Plane-wave Least-squares Reverse Time Migration

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SUMMARY

Least-squares reverse time migration is formulated with a new parameterization, where the migration image of each shot is updated separately and a prestack image is produced with common image gathers. The advantage is that it can offer stable convergence for least-squares migration even when the migration velocity is not completely accurate. To significantly reduce computation cost, linear phase shift encoding is applied to hundreds of shot gathers to produce dozens of planes waves. A regularization term which penalizes the image difference between nearby angles are used to keep the prestack image consistent through all the angles. Numerical tests on a marine dataset is performed to illustrate the advantages of least-squares reverse time migration in the plane-wave domain. Through iterations of least-squares migration, the migration artifacts are reduced and the image resolution is improved. Empirical results suggest that the LSRTM in plane wave domain is an efficient method to improve the image quality and produce common image gathers.

INTRODUCTION

Least-squares reverse time migration (LSRTM) (Dai et al., 2012) is shown to have the following advantages when the velocity model is accurate: remove migration artifacts, balance reflector amplitudes, and improve resolution. With the multisource technique, the computational cost is reduced to a level comparable to conventional reverse time migration or even lower, depending on the acquisition geometry. One of the drawbacks of multisource least-squares reverse time migration is that it is sensitive to the accuracy of the velocity model. When the velocity model contains large bulk errors, the migration images from different shots are not consistent with each other, so the stacking process become less effective in reducing crosstalk noise. Another drawback is that when many shots are blended together, it is difficult to separate them to produce common image gathers as quality control tools. In this report, the prestack migration image is computed and updated through iterations of least-square migration. To reduce the computational cost, linear time-shift encoding is used to transfer the shot domain data to the plane wave domain. The formulation is similar to differential semblance optimization (Symes and Carazzone, 1991) except that in this report only the reflectivity image is produced. Zhang et al. (2005) provided a comprehensive review of plane wave migration and Vigh and Starr (2008) implemented full waveform inversion in the plane wave domain and achieved significant computational savings. The prestack image accommodates more unknowns to allow for better fitting of the observed data. As a result, the convergence of least-squares migration is improved.

THEORY

In least-squares migration, a reflectivity model $\mathbf{m}(\mathbf{x})$ is sought to best fit the observed data with a Born modeling operator $\mathbf{L}$. Mathematically, a model $\mathbf{m}(\mathbf{x})$ is sought to minimize the misfit functional

$$f(\mathbf{m}) = \frac{1}{2} \|\mathbf{L}\mathbf{m} - \mathbf{d}\|^2 + \frac{\gamma}{2} \|\mathbf{m}\|^2, \quad (1)$$

where γ is the damping coefficient.

The following numerical scheme can be implemented with Born modeling and the reverse time migration method:

$$\begin{aligned} \mathbf{g}^{(k)} &= \mathbf{L}^T [\mathbf{L}(\mathbf{m}^{(k)}) - \mathbf{d}] + \gamma \mathbf{m}^{(k)} \\ \alpha &= \frac{(\mathbf{g}^{(k)})^T \mathbf{g}^{(k)}}{(\mathbf{L}\mathbf{g}^{(k)})^T \mathbf{L}\mathbf{g}^{(k)} + \gamma \|\mathbf{g}^{(k)}\|^2} \\ \mathbf{m}^{(k+1)} &= \mathbf{m}^{(k)} - \alpha \mathbf{g}^{(k)}. \end{aligned} \quad (2)$$

Reverse time migration is usually performed in the shot domain, and the images from each shot are then stacked together to form the final migration image. In Dai et al. (2012), only the stacked image is derived from the above numerical scheme. When the migration velocity is not accurate, the prestack images are different from different shots and the stacked image is blurred and convergence stalls. In order to improve the robustness of the LSM, we redefine all the model space variables as prestack images: $\mathbf{m} \rightarrow \mathbf{m}(s, \mathbf{x})$, where s indicates the source index. The prestack image $\mathbf{m}(s, \mathbf{x})$ contains many more unknowns than the stacked image, and provides more freedom to fit the observed data. Another advantage is that common image gathers can be extracted from prestack images as an indication of the image quality.

In Dai et al. (2012), the multisource technique is implemented with random time shifts and random source polarity encoding functions to greatly reduce the computational cost. By stacking images from different supergathers and gradients from different iterations, the coherent signal is enhanced while the crosstalk noise is reduced. In this report, the shot domain data are encoded with linear time shift encoding functions and transferred into plane waves. The encoding process can be expressed as:

$$d(p, g, t) = \sum_s d(s, g, t) * \delta(t - p \cdot s), \quad (3)$$

where the shot domain data $d(s, g, t)$ are encoded with a time shift function $\delta(t - p \cdot s)$ and stacked together. The time shift $p \cdot s$ is a linear function of shot index s , and p is the ray parameter indicating the surface shooting angle. The linear time shift encoding can be thought as a specific realization of the random encoding. Since the plane waves are coherent signals with physically realizable planar sources, the migration image of one plane wave does not contain crosstalk noise. Instead, it

contains aliasing artifacts, which can be eliminated by stacking images from many different angles. Zhang et al. (2005) provided an estimate of how many angles are needed.

In the plane wave domain, the prestack image is indexed by the ray parameter

$$\mathbf{m} = \mathbf{m}(p, \mathbf{x}). \quad (4)$$

A new misfit functional is defined in the plane wave domain

$$f(\mathbf{m}(\mathbf{x}, p)) = \frac{1}{2} \|\mathbf{L}\mathbf{m}(\mathbf{x}, p) - \mathbf{d}\|^2 + \frac{\gamma}{2} \left\| \frac{\partial \mathbf{m}}{\partial p} \right\|^2, \quad (5)$$

where γ is the damping coefficient. Then a numerical scheme similar to equation 2 can be implemented:

$$\begin{aligned} \mathbf{g}^{(k)} &= \mathbf{L}^T [\mathbf{L}(\mathbf{m}^{(k)}) - \mathbf{d}] - \gamma \frac{\partial^2 \mathbf{m}^{(k)}}{\partial p^2} \\ \alpha &= \frac{(\mathbf{g}^{(k)})^T \mathbf{g}^{(k)}}{(\mathbf{L}\mathbf{g}^{(k)})^T \mathbf{L}\mathbf{g}^{(k)} + \lambda \left\| \frac{\partial \mathbf{g}^{(k)}}{\partial p} \right\|^2} \\ \mathbf{m}^{(k+1)} &= \mathbf{m}^{(k)} - \alpha \mathbf{g}^{(k)}. \end{aligned} \quad (6)$$

NUMERICAL RESULTS

The proposed method is tested on a 2D marine dataset. There are 515 shots with a shot interval 37.5 m. Each shot is recorded by a 6 km long cable with 480 receivers and a 12.5 m interval. The nearest offset is 198 m. These 515 CSGs are transformed into common depth profiles. Then, 2D spline interpolation is done in the CDP domain to fill in the vacancy around the zero offset after normal moveout. The interpolated data are then transformed into common receiver gathers with split-spread acquisition geometry using reciprocity (Vigh and Starr, 2008). In the CRG domain, each trace is filtered with a filter $\sqrt{i/\omega}$ and then scaled by $\sqrt{i}$ to account for the 3D geometrical spreading (Zhou et al., 1997). The tau-p transform is done to each CRG to produce 31 plane-wave gathers with ray parameters (p) ranging from $-333 \mu\text{s/m}$ to $333 \mu\text{s/m}$ with an even sampling in p . The plane-wave gathers are filtered with a Wiener filter to transform its original wavelet to a Ricker wavelet with 25 Hz peak frequency. The original wavelet is obtained by stacking the ocean bottom reflections. Figure 1 shows the plane wave with zero surface shooting angle ($p = 0$).

The migration velocity is obtained by waveform inversion (Boonyasiriwat et al., 2010). Figure 2 shows: (a) the shot-domain reverse time migration image, (b) the plane-wave domain reverse time migration image and (c) the plane-wave LSRTM image. The conventional shot domain RTM image contains strong artifacts near the shallow reflectors, which are caused by head and diving waves. These artifacts are eliminated in the plane-wave RTM image because head and diving waves are not present in the plane-wave gathers (Figure 1). Similarly, the low-frequency horizontal strips in the bottom right corner of the conventional RTM image are removed in the plane-wave RTM image. In order to further improve the quality of the image, the plane-wave LSRTM algorithm is applied to these 31 plane-wave gathers and the image after 30 iterations is shown in Figure 2(c). The convergence is fast and stable (Figure 4).

Compared to the plane-wave RTM image, the LSRTM image contains fewer artifacts and shows better resolution. In the zoom view of the red boxes (Figure 3) the horizontal reflectors can be well separated in the right end, but they are interfering with each other on the left side. In the LSRTM image, these reflectors can be distinguished further to the left due to improved image resolution. In the zoom view of the blue boxes (Figure 5), the LSRTM image shows better continuity for the nearly horizontal reflectors due to suppression of the artifacts. Moreover, these reflectors are well separated from the diffractor right below them. The common image gathers can be extracted from the prestack images (not shown due to limited space) and they illustrate the migration velocity is not very accurate in the deep part and the reflectors are undermigrated.

In terms of computational costs, the original data contains 515 shots, where each shot covers about one third of the full aperture. Therefore, the computational cost of the plane-wave RTM is about $\frac{31 \times 3}{515} \approx \frac{1}{5}$ of that for conventional shot domain RTM. Each iteration of the LSRTM is assumed to cost twice that of the RTM method. For 30 iterations, the computational cost of the plane-wave LSRTM is about 12 times of the conventional RTM. When high computational efficiency is in demand, the LSRTM can be performed with the dynamic encoding approach of Krebs et al. (2009); Schuster et al. (2011), where one plane-wave gather is used for each iteration and the ray parameter p (corresponding to surface shooting angle) is dynamically changed from one iteration to another. Figure 2(d) shows the LSRTM image with dynamic encoding after 31 iterations. It is of comparable resolution to Figure 2(c) but contains more noticeable artifacts (see Figure 3(d) and 5(d)). However, the computational cost is only about 40% of that for conventional RTM. The drawback is that CIGs are not available for velocity analysis.

DISCUSSION AND CONCLUSION

In this report, we propose to store in memory the prestack image through the iterations of the least-squares reverse time migration. The advantages include (1) stable convergence even when the velocity contains bulk velocity errors and (2) the common image gathers can be used to update the velocity model. The downside is that extra memory is needed to store all the images. To reduce the computational cost, the original shot-domain data are transformed into dozens of plane-wave gathers. Numerical tests on a 2D marine dataset show that LSRTM can produce images with fewer migration artifacts, and higher resolution compared to RTM image. The method shows good convergence even when the velocity model is not accurate. The least-squares reverse time migration with multisource technique can be an efficient method to improve the quality of RTM and produce common image gathers.

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Plane-wave LSRTM

Figure 2: The migration images obtained by: (a) conventional shot-domain reverse time migration, (b) plane-wave reverse time migration, (c) plane-wave least-squares reverse time migration and (d) plane-wave LSRTM with dynamic encoding. The blue and red boxes indicate the areas for zoom view.

Plane-wave LSRTM

Figure 1: A plane-wave gather with zero surface shooting angle ($p=0 \mu\text{s/m}$).

Figure 4: The convergence curve for plane-wave LSRTM shows fast and stable convergence even when the velocity is not completely accurate.

Figure 3: The zoom view of the red boxes: (a) the conventional shot-domain RTM image, (b) the plane-wave RTM image, (c) the plane-wave LSRTM image and (d) the plane-wave LSRTM image with dynamic encoding.

Figure 5: The zoom view of the blue boxes: (a) the conventional shot-domain RTM image, (b) the plane-wave RTM image, (c) the plane-wave LSRTM image and (d) the plane-wave LSRTM image with dynamic encoding.

EDITED REFERENCES

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