Insights into the data dependency on anisotropy: an inversion prospective

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Running head: **Anisotropy dependency**

ABSTRACT

While velocity contrasts are responsible for most of the events recorded in our data, the long wavelength behavior of the velocity model is responsible for the geometrical shape of these events. For isotropic acoustic materials the wave dependency on the long (wave propagation) and short (scattering) wavelength velocity components is stationary with propagation angle. On the other hand, in representing a transversely isotropic with a vertical symmetry axis (VTI) medium with the NMO velocity, the anellepticity parameter η , the vertical scaling parameter δ , the sensitivity of waves varies with polar angle for both the long and short wavelength features of the anisotropic dimensionless medium parameters (δ and η). For horizontal reflectors at reasonable depths, the long wavelength features of the η model is reasonably constrained by the long offsets, while the short wavelength features, produce very weak reflections at even reasonable offsets. Thus, for surface acquired seismic data, we could mainly invert for smooth η responsible for the geometrical shape of reflections. On the other hand, while the δ long wavelength components mildly effects the recorded data, its short wavelength variations can produce reflections at even zero offset, with a behavior pattern synonymous to density. The lack of the long wavelength δ information

21 will mildly effect focussing, but will cause misplacement of events in depth. With low enough
22 frequencies (very low), we may be able to recover the long wavelength δ using full waveform
23 inversion (FWI). However, unlike velocity, the frequencies needed for that should be ultra
24 low to produce long wavelength scattering based model information, as δ perturbations do
25 not exert scattering at large offsets. For a combination given by the horizontal velocity, η ,
26 and ϵ , the diving wave influence of η is absorbed by the horizontal velocity, severely limiting
27 the η influence on FWI. As a result, with a good smooth η estimation, for example, from
28 tomography, we can focus FWI to invert for only the horizontal velocity, and maybe ϵ as
29 a parameter to fit the amplitude. This is possibly the most practical inversion for surface
30 seismic data in VTI media.

INTRODUCTION

31 The short wavelength components of the velocity model are responsible for most of the
32 events recorded in our seismic data, referred to them as reflections. On the other hand,
33 the long wavelength components control the shape of these events. In isotropic, constant
34 density, acoustic materials, the wave sensitivity to both the long and the short wavelength
35 components influence are angular independent. This feature does not change for velocity
36 in the acoustic anisotropic representation, if such representation is based on a velocity
37 and dimensionless parameters. In this case, the angular dependence is provided by the
38 anisotropic parameters. This dependence also varies between the short and long wavelength
39 components of these parameters.

40 Thus, multi-parameter inversion, needed in anisotropic materials (Burrige et al., 1998;
41 Plessix and Cao, 2011; Prieux et al., 2011; Operto et al., 2013), benefits from an appropriate
42 choice of parameters to represent the model. Finding a minimum set of parameters that can
43 explain the data can lead to a better inversion. Alkhalifah and Plessix (2014) analytically
44 analyzed the radial dependency (radiation pattern) of the anisotropic parameter pertur-
45 bation in acoustic transversely isotropic media with vertical symmetry axis (VTI). They
46 advocated using certain combinations of parameters for various FWI strategies, including
47 those that start with a model obtained from MVA, and those obtained from inverting div-
48 ing wave energy. Choosing the right inversion setup for resolving anisotropy can make the
49 difference between interpretable high-resolution results and results that may not make a lot
50 of sense (too smooth or too biased). Representing the VTI model using the NMO velocity,
51 v_n (Thomsen, 1986), and the anisotropy parameters, δ and η offers the proper perturbation
52 radiation pattern for an inversion that includes reflections and diving waves (Alkhalifah,

2015). Since δ mildly influences the geometrical aspects of the recorded wavefield, it can serve as a secondary parameter to fit the amplitude to compensate for the shortcomings of the acoustic model in representing the true elastic Earth. For an inversion with a hierarchical implementation in which diving waves are used first, a VTI model represented by the horizontal velocity, v_h , and the anisotropy parameters, η ($v_h = v_n\sqrt{1+2\eta}$) and ϵ , offers a practical set necessary to reduce the tradeoff and provide reasonable resolution. In this case, ϵ plays the role of amplitude fitting as it mildly effects the kinematics in the recorded wavefield (or in other words the horizontal projection of the propagator). In fact, I find this combination to be the most practical for FWI for reasons explained in this letter.

The angular influence of every point in the model can be constrained by the wave path angles of the recorded waves (it's plane wave component) that either went through it or scattered from it. The transmission provides the long wavelength information in the direction normal to the wave propagation, while the scattering defines the short wavelength in the direction normal to a potential reflector. If we are missing the needed propagation angles to constrain the anisotropic description at any wavelength scale, the description will be limited at that scale (Alkhalifah, 2015). Thus, missing the short wavelength of η or the long wavelength of δ , as is the case for a parameterization given by v_n , η , and δ , and a surface seismic P-wave experiment setup, means that we end up with a smooth η and a sharp δ , representing the δ reflectivity only. Since most of the Earth is elastic and the difference in the wavefield using the acoustic representation is given by the reflectivity, the δ reflectivity will carry in this case a lot of the amplitude residual. In this letter, I focus on the angular sensitivity of the wavefield to the short and long wavelength perturbations of the anisotropic parameter and explain what it means for FWI and parameter estimation in general using any wavefield-based method. The parameter analysis at the scattering point

77 is independent of the process used to constrain their influence. Inspired by the original
78 diagram of Claerbout (1985), shown in Figure 1, illustrating the influences of the different
79 scales of velocity, I develop such diagrams for anisotropic materials using the most feasible
80 parameter combination. The focus of the discussions and conclusions in this letter apply
81 mostly to surface seismic data acquisition for exploration or regional seismology. This
82 includes a focus on only P -waves, thus, justifying the acoustic assumption, with conclusions
83 that may apply more to the phase aspects of the inversion.

THE ANGULAR SENSITIVITY OF WAVES

84 Both the angular dependence of the kinematic and scattering components of waves have been
85 derived for purposes of traveltimes calculation and reflection amplitude. Since we are working
86 in the local (with respect to wavelength) homogeneous and linear regime, the kinematics can
87 be represented by the moveout equations (Hake et al., 1984) and the scattering by linearized
88 approximations of the reflection coefficient (Ruger, 1997). I will, however, analyze those
89 with respect to the parametrization suggested and for the purpose of inversion.

90 The kinematic-transmission components

91 The long wavelength component of the velocity model, or the anisotropic parameters, typ-
92 ically control the majority of wave propagation characteristics, effecting the shape of the
93 waves, which is conveniently described, in the high frequency asymptotic limit, by trav-
94 eltimes. To analyze the behavior of this component for reflections, and since the angular
95 dependency is our focus, I utilize derived moveout approximations to describe the angular
96 dependency. These moveout equations are derived for sources and receivers on the Earth

97 surface for an effectively homogeneous medium, but we can always bring the sources and
 98 receivers down to the grid point of interest where the medium is locally homogenous.

99 Using the Taylor's series expansion, we end up with the following 3-term approximation
 100 of the traveltime for VTI media as a function of offset, X , for the homogenous medium case
 101 (Hake et al., 1984; Alkhalifah and Tsvankin, 1995):

$$t^2 \equiv t_0^2 + \frac{X^2}{v_n^2} - \frac{2\eta X^4}{t_0^2 v_n^4}, \quad (1)$$

102 where t_0 is the zero-offset two-way time, or for horizontal events, the two-way vertical time
 103 ($t_0 = \frac{2Z\sqrt{1+2\delta}}{v_n}$, where the depth Z is an unknown quantity in surface acquired seismic data).

104 Thus, the sensitivity of traveltime to velocity can be expressed approximately as

$$\frac{\partial t}{t \partial v_n} = -\frac{X^2}{t^2 v_n^3} + \frac{8\eta X^4}{t^2 t_0^2 v_n^5} \approx -\frac{\tan^2 \frac{\theta}{2}}{v_n}, \quad (2)$$

105 stopping at the first term (leading-order influence), and

$$\frac{\partial t}{t \partial \eta} = -\frac{X^4}{t^2 t_0^2 v_n^4} \approx 4 \sin^2 \frac{\theta}{2} \tan^2 \frac{\theta}{2}, \quad (3)$$

106 where θ is the scattering angle (twice the reflection angle) between the source and receiver
 107 wavefields (or rays). Since the depth of the reflector is unknown, both parameters gain
 108 resolvability away from vertical ray path, in which the vertical path (zero offset) is used
 109 as a reference. In this case, the sensitivity of v_n is proportional to $-\tan^2 \frac{\theta}{2}$, while the
 110 sensitivity of η is proportional to $4 \sin^2 \frac{\theta}{2} \tan^2 \frac{\theta}{2}$. In the above approximation, the traveltime
 111 is insensitive to δ .

112 Finally, if we use v_h and η instead in equation 1, we obtain the following sensitivities:

$$\frac{\partial t}{t \partial v_h} = -\frac{(1+2\eta)X^2}{t^2 v_h^3} \approx -\frac{(1+2\eta) \tan^2 \frac{\theta}{2}}{v_h}, \quad (4)$$

113 stopping at the first order influence, and

$$\frac{\partial t}{t \partial \eta} = 2 \frac{X^2}{t^2 v_h^2} \approx 2 \tan^2 \frac{\theta}{2}. \quad (5)$$

114 Note in this case, v_h and η have similar dependencies at the second-order term, which is a
115 source of tradeoff.

116 The scattering components

117 In the scattering case, the angular dependency (radiation pattern) is extracted from the
118 Born approximation. For acoustic VTI media, Alkhalifah and Plessix (2014) derived such
119 patterns for two sets of anisotropic parameter combinations that they deem to be the most
120 practical. Those are v_n , η , and δ in the case in which we have the opportunity to resolve
121 long wavelength v_n first, using for example travetimes, and v_h , η , and ϵ in the case in
122 which we have the opportunity to invert diving waves first. We focus initially on the first
123 combination since it is utilized often.

124 Considering the asymptotic Greens function (without multi-pathing) for a plane wave
125 approaching location $\mathbf{x}$ in the subsurface expressed in the frequency, ω , domain

$$G(\mathbf{x}, \mathbf{k}, \omega) = A(\mathbf{x}, \mathbf{k}) \exp(i\mathbf{k} \cdot \mathbf{x}),$$

126 where $\mathbf{k}$ is the wavenumber vector for either the source or receiver wavefields, i is equal

127 to $\sqrt{-1}$, and A is the geometrical amplitude between location $\mathbf{x}$ and a potential source or
 128 receiver. We can then write the single-scattered wavefield (Alkhalifah and Plessix, 2014)

$$u_s(\mathbf{k}_s, \mathbf{k}_r, \omega) = -\omega^2 s(\omega) \int d\mathbf{x} \frac{G(\mathbf{k}_s, \mathbf{x}, \omega) G(\mathbf{k}_r, \mathbf{x}, \omega)}{v_0^2(\mathbf{x}) \rho(\mathbf{x})} \mathbf{a}_1(\mathbf{x}) \cdot \mathbf{r}_1(\mathbf{x}) \quad (6)$$

129 with s as the source function, v_0 is the background isotropic velocity, and

$$\mathbf{r}_1 = \begin{pmatrix} r_{v_n} \\ r_\eta \\ r_\delta \end{pmatrix} ; \quad \mathbf{a}_1 = \begin{pmatrix} 2 \\ 2n_{sh}^2 n_{rh}^2 \\ -(n_{sz}^2 + n_{rz}^2) \end{pmatrix}. \quad (7)$$

130 The vector $\mathbf{r}_1$ includes the perturbations of the individual parameters, v_n , η , and δ , from
 131 top to bottom. Thus, the coefficients of $\mathbf{a}_1$ define the radiation patterns of each parameter
 132 for the given parameterization (Aki and Richards, 1980). The unit vectors $\mathbf{n}_s$ and $\mathbf{n}_r$ with
 133 the source incident angle, θ_s , and the reflector dip angle ϕ , are given by

$$\mathbf{n}_s = \begin{pmatrix} n_{sh} \\ n_{sz} \end{pmatrix} = \begin{pmatrix} \sin(\theta_s) \\ \cos(\theta_s) \end{pmatrix}; \quad \mathbf{n}_r = \begin{pmatrix} n_{rh} \\ n_{rz} \end{pmatrix} = \begin{pmatrix} -\sin(\theta_s + 2\phi) \\ \cos(\theta_s + 2\phi) \end{pmatrix}. \quad (8)$$

134 For a horizontal reflector in a VTI medium, the source and receiver wave angle are the same,
 135 and thus, the η scattering potential is proportional to $\sin^4 \frac{\theta}{2}$, whereas, the velocity scattering
 136 is angle independent. The scattering potential of δ is proportional to $\cos^2 \frac{\theta}{2}$. Compared
 137 to the kinematics, which suffers from the depth ambiguity of the reflector near zero offset,
 138 minimizing the influence of δ in that case, the scattering has its highest resolution (as we
 139 see next) at zero offset, with δ perturbations playing a major role.

140 On the other hand, the radiation patterns of the parameterization (v_h, η, ε) are given

141 by (Alkhalifah and Plessix, 2014)

$$\mathbf{r}_2 = \begin{pmatrix} r_{v_h} \\ r_\eta \\ r_\epsilon \end{pmatrix} ; \quad \mathbf{a}_2 = \begin{pmatrix} 2 \\ -n_{sz}^2 n_{rh}^2 - n_{rz}^2 n_{sh}^2 \\ -(n_{sz}^2 + n_{rz}^2) \end{pmatrix} \quad (9)$$

142 Thus, for a combination given by v_h , η , and ϵ , the η influence is generally small and is
 143 predominantly located at about 90 degrees scattering angle, reasonably beyond the conven-
 144 tional offset of reflection seismic data and no where near the scattering angles associated
 145 with diving waves. Thus, the scattering potential of η in this case is weak for surface seismic
 146 data. As a result, we may ignore η in FWI, which allows us to focus on v_h .

THE WAVENUMBER CONTENT FROM THE KINEMATICS AND SCATTERING

147 The wavenumber content in the derived velocity model from the kinematics of the wavefield
 148 depends mainly on the density of events (reflections or diffractions) in which the kinematic
 149 influence can be measured (and constrained). Higher resolution models are obtained from
 150 data that have more events to constrain the kinematics of the wavefield. So the resolution
 151 of the model is dependent on the model and corresponding reflections. However, within the
 152 layer, the extracted model wavenumber is governed by the first Fresnel zone region of the
 153 gradient. In wavefield tomography and migration velocity analysis methods, our gradients
 154 are based on the adjoint state method, and thus, the model wavenumber is governed by
 155 diffraction tomography principles of transmission. In complex media, MVA methods are
 156 governed by the transmission components between our recording surface and the source of

157 events in our model, usually under the single scattering assumption. These wavenumbers
 158 are generated by diffraction tomography principles, and thus, extracted from the plane wave
 159 decomposition of the Born approximation update kernel. In fact, in isotropic media, the
 160 update wavelength at a model point, is governed by the dip of a potential reflector and the
 161 scattering angle. Specifically, the local model wavenumber vector with respect to a potential
 162 scatterer in our model (Miller et al., 1987; Jin et al., 1992; Thierry et al., 1999) is given by,

$$\mathbf{k}_m = \mathbf{k}_s + \mathbf{k}_r = 2 \frac{\omega}{v_0} \cos \frac{\theta}{2} \mathbf{n}, \quad (10)$$

163 which depends on, among other things, the angular frequency in a direction guided by a
 164 unit vector, $\mathbf{n}$, normal to a potential reflector. Here, $\mathbf{k}_s$ and $\mathbf{k}_r$ are the source and receiver
 165 (or state and adjoint state) wavefield wavenumbers, respectively, at the model point. Both
 166 diving waves and image based reflections provide updates along the wave path with very low
 167 wavenumbers. It is usually limited to the first Fresnel zone, with a maximum wavenumber
 168 dependent on the frequency of the data, and resolution provided in the direction normal
 169 to the wave path. For diving waves that direction is mainly vertical, and for imaging
 170 wave path that direction is mainly horizontal. Therefore, the range of model wavenumbers
 171 extracted from transmitted waves start from zero, to theoretically the following limit in a
 172 homogenous isotropic background:

$$k_{max} = \frac{2\omega}{v_0} \frac{\sqrt{1 + 4l \frac{\omega}{v_0}}}{1 + 2l \frac{\omega}{v_0}}, \quad (11)$$

173 where l is the distance between the source and receiver. As l tends to zero (zero-offset)
 174 $k_{max} = \frac{2\omega}{v_0}$, and as l goes to infinity $k_{max} = 2\sqrt{\frac{\omega}{lv_0}}$, which is much smaller than for zero
 175 offset and tends to zero. This is expected as we get higher resolution if our wave path is

176 shorter.

177 Conversely, the scattering part are described by the diffraction tomography components
178 that are beyond the first Fresnel zone. For typical reflection events, equation 11 holds, but
179 the scattering angle is low, resulting in high wavenumber components normal to a potential
180 reflector and a maximum given by $k_{max} = \frac{2\omega}{v_0}$ at zero offset.

MAPS OF RESOLVABILITY

181 I will discuss the anisotropy parameter resolvability utilizing Claerbout's epic diagram (Fig-
182 ure 1) that schematically explains the data sensitivity to various velocity scales. Similar
183 graphs can be constructed for the anisotropic parameters based on the above discussed
184 angular behavior of the sensitivity of the data. In classic imaging, we use the anisotropic
185 parameters given by the long wavelength components of the model description to form the
186 velocity needed for migration, or, in other words, to invert for the single scattering compo-
187 nent exclusively. In inversion, we seek both components (short and long wavelengths), to
188 produce data that include all events, including multi scattering. Since anisotropy induces
189 angular variation, we add to Claerbout's diagram an angle dimension.

190 Using the v , η , and δ combination, I show in Figure 2a the sensitivity of data to the
191 scattering potential of velocity at various angles. Since the scattering angle is used, I assume
192 that the depth of the horizontal reflector is unknown so knowledge of the velocity at low
193 wavenumbers are gained through their dependency on angle. Also, since the velocity has an
194 isotropic radiation pattern with this combination, it is similar to the isotropic case, and thus,
195 as the scattering angle increases we obtain lower wavenumbers from the FWI process. Large
196 scattering angles are usually available more up shallow, and thus, FWI performs better on

197 the shallow subsurface. For an η perturbation, such dependency, as shown in Figure 4 occurs
 198 at high scattering angles, and more so for the low wavenumber components, as it is, as we
 199 saw earlier, proportional to $\tan^2 \frac{\theta}{2}$ as opposed to $\sin^2 \frac{\theta}{2}$ for scattering. Meanwhile, Figure 2c
 200 shows that the low wavenumbers of δ are not usually attainable from surface seismic data,
 201 and only the scattering components are available. These δ perturbations can be used to fit
 202 the amplitude at conventional reflection offsets as we try to fit reflections from an acoustic
 203 model to elastic data. However, in the rare case we have ultra low frequencies in the data
 204 (inducing model wavelengths that cover the whole model), we may be able to invert for low
 205 wavenumbers δ , needed to place the reflections at their accurate depth (Figure 2d).

206 For a v_h , η , and ϵ combination, the main difference is in the η influence. The scattering
 207 potential v_h (Figure 3a) has the same radiation pattern as v_n , but the η scattering potential
 208 is much smaller (Figure 3b). In fact, η in this case does not effect diving waves as well. On
 209 the low wavenumber side, the η influence is now shared with v_h in the second order term,
 210 as evident from the moveout equations 4 and 5. This introduces a potential trade off in
 211 any MVA or MVA based tomography inversion, and thus, for this I recommend sticking
 212 with the first combination for the tomographic part of the velocity model building process.
 213 However, for the FWI part the v_h based combination provides interesting features. The low
 214 influence of η at low and high scattering angles, means that FWI can be performed for v_h
 215 only, or with ϵ to absorb the potential amplitude misfit in reflections between the acoustic
 216 assumption and the elastic nature of our data.

A SIMPLE EXAMPLE OF INFLUENCE

217 Many of the angular dependency arguments mentioned above are drawn from a plane wave
 218 decomposition of the Born scattering approximation. Let us observe whether such argu-

219 ments hold using numerical calculations that goes beyond the Born approximation or the
 220 plane wave representation. For a homogenous background NMO velocity, I consider a two-
 221 layer anisotropic parameter model in which the first layer has three scatterers located at the
 222 same depth (see Figure 4). For the first example, this anisotropic model corresponds to η ,
 223 while δ is set to zero. In this case, Figure 5a shows a shot gather for a shot located at 1280
 224 m near the surface and wavefields computed using spectral methods (Wu and Alkhalifah,
 225 2014). We first note that the η scatterer below the shot has nearly no amplitude despite the
 226 magnitude of the scatterer. A similar behavior occurred from the reflection corresponding
 227 to the η interface for small scattering angles. Only when the offset-to-depth ratio exceeds
 228 one that we start to see some reasonable amplitude. For the other scatterers, the minimum
 229 time wave path forms an angle with the vertical for either the incident or reflected wave,
 230 and thus, we get scattering from η perturbations since the scattering angle is not zero. On
 231 the other hand, if we set η constant equal 0.2 and make the model in Figure 4 represent
 232 δ , we obtain the shot gather in Figure 5b. Clearly, as expected, a δ perturbation induces
 233 zero-offset scattering and reflections. However, the amplitude of these events decrease con-
 234 siderably with offset. Also, with an $\eta = 0.2$ background, the reflection in Figure 5b arrives
 235 at earlier times at far offset, compared to the case when $\eta = 0$ in Figure 5a. Thus, despite
 236 that η does not produce a lot of scattering, it clearly effects wave propagation. The opposite
 237 conclusion can be made for δ . If we set $\eta = 0$ and add 0.2 (long wavelength variation) to
 238 the δ model in Figure 4, we obtain the shot gather in Figure 5c. Since the depth of the
 239 reflector and scatterers are unknown in our acquired data and to maintain the same time at
 240 zero offset, I adjust the depth of these events to compensate for the change in the vertical
 241 velocity caused by adding 0.2 to the δ model. Thus, Figure 5c has the same time of events
 242 at zero offset as Figures 5a and 5b, but the moveout is not effected by the added $\delta = 0.2$

243 (compared with Figure 5a). As expected and known, δ may cause considerable scattering,
244 but does not effect the wave propagation of the data we see at the Earth surface due to the
245 tradeoff with depth.

THE CASE FOR DIMENSIONLESS PARAMETERS

246 Despite our desire to have the inverted parameters share the same units, and as a result
247 have similar influence on the data, the strong angular influence of the parameters and the
248 clear bias in our acquisition (only along the Earth surface) make such a desire unattainable,
249 and thus, the utilization of the Hessian (specifically, pseudo Hessian, Shin et al. (2001)) is
250 very important to FWI. It is also important in BFGS to start the Hessian update with a
251 an initial pseudo Hessian, to speed up convergence.

252 Thus, the need for the scaling part of the Hessian in multi parameter inversion does
253 not diminish when we use parameters with the same units. These parameters have angular
254 dependence and our bias in the acquisition coverage will evidently favor certain parameters.
255 However, the dimensionless nature of the anisotropic combinations used here allows for a
256 better control of its behavior. Since the phenomena responsible for anisotropy tend to be
257 similar within a layer or a region, the anisotropy is expected to change mildly, and such
258 mild variation can be better constrained with the anisotropic parameters. This preference
259 is also highlighted in Oh and Min (2014) as they suggested using the Poisson's ratio instead
260 of the S -wave velocity in inverting elastic data. The parameter dependence shown here
261 holds for the elastic case if the Poisson's ratio is used to represent the shear wave velocity.

DISCUSSIONS

The background model considered here, for simplicity, is isotropic, but many of the results, as they are related to the physics of scattering regardless of background, still holds. The η perturbation does not exert much back scattering in the two parameter combinations addressed here. As a result, it does not share back scattering (high resolution information of such perturbations) with the data. It also means that we should be incapable of inverting for high scattering resolution of η ($\sim \frac{\omega}{v_0}$) using any method. A similar phenomena is encountered in inverting for shear wave velocity using converted waves. However, like converted waves, multi mode data may help obtain high resolution S -wave information and may also help in the elastic case to invert for η .

Considering the above arguments, for surface seismic data of conventional acquisition geometries, the most practical combination for FWI is v_h , η and ϵ . This is because the data dependency on the scattering potential of η is very mild at the angles covered by such an acquisition. Specifically, the transmission influence of η on diving waves is now absorbed by v_h . So, we can use tomography or MVA to invert for the best smooth v_h (or NMO velocity) and η . Then we use both smooth models as starting parameters in FWI to update only v_h , or v_h and ϵ . In this case, ϵ serves to improve the amplitude fit to compensate for our acoustic assumption. Afterwards, we may use the extracted high resolution horizontal velocity to improve on the η from tomography. Follow that by additional iterations of FWI to finalize v_h . This process can also be combined in a single objective function that inverts for both at different scales.

Of course, most of the conclusions drawn here are applicable mainly to surface seismic data. Other acquisition set ups like vertical seismic profiling data or cross well may require

different parameter combinations. Our analysis, however, demonstrates that any combination benefits from having one of the parameters with an isotropic radiation pattern and the others describing the deviation from that. In the anisotropic case, such parametrization is given by a velocity and dimensionless parameters. Another insight that we might propagate to other acquisition setups is the identification of the data sensitivity to the short and long wavelength model components, in which we establish a combination that makes the dependences unique to the parameters.

CONCLUSIONS

Through the analysis of the data dependency on the long and short wavelength components of the anisotropy parameters, we managed to gain knowledge on the role of the parameters in the inversion process, and understand our limitations. These limitations are driven by the physics of the wave sensitivity to the long wavelength and scattering components of the various parameters that describe the medium for a given pixel of our model space as a function of propagation (or scattering) angle. The location of this pixel with respect to the acquisition surfaces determines our ability to resolve the parameters. With a combination given by the NMO velocity, η , and δ , the inverted η will be generally smooth with information coming mainly from diving waves. We also, in this combination, can only invert for the scattering δ . Ultra low frequencies are required to obtain background δ information. The scattering δ is useful in absorbing any amplitude misfit that may come from fitting an acoustic model to an elastic Earth. In a combination given by the horizontal velocity, η and ϵ , η has minor influence on FWI for diving waves and reflections, and thus, will allow us to focus the inversion on the horizontal velocity when an accurate smooth η field is obtained from, for example, tomography. The parameter ϵ in this case is used to help fit the am-

306 plitude. Regardless of parameterization, the limiting factors of the resolution holds for all
307 parameters as the diffraction tomography principle applies to any parameter perturbation.

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LIST OF FIGURES

350 1 A diagram depicting the data dependency on the velocity wavelength in the vertical
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366 Figure 4 added to that 0.2.

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Figure 1: A diagram depicting the data dependency on the velocity wavelength in the vertical direction, represented by frequencies. It shows that our recorded seismic data depends on the low frequency velocity variation for its geometrical shape, and thus, we use Tomography or MVA to constrain it. Meanwhile, the high frequency variations in the velocity produces the reflectivity in our data.

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Figure 2: Diagrams depicting the normalized data dependency on the model parameters (for a combination given by v_n , η , and δ) as a function of vertical model wavenumber and scattering angle θ , a) for perturbations in *velocity*, b) in η , c) in δ , and d) in δ for very low frequency data.

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Figure 3: Diagrams depicting the normalized data dependency on the model parameters (for a combination given by v_h , η , and ϵ) as a function of vertical model wavenumber and scattering angle, θ , for perturbations in the a) horizontal velocity, and b) η .
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a

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