.2 GB of memory for all the gathers. This significant reduction in memory cost allows picking on computers with small memory. In addition, the visualization of the small data set is fast with minimal plotting delays. Now, we compare the cost of the proposed approach with conventional picking.

For the conventional picking approach, we display traces in sail-line slices containing approximately 360 traces for picking. In the first iteration of picking, we sequentially pick sail-line traveltimes. Picking on a sail line takes approximately the same time as picking on a time slice. We can use the proposed picking philosophy (i.e., pick as much as you need) in conventional picking. Instead of picking every sail line, we pick enough sail lines to avoid cycle skipping. Assuming that the slowest wave travels at the water velocity, the minimum wavelength for a 4 Hz wave is $1500 \text{ m/s} / 4 \text{ Hz} = 375 \text{ m}$, and therefore the spacing between picked sail lines should be at least 187 m. If we are optimistic, we can assume that picking with a sail line spacing of 150 m is sufficient. Therefore, we can pick every third line of the OBS data. In that case, only 42 of the 124 sail lines are needed for picking. We can compare the relative cost based on slice count as shown in Table 1. At a 100 ms time-slice interval, the proposed wavefront picking is still more efficient than the optimized conventional picking with 28% fewer slices to pick. Note that a sail-line slice involves $750 \text{ (samples/trace)} \times 360 \text{ (trace/sail line)} = 270,000$ samples to plot, which is larger than the $124 \text{ sail lines} \times 360 \text{ (samples/sail line)} = 44,640$ samples needed to plot a time slice. This 84% reduction of the number of samples per slice results in faster plotting of the time slices, which is crucial for QC'ing the picks quickly before tomography.

When we QC the picks using the conventional approach, we run queries on the data sets to form arbitrary lines and/or display picks on CSGs and crosslines. Generating such slices needs to be fast to QC as many of the picks as possible in a short time. Performing such queries could take a long time because the queries need data to be accessed from disk. Windowing the data for the new approach reduces the data size significantly, such that most of the data can be

stored in random access and cache memory for faster queries and visualization. See Table 2 for data sizes.

We empirically find that QC'ing wavefront picks takes far less time than QC'ing the conventional picks. This is mainly because all the previous wavefront picks within a gather are displayed while making new picks, which leaves less room for making a picking error. In addition, the smaller data size allows for more rapid plotting, and therefore, less QC time compared with the conventional method.

Traveltime inversion using pseudoreceivers

The picking and QC procedure described above will result in a set of piecewise-continuous contours of the wavefronts at different times. For ray tomography, the continuous contours are sampled uniformly in the azimuthal direction, and each sample is used as a pseudoreceiver with the first-arrival traveltime equal to that of the time slice used in picking the wavefront contour. This sampling produces pseudoreceivers, each with a first-arrival traveltime. For the OBS data, the traveltime contours are resampled uniformly with 1° angular spacing as shown in Figure 10.

We now discuss the inversion of these associated traveltimes by traveltime tomography.

TOMOGRAPHY

The early arrival traveltimes can be modeled using the eikonal equation (Aki and Richards, 2002; Cerveny, 2005)

Figure 10. Traveltime picks on pseudoreceivers sampled from the continuous wavefront contours.

Figure 11. The final 3D tomogram after 50 iterations.

$$|\nabla \tau(\mathbf{x})|^2 = \frac{1}{v^2(\mathbf{x})}, \quad (9)$$

with the boundary condition

$$\tau(\mathbf{x}_s) = 0, \quad (10)$$

where $v(\mathbf{x})$ is the velocity model and $\tau(\mathbf{x})$ is the traveltime from the source position $\mathbf{x}_s$ to the $\mathbf{x}$ position. We solve the eikonal equation using the finite-difference method proposed by Jeong and Whitaker (2008), which gives traveltime tables throughout the 3D model. From the tables, we obtain the traveltime values at the pseudoreceivers and compare them with those obtained from our traveltime picks. The traveltimes are inverted for a 3D model, with a 50 m grid spacing in the x -, y -, and z -directions.

To compute the tomogram, we find the slowness model $\mathbf{s}$ that minimizes the objective function

$$f(\mathbf{s}) = \frac{1}{2} \sum_{i=1}^{N_i} (w_i \delta \tau_i(\mathbf{s}))^2, \quad (11)$$

where $\delta \tau_i$ is the difference between the observed (from wavefront picking) and the calculated traveltimes, w_i is a positive weighting factor, and N_i is the number of data samples used in the inversion. Traveltime picks at the far offsets are subjected to higher uncertainties due to the very weak amplitudes of the first arrivals. In addition, far-offset traveltime picks often dominate the residual due to their magnitude, compared with the near offsets. To balance the residuals, we use a weighting factor

Figure 12. Depth slices from the final 3D tomogram.

$$w_i = \frac{1}{\tau_{\text{obs}}^p}, \quad (12)$$

where τ_{obs} represents the observed traveltimes and p is a positive power parameter chosen empirically; the higher the value of p , the more biased the inversion will be toward the near-offset picks. We empirically choose $p = 0.5$, which gives balanced updates to the shallow and the deep portions of the model. The objective function is minimized with Gauss-Newton iterations (Heath, 1996) by solving the preconditioned linearized system of equations for the slowness update δs :

$$\mathbf{WJPy} = \mathbf{W}\delta t, \quad (13)$$

$$\delta s = \mathbf{Py}, \quad (14)$$

Figure 13. Cross sections through the tomogram.

Figure 14. Traveltime residuals after 50 iterations of tomography. The traveltime errors are predominantly within a half a period (± 0.125 s) of the maximum frequency (4 Hz) that is used in the first iteration of FWI.

where δt is a vector containing the traveltime residuals, $\mathbf{W}$ is a diagonal matrix with $\mathbf{W}_{ii} = w_i$, $\mathbf{J}$ is the Jacobian matrix for the traveltime misfit function, $\mathbf{P}$ is a preconditioning operator (a Gaussian

Figure 15. Traveltime residuals for a given CRG. Due to strong regularization, tomographic residuals cannot fit short-wavelength features below the sea bottom.

Figure 16. Vertical slices from the (a) traveltime tomogram and (b) early arrivals FWI tomogram using the traveltime tomogram as a starting model. The fringe zones of the survey are excluded from the plots.

smoothing operator), δs is the slowness update, and y is a temporary vector.

The adjoint of the Jacobian matrix $J^\dagger$ smears the traveltime residuals along the raypaths computed using the traveltime tables (Dessa et al., 2004). This back-projection approach is preferred to the back projection using the adjoint-state method (Sei and Symes, 1994; Leung and Qian, 2006; Taillandier et al., 2009), due to the small number of rays, which is faster to trace.

The role of the preconditioner P is to ensure that the model update is very smooth. At early iterations, a Gaussian smoothing filter is chosen to span the whole model, and the width of the Gaussian filter is gradually reduced to 800 m, which is approximately twice the distance between the OBS stations to avoid aliasing. The height of the Gaussian filter is 15% of its width.

At every iteration of tomography, the slowness update is obtained using the conjugate-gradient method (three iterations), and the slowness model is updated using $s_{k+1} = s_k + \alpha \delta s$. The step length

α is obtained using the golden-section method (Chapra and Canale, 2001).

Tomography results

Figure 11 shows the final tomogram calculated from the travel-times picked from the time slices. Figures 12 and 13 depict the depth slices and cross sections of the final tomogram, respectively. The velocity gradually increases with depth, and there is a decrease in velocity at the depth of 1 km. A gentle anticline structure is prominent in the middle of the volume. There is one shallow refractor (approximately 500 m deep) and a deeper one at 1.5 km depth.

The primary objective of traveltime tomography is to compute a starting velocity model, in which the calculated and the observed waveforms are not cycle skipped. Figure 14 shows the traveltime error histogram associated with the final tomogram. Because the errors are predominately less than half the period of 4 Hz, the tomo-

Figure 17. (Left) Selected sections from observed data traces, and the corresponding traces from finite-difference solutions to the 3D wave equation using (middle) the traveltime tomogram and (right) FWI tomogram (five iterations). The 0.6 s automatic gain control is applied to the traces for display.

gram is used for FWI with a maximum frequency of 4 Hz. However, a close look at the nature of traveltime residuals in Figure 15 indicates that the residuals are mostly related to short wavelength heterogeneities that cannot be inverted due to the smoothing constraints used during the inversion. Relaxing the smoothing (i.e., unregularized/unpreconditioned inversion) will minimize the residuals, but will also give geologically unrealistic tomograms. This highlights one of the limitations of traveltime tomography and the proposed method. We rely on FWI to recover such short-scale features.

FULL-WAVEFORM INVERSION

AlTheyab et al. (2013) describe the theory of Gauss-Newton optimization for time-domain FWI. Gauss-Newton optimization (Pratt and Shin, 1998; Akcelik et al., 2002; Erlangga and Herrmann, 2009) amounts for applying least-squares reverse time migration (LSRTM) (Plessix and Mulder, 2004; Dai et al., 2012) to the waveform residuals at each nonlinear iteration to estimate the search direction. Here, we apply the same method to the OBS data, using the traveltime tomogram as a starting model. The number of iterations for the LSRTM inversion is set at five. The 2–4 Hz transmission data are inverted, and the results presented here are for the 3D model that excludes the fringe zone (4 km from the edges of the survey). To overcome aliasing caused by the large OBS node spacing, the model updates are preconditioned by Gaussian smoothing similar to the one used in traveltime tomography.

Figure 16 shows the initial velocity model and the final tomogram. It is evident that the final tomogram contains some low-wavenumber updates and more geologic details. Figure 17 compares the predicted data from the traveltime tomogram and the final FWI tomogram. The data fit is better from the FWI tomogram, and the fit is expected to get better after more FWI iterations.

CONCLUSIONS

We present an efficient method for picking first-arrival wavefronts for data with dense wavefield sampling. This method allows for picking geographically on time slices as opposed to the traditional picking of traveltimes in CSGs or common-offset gathers. The wavefronts are geometrically modeled with a minimum number of picks using Bézier splines and interpolation in polar coordinates.

The picking method is applied to 3D OBS data, and the travel-time picks are inverted to get a velocity tomogram. Using the tomograms, we can predict first arrivals that are in-phase with the observed data. The proposed method reduces the number of data slices to be picked, and the time slices are faster to load and plot compared with inline slices. In addition, the memory requirement for the proposed approach is significantly less than that required in the conventional approach. This leads to a robust implementation with modest computer capabilities.

The wavefront picking error grows with increasing trace separation and the frequency associated with picked wavefronts. Therefore, this approach is applicable to dense arrays for which targeted frequencies are not aliased spatially.

In this paper, we proposed picking fewer traveltimes along with a strongly regularized inversion. This will result in tomograms that have a lower resolution compared with the conventional approach. The proposed approach is strictly for the purpose of generating smooth initial models for multiscale FWI. We also assumed that

short-scale heterogeneities do not cause cycle skipping. If this is the case, dense picking might be unavoidable.

In our application, manual picks were inverted by traveltime tomography. An alternative approach is to use the picks as seeds for automated picking that would eventually improve the resolution of traveltime tomography. The picking approach can also be extended for picking surface waves and possibly reflection waves.

The proposed method is based on the eikonal equation, which might not be suitable for complex structures. We believe that our algorithm can work with wave-equation traveltime inversion for complex structures. However, practical issues might arise when the method is applied to data recorded over complex media with salt bodies.

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APPENDIX A

PICKING USING BÉZIER SPLINES

Bézier splines can be intuitively understood using De Casteljau's algorithm, which plots a continuous smooth curve in a 2D plane using three control points as follows:

- 1) Define three control points $\mathbf{p}$, $\mathbf{q}$, and $\mathbf{s}$ by their x - and y -positions, in which the points $\mathbf{p}$ and $\mathbf{s}$ mark the ends of the curve, and the $\mathbf{q}$ controls the curvature of the curve as shown in Figure A-1.
- 2) As demonstrated in Figure A-2, evaluate the continuous curve $\mathbf{c}(u)$ by sampling along the variable $u \in [0, 1]$, where for a given u define a new $\mathbf{p}'$ between $\mathbf{p}$ and $\mathbf{q}$, where $|\mathbf{p}' - \mathbf{p}| = u|\mathbf{q} - \mathbf{p}|$, similarly define $\mathbf{q}'$ between $\mathbf{q}$ and $\mathbf{s}$, such that $|\mathbf{q}' - \mathbf{q}| = u|\mathbf{s} - \mathbf{q}|$. Finally, define the interpolated point $\mathbf{c}(u)$ between $\mathbf{p}'$ and $\mathbf{q}'$, such that $|\mathbf{c} - \mathbf{p}'| = u|\mathbf{q}' - \mathbf{p}'|$.

The position of $\mathbf{q}$ controls the curvature and the tangent of the interpolated curves at $\mathbf{p}$ and $\mathbf{s}$.

To use the algorithm above to pick wavefronts, we pick one point along the observed wavefront by pressing the mouse button and then dragging (i.e., continue pressing while moving) to form a line tangential to the wavefront as shown in Figure A-3. Then, we release the mouse button, and the new mouse position becomes the end point for the interpolated curve. The interpolation is done in real time, such that as we move the mouse pointer, the curve changes accordingly. Therefore, we move as far as we can along the wavefront while tracking the wavefront. This picking procedure is repeated for the next segments to form a complete pick of the wavefront.

Figure A-1. Quadratic Bézier curve parameterization by three points in a 2D plane.

Figure A-2. De Castelja's algorithm for evaluating Quadratic Bézier curves for u equal to, respectively, from top to bottom, 0.25, 0.5, and 0.75.

Figure A-3. Quadratic Bézier curve parameterized by three points in a 2D plane to track the wavefront plotted in the background.

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