Common-image gathers using the excitation amplitude imaging condition

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ABSTRACT

Common-image gathers (CIGs) are extensively used in migration velocity analysis. Any defocused events in the subsurface offset domain or equivalently nonflat events in angle-domain CIGs are accounted for revising the migration velocities. However, CIGs from wave-equation methods such as reverse time migration are often expensive to compute, especially in 3D. Using the excitation amplitude imaging condition that simplifies the forward-propagated source wavefield, we have managed to extract extended images for space and time lags in conjunction with prestack reverse time migration. The extended images tend to be cleaner, and the memory cost/disk storage is extensively reduced because we do not need to store the source wavefield. In addition, by avoiding the crosscorrelation calculation, we reduce the computational cost. These features are demonstrated on a linear $v(z)$ model, a two-layer velocity model, and the Marmousi model.

INTRODUCTION

Migration velocity analysis (MVA) is used to determine the velocity model for seismic imaging, specifically in complex media. A common practice in MVA is to examine the alignment of reflection events in common-image gathers (CIGs) and use the misalignment to update the velocity. This approach, based on the surface offset domain, is used often with the Kirchhoff method. However, it is prone to artifacts in complex areas (Xu et al., 2001; Stolk and Symes, 2002, 2004). On the other hand, computing angle-domain CIGs (ADCIGs) rather than surface-offset-domain CIGs provides a better velocity quality control (QC) measure (Xu et al., 2001; Bleistein and Gray, 2002). The reflected events in ADCIGs are flat, curved up, or down depending on whether migration velocity is correct, lower, or higher, respectively. Wave-equation methods have also enjoyed increasing popularity to get CIGs, especially

subsurface offset-domain CIGs (ODCIGs) and ADCIGs (Prucha et al., 1999; Sava and Fomel, 2003). They tend to focus the reflected energy in the ODCIGs provided the migration velocity is correct. Otherwise, we end up with smiles concaved upward or downward depending on the migration velocity being high or low, respectively. Moreover, the transmitted events in ODCIGs can also be a measure of accuracy in a velocity model (Chauris et al., 2013; Shen, 2013). Their behavior is of course different than the reflected ones. A correct migration velocity focuses the transmitted events vertically at zero offset along the depth. Defocusing obviously signifies the inaccuracy in migration velocity.

Several methods have already been proposed to construct ADCIGs (de Bruin et al., 1990; Prucha et al., 1999; Mosher and Foster, 2000; Zhang et al., 2007) in relation with wave-equation migration methods. There are two possible ways to produce those, referred to as data space and image space approaches (Sava and Fomel, 2003). The first approach generates ADCIGs during migration. The second approach uses the imaging condition first and subsequently produces ADCIGs from ODCIGs or time-shift CIGs. Therefore, it can be thought of as a postprocessing step of imaging. In this paper, we consider image space methods to get CIGs. Thus, the imaging process itself is an essential ingredient of such methods necessary to obtain CIGs. However, it involves crosscorrelation in space and time, which can be costly, especially in 3D. Moreover, depending on the data and the model size, it requires a large memory space/disk storage to store the source wavefield. The boundary-saving scheme (Berkhout, 1988; Clapp, 2008) is one solution to handle such issues, but at the cost of an additional wavefield extrapolation. In addition, crosscorrelation of wavefields propagating in the same direction causes low-wavenumber events in the seismic images, which are often considered as the migration noise. However, numerous filtering techniques are available in the literature (Youn and Zhou, 2001; Guitton et al., 2007; Liu et al., 2011; Rocha et al., 2015) to remove those noises from the seismic images.

In this study, we use the excitation amplitude imaging condition (Nguyen and McMechan, 2013) in the extended domain, which helps mitigate the memory issue in an efficient manner. It considers

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the most energetic arrival of the source wavefield in the process of imaging. Thus, it preserves all the benefits mentioned in [Nguyen and McMechan \(2013\)](#). Namely, it inherits the mitigation of the need for storing the full source wavefield. Secondly, it is faster than the boundary saving scheme for conventional imaging condition. Moreover, the CIGs obtained using the excitation imaging condition method is less noisy than that of conventional methods. In the following section, we describe the excitation amplitude imaging condition to compute CIGs in conjunction with prestack reverse time migration. We demonstrate its application on a linear $v(z)$ medium, a two-layer model, and the Marmousi model.

THEORY

Conventional imaging condition

The conventional imaging condition, often referred to as UD^* ([Claerbout, 1971](#)), is the zero-lag crosscorrelation between the forward extrapolated source wavefield and the backward extrapolated receiver wavefield in time at every image point. The image I at an image point $\mathbf{x} = (x, y, z)$ for source s_j is defined as

$$I(\mathbf{x}; s_j) = \overrightarrow{U}_s(\mathbf{x}, t; s_j) * \overleftarrow{U}_r(\mathbf{x}, t; s_j), \quad (1)$$

where $*$ denotes the crosscorrelation process in time t and $U_s(\mathbf{x}, t; s_j)$ and $U_r(\mathbf{x}, t; s_j)$ represent the source and receiver wavefield at $\mathbf{x}$ for source s_j , respectively. The symbol $\overrightarrow{(\)}$ and $\overleftarrow{(\)}$ represent the forward and backward propagation of wavefield in time, respectively.

The amplitude of an image provided by the crosscorrelation imaging condition is typically incorrect ([Claerbout, 1971](#); [Kaelin and Guitton, 2006](#)). Normalizing the crosscorrelation image with the square of the source illumination strength provides us with

$$I(\mathbf{x}; s_j) = \frac{\overrightarrow{U}_s(\mathbf{x}, t; s_j) * \overleftarrow{U}_r(\mathbf{x}, t; s_j)}{\overrightarrow{U}_s(\mathbf{x}, t; s_j) * \overrightarrow{U}_s(\mathbf{x}, t; s_j)}, \quad (2)$$

which is an image with correct phase and amplitude under the assumption of a dispersion-free medium ([Chattopadhyay and McMechan, 2008](#)). The final image is then given by a summation of $I(\mathbf{x}; s_j)$ over all shots to obtain $I(\mathbf{x})$.

However, the addition of extra dimensions to the three coordinates of the physical space of the image provides the opportunity to analyze the velocity used to extrapolate the source and receiver wavefields. In general, the extended image gather is defined as ([Sava and Fomel, 2006](#))

$$I(\mathbf{x}, \mathbf{h}, \tau) = \overleftarrow{U}_r(\mathbf{x} + \mathbf{h}, t + \tau) * \overrightarrow{U}_s(\mathbf{x} - \mathbf{h}, t - \tau), \quad (3)$$

where $\mathbf{h} = (h_x, h_y, h_z)$ and τ denote the half-subsurface offset and time lag, respectively, and $I(\mathbf{x}, \mathbf{h} = \mathbf{0}, \tau = 0)$ gives the subsurface image.

Setting $\tau = 0$ in equation 3 gives the ODCIGs. In particular, we may choose, as is done commonly, $\mathbf{h} = (h, 0, 0)$ to reduce the cost ([Rickett and Sava, 2002](#)):

$$I(\mathbf{x}, h) = \overleftarrow{U}_r(x + h, y, z, t) * \overrightarrow{U}_s(x - h, y, z, t). \quad (4)$$

Setting $\mathbf{h} = \mathbf{0}$ in equation 3 gives the time-shift CIGs (equation 5). [Sava and Fomel \(2006\)](#) show that time shifting is more efficient than

space-shift imaging because it operates on one image location at a time. It also reduces the output disk storage requirement because it involves one parameter τ .

$$I(\mathbf{x}, \tau) = \overleftarrow{U}_r(\mathbf{x}, t + \tau) * \overrightarrow{U}_s(\mathbf{x}, t - \tau). \quad (5)$$

In addition, the amplitude of the image in equation 3 can be corrected by normalizing it by the source illumination:

$$I(\mathbf{x}, \mathbf{h}, \tau) = \frac{\overleftarrow{U}_r(\mathbf{x} + \mathbf{h}, t + \tau) * \overrightarrow{U}_s(\mathbf{x} - \mathbf{h}, t - \tau)}{\overrightarrow{U}_s(\mathbf{x} - \mathbf{h}, t - \tau) * \overrightarrow{U}_s(\mathbf{x} - \mathbf{h}, t - \tau)}. \quad (6)$$

Like migration, the extended image also requires storage of the source wavefield, which of course demands a huge memory/disk storage, especially in 3D to apply crosscorrelation. It can be reduced using the excitation amplitude imaging condition as described in the following section.

Excitation amplitude imaging condition

Excitation time t_{ex} to an image point $\mathbf{x}$ is the arrival time of the maximum source wavefield D_{max} ([Chang and McMechan, 1986](#)). The amplitude D_{max} is known as the excitation amplitude. The excitation imaging condition assumes that any arrivals, other than the excitation amplitude is weak and can be ignored. Hence, its simple implementation does not entertain multipathing. Mathematically, excitation wavefield $\tilde{U}_{\text{ex}}$ is an amplified impulse defined as

$$\begin{aligned} \tilde{U}_{\text{ex}}(\mathbf{x}, t) &\approx \tilde{G}(\mathbf{x}, t; s_j) \delta(t - t_{\text{ex}}(\mathbf{x})), \\ &= \begin{cases} \tilde{D}_{\text{max}}(\mathbf{x}, t_{\text{ex}}(\mathbf{x}); s_j) & \text{for } t = t_{\text{ex}}(\mathbf{x}); \\ 0 & \text{otherwise,} \end{cases} \end{aligned} \quad (7)$$

where $G(\mathbf{x}, t; s_j)$ represents the Green's function to the image point $\mathbf{x}$ from the source s_j . Therefore, it does not take into account the full signature of the source function.

Now the image with the excitation amplitude imaging condition ([Nguyen and McMechan, 2013](#)) for an image point $\mathbf{x}$ is a stable ratio defined as

$$I_{\text{ex}}(\mathbf{x}; s_j) = \frac{\overleftarrow{U}_r(\mathbf{x}, t_{\text{ex}}(\mathbf{x}); s_j)}{\tilde{D}_{\text{max}}(\mathbf{x}, t_{\text{ex}}(\mathbf{x}); s_j)}, \quad (8)$$

where $D_{\text{max}}(\mathbf{x}, t_{\text{ex}}(\mathbf{x}); s_j)$ is the source wavefield at the corresponding excitation time step $t_i = t_{\text{ex}}$ for the corresponding source s_j . Therefore, it scales down the receiver wavefield at the time of imaging by the peak source amplitude. The excitation wavefield to the image point $\mathbf{x}$ is the most energetic arrival, which assures more stability of the method compared with the deconvolution imaging condition (equation 9) often referred to as U/D ([Claerbout, 1971](#)):

$$I(\mathbf{x}; s_j) = \frac{\overleftarrow{U}_r(\mathbf{x}, t_0(\mathbf{x}); s_j)}{\overrightarrow{U}_s(\mathbf{x}, t_0(\mathbf{x}); s_j)}, \quad (9)$$

where t_0 is the onset time defined as the first-arrival time of the source wavefield $U_s(\mathbf{x})$ to the image point $\mathbf{x}$ for the source s_j . However, the excitation imaging condition, being deconvolution in

nature, may show ill behavior, especially in a very low source illumination area.

We extend the excitation amplitude imaging condition to obtain space extended images and thereafter ADCIGs. Thus, we define the extended space domain using the excitation imaging condition as

$$I_{\text{ex}}(\mathbf{x}, \mathbf{h}) = \frac{\overleftarrow{U}_r(\mathbf{x} + \mathbf{h}, t_{\text{ex}}(\mathbf{x} - \mathbf{h}))}{\overrightarrow{D}_{\text{max}}(\mathbf{x} - \mathbf{h}, t_{\text{ex}}(\mathbf{x} - \mathbf{h}))}. \quad (10)$$

The ADCIG can easily be constructed in the Fourier domain using the simple equation (Sava and Fomel, 2000)

$$\tan \theta = -\frac{|\mathbf{k}_h|}{k_z}, \quad (11)$$

where θ and $(\mathbf{k}_h, k_z)$ represent the reflection angle and the offset and depth wavenumbers, respectively. It is a radial-trace transform and is equivalent to the slant-stack method (Ottolini, 1982).

The denominator in equation 10 being the maximum amplitude of the source wavefield at the image point $\mathbf{x} - \mathbf{h}$ assures the stability of $I(\mathbf{x}, \mathbf{h})$. Being deconvolution in nature, it takes care of the phase and amplitude of the CIG. In fact, equation 10 is a special case of equation 6:

$$\begin{aligned} I(\mathbf{x}, \mathbf{h}, \tau) &= \frac{\overleftarrow{U}_r(\mathbf{x} + \mathbf{h}, t + \tau) * \overrightarrow{U}_s(\mathbf{x} - \mathbf{h}, t - \tau)}{\overrightarrow{U}_s(\mathbf{x} - \mathbf{h}, t - \tau) * \overrightarrow{U}_s(\mathbf{x} - \mathbf{h}, t - \tau)}, \\ &= \frac{\sum_{i=1}^{\text{nt}} \overleftarrow{U}_r(\mathbf{x} + \mathbf{h}, t_i + \tau) \overrightarrow{U}_s(\mathbf{x} - \mathbf{h}, t_i - \tau)}{\sum_{i=1}^{\text{nt}} \overrightarrow{U}_s(\mathbf{x} - \mathbf{h}, t_i - \tau)^2}, \end{aligned} \quad (12)$$

where nt is the total number of time samples. Now, setting $\tau = 0$ in equation 12 and assuming the excitation imaging condition (equation 7), the CIG at image point $\mathbf{x} = \psi$ is given by

$$\begin{aligned} I(\psi, \mathbf{h}) &= \frac{\overleftarrow{U}_r(\psi + \mathbf{h}, t_{\text{ex}}(\psi - \mathbf{h})) \overrightarrow{U}_s(\psi - \mathbf{h}, t_{\text{ex}}(\psi - \mathbf{h})) + \sum_{i=1}^{\text{nt}} \overleftarrow{U}_r(\psi + \mathbf{h}, t_i) \overrightarrow{U}_s(\psi - \mathbf{h}, t_i)}{\overrightarrow{U}_s(\psi - \mathbf{h}, t_{\text{ex}}(\psi - \mathbf{h}))^2 + \sum_{i=1}^{\text{nt}} \overrightarrow{U}_s(\psi - \mathbf{h}, t_i)^2}, \\ &\approx \frac{\overleftarrow{U}_r(\psi + \mathbf{h}, t_{\text{ex}}(\psi - \mathbf{h})) \overrightarrow{U}_s(\psi - \mathbf{h}, t_{\text{ex}}(\psi - \mathbf{h}))}{\overrightarrow{U}_s(\psi - \mathbf{h}, t_{\text{ex}}(\psi - \mathbf{h}))^2}, \\ &= \frac{\overleftarrow{U}_r(\psi + \mathbf{h}, t_{\text{ex}}(\psi - \mathbf{h}))}{\overrightarrow{U}_s(\psi - \mathbf{h}, t_{\text{ex}}(\psi - \mathbf{h}))}, \\ &= I_{\text{ex}}(\psi, \mathbf{h}). \end{aligned} \quad (13)$$

The essence of this method lies in the fact that for a particular half-subsurface offset $\mathbf{h}$, it involves only one floating-point operation (FLOP), namely, just one division only at the time of imaging. On the other hand, equation 6 with $\tau = 0$ involves twice nt multiplications, twice $(\text{nt} - 1)$ additions, and one division at the time of imaging. Therefore, the implementation of excitation amplitude method is inherently faster than that of the conventional source-normalized crosscorrelation imaging method.

This method has additional benefits. It requires the source wavefield at an image point to be stored at only one time step, i.e., at the excitation time. Hence, it drastically reduces the storage requirements compared with the conventional implementation. Boundary saving scheme in the conventional approach also does not need to store the full history of the source wavefield, but at the cost of an

additional wavefield extrapolation. Although the excitation method reduces low-wavenumber artifacts that appear as a result of spatial accumulation of energy over all the time in the conventional method (Nguyen and McMechan, 2013), not all of them can be eradicated, especially in the presence of a positive velocity gradient (Chang and McMechan, 1986). However, its main weakness is in handling multipathing in the source wavefield. This can be addressed by storing more than one maximum of the source wavefield per image point to apply the imaging condition (Jin et al., 2015). However, in very complex velocity models this can be a challenge. Conventional imaging condition can intrinsically handle this issue by using the full source wavefield with all its complexity, but at the expense of increasing the background noise in the image and higher storage cost. In addition, an intermediate solution involving sparse crosscorrelation exploits the inherent strength of the conventional approach to ameliorate the excitation method in the presence of multipath (Nguyen and McMechan, 2015).

We can easily extend the excitation amplitude imaging concept to extract the time-shift CIGs:

$$I_{\text{ex}}(\mathbf{x}, \tau) = \frac{U_r(\mathbf{x}, t_{\text{ex}}(\mathbf{x}) + 2\tau)}{D_{\text{max}}(\mathbf{x}, t_{\text{ex}}(\mathbf{x}))}, \quad (14)$$

and the ADCIG as well thereafter as shown in Sava and Fomel (2006)

$$\cos^2 \theta = \frac{c^2(\mathbf{x}) |\mathbf{k}_x|^2}{4 \omega^2}, \quad (15)$$

where $(\mathbf{k}_x, \omega)$ is the wavenumber-frequency domain. Equation 14 is also a special case of equation 6.

The involvement of the nonstationary velocity term $c(\mathbf{x})$ in the ADCIG construction (equation 15) prevents us from an efficient Fourier-type implementation. This burden can easily be alleviated by replacing τ with the variable $\xi = (\tau/2)c(\mathbf{x})$ (Khalil et al., 2013; Alkhalifah, 2015).

Algorithm to get CIGs using excitation amplitude imaging condition

The main novelty of this study is the computation of ODCIGs or time-shift CIGs using a single, but the most energetic, arrival of the source wavefield. Transformation to ADCIGs from them can be considered as a postprocessing step of imaging. Therefore, it is exactly similar to the conventional approach.

With the goal of obtaining CIGs, the algorithm can be divided into three steps:

- 1) Computation of the source excitation: At each time step of the forward propagation of the source wavefield, we update the excitation amplitude D_{max} and time t_{ex} for each grid point in the model only if there is an amplitude increase.
- 2) Application of the imaging condition: At each time step of the backward propagation, we normalize the receiver wavefield by excitation amplitude D_{max} only at the grid points that satisfy the imaging time t_{ex} . The result is imaging only once for each subsurface offset or time lag extension at every grid point.
- 3) Transformation of ODCIGs or time-shift CIGs to ADCIGs.

Figure 1. Panel (a) $v(z)$ medium: the surface velocity is 1.5 km/s with velocity gradient 0.5 1/s in depth. ODCIG at $x = 8$ km using (b) the source normalized conventional imaging condition and (c) the excitation amplitude imaging condition.

Figure 2. (a) Wrong velocity medium: the surface velocity is 1.5 km/s with velocity gradient 0.6 1/s in depth. ODCIG at $x = 8$ km using (b) the source normalized conventional imaging condition and (c) the excitation amplitude imaging condition.

EXAMPLES

We demonstrate the versatility of the excitation extended imaging condition on a linear $v(z)$ medium, a two-layer velocity model, and the Marmousi model.

Figure 3. (a) A two-layer velocity model: the first-layer velocity ($v = 2$ km/s) is used for migration. (b) Excitation time map and (c) excitation amplitude map for the source at $x = 4.0$ km.

Figure 4. The figure demonstrates the ODCIGs at $x = 4$ km for model shown in Figure 3a using excitation amplitude imaging condition (equation 8) and conventional imaging condition (equation 6), respectively, with different velocities: (a and d) with true velocity, (b and e) with 5% higher, and (c and f) with 5% lower than the true velocity.

Linear $v(z)$ medium

The velocity model (Figure 1a) contains 811 and 201 grid points in x - and z -directions, respectively. The surface velocity is 1.5 km/s with velocity gradient 0.5 km/s per km in depth. We use 49 sources placed at the surface with an interval of 0.32 km. The transmitted waves produced by the model are recorded at 811 receivers located at the surface. The source wavelet is a Ricker wavelet with a peak frequency of 5 Hz. The bottom panels of Figure 1 show the ODCIG at $x = 8$ km using conventional (Figure 1b) and excitation method (Figure 1c). They look similar for both imaging conditions.

We then compare the behavior of the transmitted events using an incorrect migration velocity. Figure 2a displays an incorrect migration velocity with velocity gradient 0.6 1/s in depth. The inaccuracies in the migration velocity expectedly induce the defocusing in the transmitted events in the ODCIGs in a similar manner for both imaging conditions as shown in Figure 2b and 2c. However, as described in Shen (2013), the transmitted waves in ODCIGs behave differently than the reflected events. They focus vertically

rather than in the typical x shape that is prominent to the reflected events.

Two-layer velocity model

The velocity model (Figure 3a) contains 811 and 251 grid points in x - and z -directions, respectively. The reflector is at $z = 1.5$ km depth. We use 49 sources placed at the surface with an interval of 0.16 km. Data are simulated using a Ricker source wavelet with a peak frequency of 10 Hz recorded at 811 receivers located at the surface. The excitation time map as shown in Figure 3b represents the imaging time for a source located at $x = 4$ km. The corresponding amplitude map (Figure 3c) represents the denominator in equation 8. Figure 4a shows the ODCIG at $x = 4$ km using the excitation amplitude imaging condition with migration velocity $v = 2$ km/s. The energy is mainly focused at zero offset. The visible artifacts are because of the limited acquisition aperture and frequency bandwidth. Energy leaks to nonzero offsets with the wrong velocity as demonstrated in Figure 4b and 4c. The signature smile is upward or downward with the migration velocity being higher or lower than the correct one, respectively. Figure 4d, 4e, and 4f confirms the similarity between ODCIGs using source normalized conventional and excitation amplitude imaging condition.

The Marmousi model

The velocity model (Figure 5a) contains 2301 and 751 grid points in x - and z -directions, respectively. We use 170 sources placed with an interval of 0.05 km. Data are simulated using a Ricker source wavelet with a peak frequency of 15 Hz and recorded at receivers located at all grid points of x at the surface for a period of 5.0 s. We use a smooth version of the Marmousi model as shown in Figure 5b

Figure 5. (a) The Marmousi velocity model, (b) a smooth version of the Marmousi, used as a migration velocity, (c) excitation time map, (d) excitation amplitude map for source at $x = 4.5$ km, (e) the stacked migration image (extracted for $h = 0$ from ODCIG cube) using the excitation amplitude imaging condition.

Figure 6. For the Marmousi model (a) the ODCIGs at $x = \{2, 5, 8\}$ km, respectively and (b) the corresponding ADCIGs, using the excitation amplitude imaging condition for true migration velocity (Figure 5b).

for migration. Figure 5c and 5d shows the excitation time and amplitude maps, respectively, for a source at $x = 4.5$ km. We display the migration image using the excitation amplitude imaging condition in Figure 5e.

Figure 7. For the Marmousi model (a) the ODCIGs at $x = \{2, 5, 8\}$ km, respectively and (b) the corresponding ADCIGs, using a conventional source normalized cross-correlation imaging condition using the true migration velocity (Figure 5b). Compare with Figure 6.

Figure 8. From left to right, the panel depicts a thin slice of Marmousi velocity model, the corresponding slice of migration image at approximately $x = 6$ km, the ODCIG, and the corresponding ADCIG at $x = 6$ km. Panel (a) results are obtained using the excitation amplitude imaging condition. Panel (b) associates with conventional source normalized cross-correlation imaging condition.

Figure 9. Comparison of depth profiles of CIGs using excitation amplitude (blue) and conventional (red) imaging condition at (a) $x = 4$ km, angle = 20°, (b) $x = 4$ km, angle = 30°, (c) $x = 4$ km, $h = 0$ km, and (d) $x = 4$ km, $h = 0.1$ km.

We then compare the ODCIGs at $x = \{2, 5, 8\}$ km between those obtained using the excitation method (Figure 6a) and conventional source normalized imaging condition (Figure 7a), and they look similar. Like the conventional approach, the leakage of energy to nonzero subsurface offset is severe with increasing complexity in the model, even with the overall correct migration velocity. The deeper events defocus more because of the larger width of the Fresnel zone (Jin and McMechan, 2015). Then, we display the corresponding ADCIGs, another QC measure of velocity, in Figures 6b and 7b. Events are flat in ADCIGs. However, a closer look at the CIGs attracts our attention to some subtle differences between the two methods. We encircle some of them with red ellipses in Figure 8. It reveals that the migration noise in CIGs using our method is less than its conventional counterpart. The enhancement in the signal-to-noise ratio in the CIGs using our method focuses the energy to zero offset. However, the amplitude, as expected, is different than its conventional counterpart, as we lost some multipath energy. But in this case, the difference is negligible as suggested by the depth profile of the CIGs for different angle and subsurface offsets at $x = 4$ km in Figure 9. We also display the CIGs based on shifting the time axis as defined in equation 14. As expected, the correct velocity flattens the events in ADCIG (Figure 10).

Then, we demonstrate the effect of an erroneous migration velocity on the CIGs. The errors in migration velocity are introduced by perturbing the smooth velocity (Figure 5b) by 10% from a depth of $z = 1$ km downward (Figures 11a and 12a). The signature smiles in ODCIGs (Figure 11b) concave upward with +10% error in the migration velocity (Figure 11a). The corresponding events in the ADCIGs (Figure 11c) deflect from being flat to downward. On the other hand, -10% error in the velocity model (Figure 12a) makes the signature smiles in ODCIGs (Figure 12b) concave downward and the corresponding events in ADCIGs (Figure 12c) curve up-

ward. Therefore, the velocity inaccuracies spread the energy in the ODCIGs from zero offset to finite offsets. In addition, they make the events nonflat in the ADCIGs.

DISCUSSION

The excitation imaging condition uses part of the source wavefield in the reverse time migration approach. It specifically extracts the energy corresponding to the maximum amplitude source wavefield at every image point, and thus, ignores the contribution of other arrivals. In the Marmousi example, where multipathing is prevalent, such limitation had minor effect on the quality of the image or even the extended images. Most of the image energy is usually extracted from a single scattering optimal (energy wise) path of the waves. In some cases, mutipathing must be addressed accurately. However, the excitation imaging condition also offers the opportunity to apply the deconvolution imaging condition without the worry of dividing over zero, a common limitation in the conventional U/D (Claerbout, 1971; Chattopadhyay and McMechan, 2008) approach that requires a stabilization step. In extended images, because of the involvement of multiple lags, such features prove to be even more helpful.

Alternatively, we can have even a more efficient implementation by extracting the source wavefield excitation information from solv-

Figure 10. For the Marmousi model: (a) the time-shift CIGs at $x = \{2, 5, 8\}$ km and (b) the corresponding ADCIGs, respectively, using the true migration velocity (Figure 5b).

Figure 11. The figure depicts (a) an incorrect migration velocity model (10% higher than the true one from depth $z = 1$ km downward) for the Marmousi data set, (b) ODCIGs at $x = \{2, 5, 8\}$ km, and (c) the corresponding ADCIGs, respectively, using the excitation amplitude imaging condition.

ing the eikonal equation or ray tracing (Chang and McMechan, 1986; Nichols, 1996; Sava and Fomel, 2001). Despite the high-frequency asymptotic nature of such approximations, the often simple nature of the source wavefield, compared with the receiver wavefield, allows us to use such approximations. However, in reasonably complex media, we might need to extrapolate the wavefield for sources and receivers to obtain an accurate image.

We compare the computational aspects of the two methods in Figure 13. As discussed in the “Theory” section, the excitation amplitude imaging condition reduces the number of FLOPs, hence the computational cost. It involves only one FLOP per subsurface image point, rather than that of the order of nt , at the time of imaging. Therefore, the gain increases significantly in the case of large data sets. For the Marmousi model, it is almost 12 times faster than the boundary-saving scheme of the conventional method in getting the ODCIGs. Again, we cannot store its full history of the source wavefield because of the memory-storage limitation, thus, end up using only the boundary-saving scheme in the conventional method. In general, excitation amplitude method is much faster and cheaper compared with conventional methods with savings relative to either the source wavefield in the boundary or using its full history. However, transformation of ODCIGs to ADCIGs in this study shares the

Figure 12. The figure depicts (a) a wrong migration velocity model (10% lower than the true one from depth $z = 1$ km downward) for Marmousi data set, (b) ODCIGs at $x = \{2, 5, 8\}$ km, and (c) the corresponding ADCIGs, respectively, using the excitation amplitude imaging condition.

Figure 13. Summary of computational time and memory usage of source-normalized conventional with respect to the excitation amplitude method to get the ODCIGs for a single source. The computation efficiency of the latter method is supported by the fact that all residuals are positive. Note that we cannot store the enormous source wavefield history of the Marmousi model in the limited available computational memory.

same procedure, hence having the same cost as its conventional counterpart.

CONCLUSION

We propose to use the excitation amplitude imaging condition to extract CIGs. It requires the source wavefield information for a single arrival, and thus, it reduces the enormous storage requirements for the source wavefield in the conventional implementation. This also has the additional advantage of reducing the noise. However, handling multipathing in the source wavefield is a shortcoming of this method. Unlike conventional imaging condition, ODCIG requires a shift in the source wavefield at the excitation time only, rather than all the time steps. Similarly, time-shift-based CIGs require the receiver wavefield to be shifted only at the excitation time. This reduces the number of FLOPs in generating the CIGs and hence the computational cost.

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