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A Sparsity-Regularized Born Iterative Method for Reconstruction of Two-Dimensional Piecewise Continuous Inhomogeneous Domains

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***Abstract*— A sparsity-regularized Born iterative method (BIM) is proposed for efficiently reconstructing two-dimensional piecewise-continuous inhomogeneous dielectric profiles. Such profiles are typically not spatially sparse, which reduces the efficiency of the sparsity-promoting regularization. To overcome this problem, scattered fields are represented in terms of the spatial derivative of the dielectric profile and reconstruction is carried out over samples of the dielectric profile's derivative. Then, like the conventional BIM, the nonlinear problem is iteratively converted into a sequence of linear problems (in derivative samples) and sparsity constraint is enforced on each linear problem using the thresholded Landweber iterations. Numerical results, which demonstrate the efficiency and accuracy of the proposed method in reconstructing piecewise-continuous dielectric profiles, are presented.**

Index Terms— Born iterative method, domain sparsification, iterative shrinkage thresholding, Landweber iterations microwave imaging, regularization.

I. INTRODUCTION

Electromagnetic inverse scattering problem can be defined as determining the material properties of an object from the scattered fields measured away from the investigation domain [1]. Development of numerical tools, which are capable of efficiently and accurate solving electromagnetic inverse scattering problems, are called for in various areas of engineering including remote sensing, non-destructive testing, medical imaging, through-the-wall imaging, and hydrocarbon reservoir exploration [1]. However, this is a challenging task since the inverse scattering problems are inherently nonlinear and ill-posed [2]. A common way of tackling the nonlinearity is through the Born iterative method (BIM) [3], where the nonlinear problem is turned into a sequence of linear ones. Consequently, the ill-posedness is transferred onto the linear problem of each iteration and tackled via a regularization scheme that minimizes a cost function weighted between measurement data misfit and a penalty term.

Recently, sparsity-promoting regularization schemes [4]-[5] have been incorporated into the BIM and the efficiency and accuracy of the resulting frameworks in reconstructing two-dimensional (2D) sparse domains have been demonstrated [6]-[8]. In these papers, "sparse domain" means that scatterers occupy only a small fraction of the investigation domain, i.e., sparseness is defined in the spatial domain. Even though this approach results in sharper and more accurate reconstructions, its applicability is clearly limited to spatially sparse domains. This limitation can be overcome by transforming non-sparse domains into sparser ones, where regularization can be efficiently applied, for example through the use of wavelet or cosine transform [9].

In this work, following the idea of sparsification transformations, spatial derivative of the material properties is defined as the unknown to be reconstructed. More specifically, the scattered fields are represented in terms of the derivatives of the material properties. As expected, this approach results in significant sparsification when the material properties are piecewise continuous, i.e., the spatially differentiated investigation domain is sparser than the original domain. This allows for very efficient use of sparsity-promoting regularization schemes.

The proposed sparsification approach is implemented within the BIM, similar to the schemes described in [6]-[8]. Investigation domain is discretized into square elements. Spatial differentiation is computed by applying finite differences to the samples of the dielectric permittivity at the boundaries of the consecutive discretization elements. Then, the scattered fields are represented in terms of these derivative samples using an integral equation formulation. The nonlinear problem is iteratively converted into a sequence of linear problems (where unknowns are derivative samples). Sparsity constraint is enforced on each linear problem using the thresholded Landweber iterations [4].

Numerical results, which demonstrate the efficiency and accuracy of the proposed method in reconstructing piecewise-continuous dielectric profiles, are presented.

II. FORMULATION

A. Scattering Equations and Their Discretization

Let S represent the support of a 2D piecewise continuous inhomogeneous investigation domain that resides in an unbounded background medium. The permittivity is constant in the background, i.e., $\varepsilon(\mathbf{r}) = \varepsilon_b$ while $\varepsilon(\mathbf{r})$ is the unknown to be determined for $\mathbf{r} \in S$. Permeability is constant everywhere, i.e., $\mu(\mathbf{r}) = \mu_b$ for all $\mathbf{r}$. It is assumed that S is excited by a line source, which generates only TM fields and operates at frequency ω, and scattered fields are measured at

N^R number of receivers located at $\mathbf{r}_m^R$, $m=1,...,N^R$ around S. The following integral relation can be written for the incident, scattered, and total electric fields, $E_z^{inc}(\mathbf{r})$, $E_z^{sca}(\mathbf{r})$, and $E_z(\mathbf{r})$:

$$\begin{aligned} E_z(\mathbf{r}) &= E_z^{inc}(\mathbf{r}) + E_z^{sca}(\mathbf{r}) \\ &= E_z^{inc}(\mathbf{r}) - k_b^2 \int_S \tau(\mathbf{r}') E_z(\mathbf{r}') G(\mathbf{r},\mathbf{r}') ds'. \end{aligned} \tag{1}$$

Here, $\tau(\mathbf{r}) = \varepsilon(\mathbf{r})/\varepsilon_b - 1$ is the contrast and $k_b = \omega\sqrt{\mu_b \varepsilon_b}$ and $G(\mathbf{r},\mathbf{r}')$ are the wavenumber and the 2D Green function in the background medium. To discretize (1), S is divided into N square cells with dimension Δd, such that S_k and $\mathbf{r}_k$, represent the support and centers of the kth cell respectively. Unknowns $\tau(\mathbf{r})$ and $E_z(\mathbf{r})$ for $\mathbf{r} \in S$ are expanded in terms of pulse basis functions, each of which is defined to have unit amplitude for only $\mathbf{r} \in S_k$. Inserting these expansions into (1) and evaluating the resulting equation at $\mathbf{r}_k, k=1...,N$ and $\mathbf{r}_m^R$, $m=1....N^R$ yield

$$\bar{E} = \left[\bar{I} - \bar{G} D\{\bar{\tau}\}\right]^{-1} \bar{E}^{inc} \tag{2}$$

$$\bar{E}^{sca} = \bar{G}^R D\{\bar{E}\}\bar{\tau} \tag{3}$$

respectively. Here, $\bar{I}$ is an identity matrix, $\{\bar{E}\}_k = E_z(\mathbf{r}_k)$, $\{\bar{\tau}\}_k = \tau(\mathbf{r}_k)$, $\{\bar{E}^{inc}\}_k = E_z^{inc}(\mathbf{r}_k)$, $\{\bar{E}^{sca}\}_m = E_z^{sca}(\mathbf{r}_m^R)$, and

$$\{\bar{G}\}_{k,k'} = k_b^2 \int_{S_{k'}} G(\mathbf{r}_k,\mathbf{r}') ds' \ , \ \{\bar{G}^R\}_{m,k'} = k_b^2 \int_{S_{k'}} G(\mathbf{r}_m^R,\mathbf{r}') ds' \tag{4}$$

and operator $D\{\cdot\}$ reshapes its argument into a diagonal matrix.

B. Spatial Sparsification

Spatial sparsification is carried out by taking the derivative of $\tau(\mathbf{r})$ in $\hat{x}$ and $\hat{y}$ directions. Derivatives are computed along the edges of the discretization elements by applying central difference to samples of $\tau(\mathbf{r})$ at the centers of the two elements "touching" that edge. Following this description, one can write

$$\bar{g} = \bar{M}\bar{t} \tag{5}$$

where the first N entries of the vector $\bar{g}$ store the samples of the derivative along $\hat{x}$ direction and the second N entries store the samples of the derivative along $\hat{y}$ direction and $\bar{M}$ is a sparse matrix that carries out finite difference operation. "Inverse" of (5) can be written as

$$\bar{t} = \left(\bar{M}^T \bar{M}\right)^{-1} \bar{M}^T \bar{g} \tag{6}$$

where superscript "T" implies transpose operation. Using (6) in (3), one can obtain

$$\bar{E}^{sca} = \bar{G}^R D\{\bar{E}\}\left(\bar{M}^T \bar{M}\right)^{-1} \bar{M}^T \bar{g} = \bar{H}\bar{g} \tag{7}$$

C. Sparsity-regularized BIM

Sparsity-regularized BIM minimizes a cost function that is weighted between the data misfit and the L_1 norm of the solution as the penalty term:

$$\begin{aligned} \bar{g} &= \arg\min_{\bar{g}} \left\| \bar{E}^{sca} - \bar{E}^{mea} \right\|_2 + \gamma \left\| \bar{g} \right\|_1 \\ &= \arg\min_{\bar{g}} \left\| \bar{H}\bar{g} - \bar{E}^{mea} \right\|_2 + \gamma \left\| \bar{g} \right\|_1 \end{aligned} \tag{8}$$

Here, $\bar{E}^{mea}$ stores the scattered fields measured at $\mathbf{r}_m^R$, $m=1....N^R$ and γ is the weight of the penalty term. Sparsity-regularized BIM converts the nonlinear problem in (8) into a sequence of linear problems and enforces the sparsity constraint on each linear problem via thresholded Landweber iterations [6]-[8]. This leads to the following algorithm:

Set $\bar{E}^0 = \bar{E}^{inc}$
for $p = 1,...,N^{BIM}$
  Compute $\bar{H}^{(p)}$ using $\bar{E}^{(p)}$
  for $i = 1,...,N^{LW}$
    $\bar{g}_{(i+1)}^{(p)} = T^{\gamma}(\bar{g}_{(i)}^{(p)} - \beta \bar{H}^{(p)\dagger}[\bar{H}^{(p)} \bar{g}_{(i)}^{d(p)} - \bar{E}^{mea}])$
  end for
  Compute $\bar{t}^{(p)}$ using $\bar{g}_{(i+1)}^{(p)}$ in (6)
  Compute $\bar{E}^{(p+1)}$ using $\bar{t}^{(p)}$ in (3)
end for

In the above algorithm, variables with superscript (p) belong to pth Born iteration and those with subscript (i) belong to the ith Landweber iteration. β is the Landweber iteration step size, which should satisfy $0 < \beta < 2/\sigma_{\max}$, where $\sigma_{\max}$, is the largest singular value of $\bar{H}^{(p)}$ for convergence. A method to choose the maximum iteration numbers N^{BIM} and N^{LW} is described in [8]. The expression for the (complex) soft thresholding function T^{γ} can be found in [7]-[10].

III. NUMERICAL RESULTS

The accuracy and efficiency of the proposed scheme are demonstrated using a numerical example. The contrast profile of the investigation domain is shown in Fig. 1(a). The investigation domain is discretized using 900 square cells with dimension $\Delta d = 0.15$ m. The sparseness level of $\bar{t}$ and $\bar{g}$ are 65% and 93%, respectively. The numbers of receivers and transmitters are 32 and 16. The transmitters operate at 150MHz. The noise level in $\bar{E}^{mea}$ is - 25 dB.

Three different solvers are used for reconstructing the contrast: (i) BIM with Tikhonov regularization operated on $\bar{t}$, (ii) BIM with sparsity regularization operated on $\bar{t}$, and (ii) BIM with sparsity regularization operated on $\bar{g}$. Fig. 2 compares the relative L_2 -norm error in the contrast reconstructed during the Born iterations. It is clearly shown that the proposed method converges faster. Additionally, images obtained by the three different solvers are compared. Figs. 1(b), (c), and (d) present the contrasts reconstructed by BIM (i), (ii), and (iii) at iterations $i = 50$, $i = 12$, and $i = 50$, respectively. Clearly, the reconstruction obtained by the proposed method is more accurate.

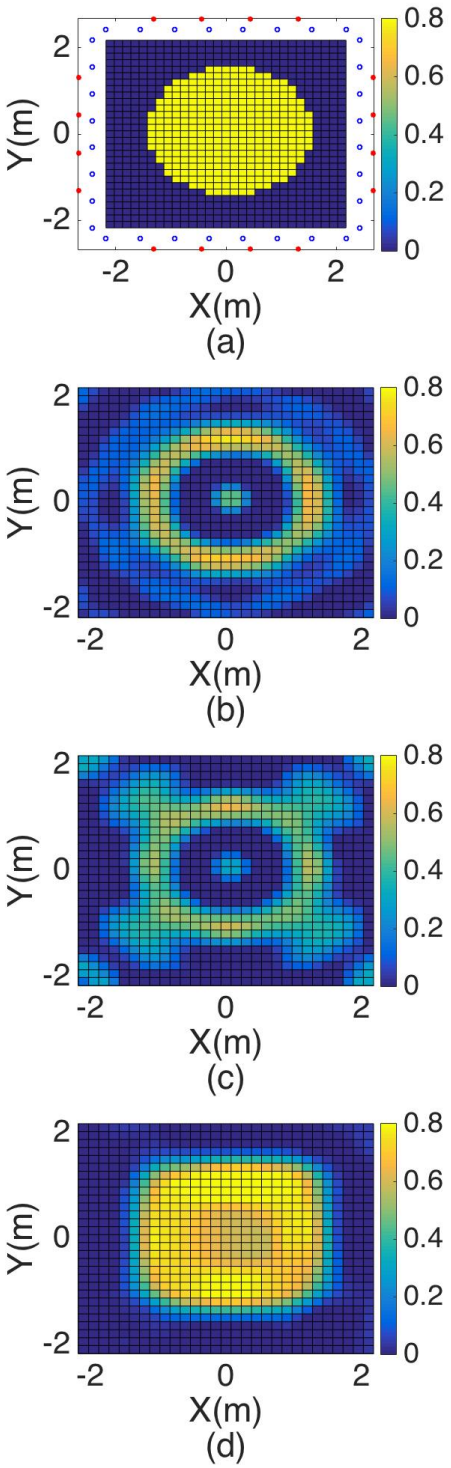

Fig. 1. (a) Original contrast profile. The red stars and blue circles around the investigation domain represent the transmitters ($N^T = 16$) and receivers ($N^R = 32$) respectively. Profiles reconstructed by BIM (i) [in (b)], BIM (ii), [in (c)], and BIM (iii) [in (d)] at iterations $i = 50$, $i = 12$, and $i = 50$, respectively.

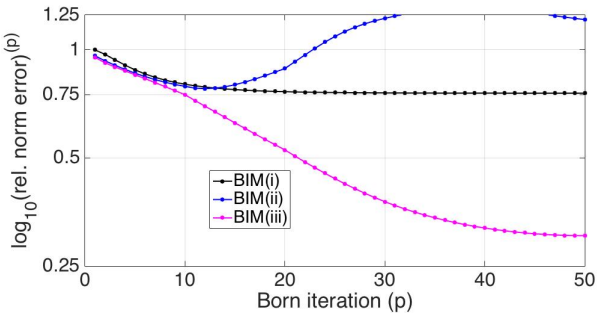

Fig. 2. Relative L_2 -norm error in the contrast reconstructed during the Born iterations.

IV. Conclusion

A sparsity-regularized BIM is described for efficiently and accurately reconstructing 2D piecewise-continuous inhomogeneous dielectric profiles. The investigation domain is sparsified using the spatial derivative of dielectric profile as the unknown to be reconstructed to increase the efficiency of the sparsity-promoting regularization. Numerical results demonstrate the efficiency and accuracy of the proposed method in reconstructing piecewise-continuous dielectric profiles.

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