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Curvature effects in two-dimensional optical devices inspired by transformation optics

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Light transport in curved quasi two-dimensional waveguides is considered theoretically. Within transformation optics and tensor theory, a concise description of curvature effects on transverse electric and magnetic waves is derived. We show that the curvature can induce light focusing and photonic crystal properties, which are confirmed by finite element simulations. Our results indicate that the curvature is an effective parameter for designing quasi two-dimensional optical devices in the fields of micro and nano photonics. *Published by AIP Publishing.*

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Negative refraction,^{1–5} electromagnetic invisibility,^{6–8} perfect lenses,^{2,4} and microscopy with super-resolution^{9–13} are noteworthy discoveries in optics. Advances in micro and nano technology nowadays facilitate an ongoing revolution in the control of light^{14,15} and can be expected to enter daily life in the not too distant future. The key challenge to accurately design the microscopic response of light in photonic devices is addressed by light fibers,¹⁶ dielectric or metallic gratings,¹⁷ photonic crystals,^{18,19} and metallic lines or rings,^{20–22} which make use of spatial distributions of permeability and permittivity. In the present work, we consider another parameter, the curvature of a quasi two-dimensional (2D) waveguide. Curvature effects are well known in physics since Albert Einstein's theory of general relativity, one of the most important evidences being the observation of light bending around the sun in 1919.²³ The idea that a curved space can bend light is the basic principle of gravitational lenses,²⁴ while it is not possible to design a curved three-dimensional (3D) space.

This limitation can be overcome by considering a quasi 2D structure, being stretchable along the third dimension, as shown in Fig. 1(a). The curvature of the space can be transformed into an effective permeability and an effective permittivity. Using this principle, the theory of transformation optics (TO) has been proposed to formulate schemes for manipulating electromagnetic waves and controlling light rays.^{25–33} However, devices designed by TO require complex permeability and permittivity distributions, which is an experimental challenge. We aim to avoid these complications by studying the curvature effects in quasi 2D waveguides. We will show that it is possible to design lenses, gratings, and photonic crystals. In addition, a concise 2D scalar equation, instead of the 3D vector Maxwell equations, is derived to describe the light transport, which strongly reduces the computational complexity.

Let the bottom and top surfaces of a quasi 2D waveguide of thickness h [Fig. 1(a)] be described by the function $f(x, y)$ in system O

$$z_{bot} = f(x, y), \quad (1a)$$

$$z_{top} = f(x, y) + h. \quad (1b)$$

A quasi 2D waveguide in that electromagnetic waves are transmitted between the bottom and top surfaces could be achieved by a metal/insulator/metal structure with specific curved surfaces and h smaller than the wavelength λ of the electromagnetic wave. The curved space in Fig. 1(a) is transformed into a system O' , see Fig. 1(b), using the transformation

$$x' = x, \quad y' = y, \quad z' = z - f(x, y). \quad (2)$$

Since the bottom and top surfaces in O' are given by $z'_{bot} = 0$ and $z'_{top} = h$, respectively, the waveguide is flat, but has a non-trivial metric tensor

$$\mathbf{G} = \begin{pmatrix} 1 + \left(\frac{\partial f}{\partial x'}\right)^2 & \frac{\partial f}{\partial x'} \frac{\partial f}{\partial y'} & \frac{\partial f}{\partial x'} \\ \frac{\partial f}{\partial x'} \frac{\partial f}{\partial y'} & 1 + \left(\frac{\partial f}{\partial y'}\right)^2 & \frac{\partial f}{\partial y'} \\ \frac{\partial f}{\partial x'} & \frac{\partial f}{\partial y'} & 1 \end{pmatrix}. \quad (3)$$

The Maxwell equations in O'

$$\nabla' \times \mathbf{E}' - i\omega\mu_0\boldsymbol{\mu}_r' \cdot \mathbf{H}' = 0, \quad \nabla' \times \mathbf{H}' + i\omega\epsilon_0\boldsymbol{\epsilon}_r' \cdot \mathbf{E}' = 0, \quad (4)$$

have the same form as those in O

$$\nabla \times \mathbf{E} - i\omega\mu_0\boldsymbol{\mu}_r \cdot \mathbf{H} = 0, \quad \nabla \times \mathbf{H} + i\omega\epsilon_0\boldsymbol{\epsilon}_r \cdot \mathbf{E} = 0, \quad (5)$$

where μ_0 and ϵ_0 ($\boldsymbol{\mu}_r$ and $\boldsymbol{\epsilon}_r$) are the vacuum (relative) permeability and permittivity, respectively. $\mathbf{E}$ and $\mathbf{H}$ ($\mathbf{E}'$ and $\mathbf{H}'$)

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FIG. 1. Schematic diagrams of (a) a curved quasi 2D waveguide and (b) the corresponding transformed space. The thickness h of the quasi 2D waveguide is much less than the wavelengths of the electromagnetic waves considered.

represent the electric and magnetic fields with frequency ω . We have $\mathbf{E}' = (\Lambda^T)^{-1}\mathbf{E}$ and $\mathbf{H}' = (\Lambda^T)^{-1}\mathbf{H}$ with the transpose of the Jacobian matrix

$$\Lambda^T = \begin{pmatrix} 1 & 0 & -\frac{\partial f}{\partial x'} \\ 0 & 1 & -\frac{\partial f}{\partial y'} \\ 0 & 0 & 1 \end{pmatrix}. \quad (6)$$

The second order tensors μ'_r and ϵ'_r (transformed relative permeability and permittivity) are given by

$$\mu'_r = \epsilon'_r = \begin{pmatrix} 1 & 0 & -\frac{\partial f}{\partial x'} \\ 0 & 1 & -\frac{\partial f}{\partial y'} \\ -\frac{\partial f}{\partial x'} & -\frac{\partial f}{\partial y'} & 1 + \left(\frac{\partial f}{\partial x'}\right)^2 + \left(\frac{\partial f}{\partial y'}\right)^2 \end{pmatrix}. \quad (7)$$

Because the electromagnetic wave is strongly confined along the z' direction, the dependence on z' can be neglected. After substituting Eqs. (6) and (7) into (4), we can approximate the 3D vector Eq. (4) by the 2D scalar equation

$$\begin{aligned} & \left[1 + \left(\frac{\partial f}{\partial y'}\right)^2\right] \frac{\partial^2 U}{\partial x'^2} - \left[1 + \left(\frac{\partial f}{\partial x'}\right)^2\right] \frac{\partial^2 U}{\partial y'^2} \\ & - 2 \frac{\partial f}{\partial x'} \frac{\partial f}{\partial y'} \frac{\partial^2 U}{\partial x' \partial y'} + \left(\frac{\partial f}{\partial y'} \frac{\partial^2 f}{\partial x' \partial y'} - \frac{\partial^2 f}{\partial y'^2} \frac{\partial f}{\partial x'}\right) \frac{\partial U}{\partial x'} \\ & + \left(\frac{\partial f}{\partial x'} \frac{\partial^2 f}{\partial x' \partial y'} - \frac{\partial^2 f}{\partial x'^2} \frac{\partial f}{\partial y'}\right) \frac{\partial U}{\partial y'} \\ & + \left[1 + \left(\frac{\partial f}{\partial x'}\right)^2 + \left(\frac{\partial f}{\partial y'}\right)^2\right] k_0^2 U = 0, \end{aligned} \quad (8)$$

where U denotes the transverse component of the electric or magnetic field. With the 2D metric tensor

$$g = \begin{pmatrix} 1 + \left(\frac{\partial f}{\partial x'}\right)^2 & \frac{\partial f}{\partial x'} \frac{\partial f}{\partial y'} \\ \frac{\partial f}{\partial x'} \frac{\partial f}{\partial y'} & 1 + \left(\frac{\partial f}{\partial y'}\right)^2 \end{pmatrix}. \quad (9)$$

Eq. (10) can be simplified as

$$\frac{1}{|g|} (|g| g^{ij} U_{,i})_{,j} + k_0^2 U = 0, \quad (10)$$

where $i, j = x', y'$, the Einstein sum rule is adopted, and k_0 is the wavevector of the light in vacuum. Commas represent derivatives with respect to the corresponding coordinates and $|g|$ is the determinant of the 2D metric tensor. Note that the derivative in Eq. (10) is different from the 2D Laplacian in a curved space (for which $|g|$ is replaced by $\sqrt{|g|}$). Equations (9) and (10) allow us to evaluate the curvature effects on the electromagnetic wave in the quasi 2D waveguide, which cannot support the transversal electric and magnetic waves simultaneously. For example, surface plasmon polaritons that travel along a metal-dielectric interface are transversal magnetic waves. Since they are already well studied,^{25,28} we focus on the transversal electric waves in metal/dielectric/metal structures in the following. Without loss of generality, we assume the material outside the top and bottom surfaces to be a perfect electric conductor and the interlayer to be air, see Fig. 1. We study light with a wavelength of $\lambda = 1.55 \mu\text{m}$ to demonstrate the curvature effects.

With U being the z' component of the electric field, the approximation of Eq. (10) is based on the fact that the field components $E'_{x'}$ and $E'_{y'}$ are very small with respect to the incident field E'_{in} (along the z' direction). We can thus measure the inaccuracy of Eq. (10) by the relative error

$$\eta = \max \left[(|E'_{x'}|^2 + |E'_{y'}|^2) / |E'_{in}|^2 \right], \quad (11)$$

where the maximum is determined over the whole waveguide. Equation (10) is valid only for $\eta \ll 1$. In the following we call Fig. 1(a) the 3D model [described by Eq. (5)], Fig. 1(b) the TO model [described by Eq. (4)], and Eq. (10) the 2D model, for simplicity. The former two models are connected by the transformation of Eq. (2) and therefore give the same results.

We first confirm the validity of the 2D model by comparing numerical results with the 3D and TO models. Using the finite element method, all three models are solved by the COMSOL Multiphysics code. Figure 2 shows the electromagnetic energy densities obtained for the 3D, TO, and 2D models in the first, second, and third column, respectively. For both Gaussian (f_G) and cosine (f_C) surface functions, as defined in the caption of Fig. 2, we obtain very similar results for the three models. Minor differences between the 3D and TO models are due to the numerical precision mainly determined by the finite element grid. We estimate errors of 0.6% and 1.4%, respectively, for f_G and f_C by comparing Figs. 2(a) and 2(d) and Figs. 2(b) and 2(e). Since errors of 2.2% and 6.0% are obtained for the 2D model by comparing Figs. 2(c) and 2(f) with 2(b) and 2(e), see also the effective permittivity plotted in Fig. S1 of the [supplementary material](#), neglecting $E'_{x'}$ and $E'_{y'}$ is acceptable. Because the radius of

FIG. 2. (a)–(f) Maps of the electromagnetic energy density for $\lambda = 1.55$ μm plane wave light incident from the negative x direction ($h = 200$ nm): (a) and (d) 3D model, (b) and (e) TO model, and (c) and (f) 2D model. The energy densities are normalized with respect to their maxima (given in each map). For the 3D and TO models, the energy densities are plotted for the plane centered between the top and bottom surfaces. We use in (a)–(c) $f_G(x, y) = -A \exp\left[-\left(\frac{x^2}{w_x^2} + \frac{y^2}{w_y^2}\right)\right]$ with $A = 1.5$ μm , $w_x = w_y = 2.0$ μm and in (d)–(f) $f_C(x, y) = -A \cos\left[\frac{2\pi}{L} \sqrt{x^2 + y^2}\right]$ for $\sqrt{x^2 + y^2} \leq 0.5L$ and $f_C(x, y) = A$ elsewhere with $A = 1.5$ μm and $L = 4.0$ μm . The relative error η as a function of A is given in (g) for f_G and in (h) for f_C .

curvature is larger for f_G (0.86λ) than f_C (0.17λ), the error is larger in Fig. 2(f) than in Fig. 2(c). According to Figs. 2(g) and 2(h), we have $\eta = 0$ for $A \rightarrow 0$ (no curvature), indicating that the 2D model is accurate. Since η increases for increasing A weaker for $h = 200$ nm than for $h = 400$ nm, the 2D model is a good approximation when h remains below some 10% of λ .

Being a 2D scalar wave equation, Eq. (10) saves plenty of computational resources as compared to the 3D and TO models (3D vector wave equations). Using Eq. (10), we calculate the curvature effects for several quasi 2D waveguides, as shown in Fig. 1(a), see the results in Fig. 3, employing f_G with a radii of curvature larger than 0.86λ . We observe that the curvature of the waveguide can be used to focus the light via the effective permittivity (compare Fig. S2 of the [supplementary material](#)). Since the waveguide is trough-like along the x direction for $w_x > w_y$, the map in Fig. 3(a) shows a fountain pattern. For increasing w_y , the incident wave is focused to a spot, see Fig. 3(b). The effect can be enhanced by decreasing w_x , see Fig. 3(c), as this implies a curvature increase. For decreasing A , the curvature decreases and therefore the width of the focal spot increases, see Fig. 3(d). Distributions of the electromagnetic energy density along lines crossing the focal spots [Figs. 3(a)–3(d)] show very narrow peaks, see Figs. 3(e)–3(h). The peak value and width, respectively, increases and decreases for decreasing w_x (increasing curvature). On the contrary, the peak value increases for increasing w_y , as the spatial region participating in the focusing of the light grows. Since the curvature tends to be zero for $w_y \rightarrow 0$, the focal spot shifts out of the device. We conclude that a curvature decrease normal to the incident light focuses more light and a curvature increase parallel to the incident light reduces the width of the focal spot.

Figure 4 shows the dependencies of the position and width of the focal spot on the curvature of the quasi 2D waveguide. As the center of the curved region is located at the origin of the coordinate system, the position represents the focal length. The focal length increases for increasing w_x or decreasing A , because the curvature decreases, see Fig. 4(a). For the same reason this behavior is also found for the width of the focal spot, see Fig. 4(b), which is defined as the full width at half maximum of the distribution of the electromagnetic energy density along the line crossing the focal spot (Fig. 3). For $A \geq 30$ μm the focal length is less than w_x and the focal spot thus is located in the curved region of the waveguide, while for $A \leq 30$ μm , it is located in the flat region. We find that the position of the focal spot can be well controlled by the curvature and that its width can approach the diffraction limit. This demonstrates very effective light focusing, without requiring complex distributions of the permeability and permittivity, which is key from a fabrication point of view.

In order to highlight the potential of our approach, we build a photonic crystal by designing the surfaces of the quasi 2D waveguide, as shown in Fig. 1(a). We use the surface function f_{PC} defined in the caption of Fig. 5, which has the same maximal radius of curvature as f_C so that the 2D model is applicable. Figure 5(a) shows a typical photonic band structure and Figs. 5(b) and 5(c) related transmission spectra. Along the directions Γ -X and Γ -M, respectively, the band gaps extend from 0.19 to 0.26 eV and from 0.25 to 0.31 eV. Because of Eq. (10), the translation period is $\sqrt{2}/2$ times that of f_{PC} . Similar to the demonstrated lenses and photonic crystals, other optical devices can also be designed by controlling the curvature, in particular, gratings and beam splitters. For confirming the validity of our theory, we show in Fig. S3 of the [supplementary material](#) for a silicon

FIG. 3. Maps of the electromagnetic energy density for a $\lambda = 1.55 \mu\text{m}$ incident Gaussian wave with transversal width $40 \mu\text{m}$. (a)–(d) The energy densities are normalized with respect to their maxima (given in each map). (e)–(h) Distributions of the electromagnetic energy density along the lines shown in the left panels (across the focal spots). The surface function is $f_G(x, y) = -A \exp\left[-\left(\frac{x^2}{w_x^2} + \frac{y^2}{w_y^2}\right)\right]$ with the parameters given in each map.

FIG. 4. (a) Position and (b) width of the focal spot as functions of w_x for f_G , $w_y = 40 \mu\text{m}$, and different A .

FIG. 5. (a) Band structure of a photonic crystal (designed by tuning the curvature of a quasi 2D waveguide) and transmission spectra along the (b) Γ -X and (c) Γ -M directions, for $h = 200 \text{ nm}$ and surface function $f_{PC}(x, y) = A \cos\left(\frac{2\pi x}{L}\right) \cos\left(\frac{2\pi y}{L}\right)$ with $A = 1.5 \mu\text{m}$ and $L = 4 \mu\text{m}$. The insets in (b) and (c) show contour maps of f_{PC}^2 .

waveguide that the TO and 2D models give almost identical distributions of the electromagnetic energy density.

To summarize, the curvature is found to be an effective parameter for controlling the light traveling in a quasi 2D waveguide. Using the 3D vector Maxwell equations and tensor theory, we have derived a concise 2D scalar model to save computational resources and have demonstrated that this model is accurate when the thickness of the waveguide does not exceed some 10% of the light's wavelength. For quasi 2D waveguides with surfaces described by Gaussian and cosine functions, lenses and photonic crystals have been designed by employing curvature effects, which show that curvature-based photonic devices can be achieved.

See [supplementary material](#) for more calculation details of the study.

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