Exploring imaging capabilities of the extended prestack wavefield

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SUMMARY

The dynamic form of the double square-root (DSR) equation provides a mechanism to extrapolate prestack wavefields by moving the sources and receivers in space with respect to a potential image (scatterer) point. Its classical implementation for imaging often assumes sources and receivers are at the same horizontal surface. Reverse time migration (RTM), as well as common shot migrations in general, through its separate treatment of the sources and receivers, allows for more flexibility in source and receiver configurations. A simple modification to the classical DSR equation provides such flexibility. Specifically, we define a 7-dimensional prestack wavefield for 3-D media that includes the vertical source and receiver offset. The corresponding dispersion relation can be used to extrapolate such wavefields. However, the cost for such a definition and extrapolation can be prohibitive, considering the high dimensionality of the problem. We reduce the dimensionality by recognizing that the sources and receivers often share the same horizontal plane, and thus, obtain the conventional DSR formulation. An efficient implementation of DSR in time yields extrapolation speeds that nominally exceed those obtained from reverse time migration. We can also reduce the dimensionality by setting the horizontal offset between the source and receiver at the image point to zero, or using a DSR-like formulation to correct for source-receiver vertical offset or topography.

INTRODUCTION

Seismic data are acquired using layouts of sources and receivers that are designed to provide the maximum information for imaging the Earth's subsurface at a reasonable cost. However, the process of recording the wavefield scattered from the subsurface along only one surface of the domain of interest produces challenges as we need to both estimate the properties of the subsurface and image it (Claerbout, 1985; Berkhout, 1985). The double square-root (DSR) equation describes the singularity of a wavefield for a non-coincident sources and receivers as they relate to a potential image point. In this extended, but not physical, domain prestack wavefields are characterized by sources and receivers treated as independent variables despite their common physical space, but they are constrained to share the same horizontal plane (Yilmaz, 2001; Biondi, 2002). In this case, the extrapolation of the prestack wavefield allows for an efficient extrapolation of the full data, faster than those obtained from reverse time migration (RTM). DSR also provides subsurface offsets naturally, which allows for migration velocity analysis (MVA). The

downward continuation approach suffers, however, from a one-way wavefield extrapolation direction and requires that the sources and receivers share the same recording surface. The first challenge was addressed by Alkhalifah and Fomel (2010) by implementing time extrapolation of the prestack wavefield using another form of the DSR equation allowing waves to move up and down. The second challenge was partially addressed by Liu et al. (2011) by using the DSR formulation to extrapolate data recorded on irregular surfaces to a datum, but with a separate extrapolation for sources and receivers.

An even more expanded domain of the prestack wavefield allows for independent depth wavenumbers for the sources and receivers, and thus, alleviates some of the restrictions in the conventional DSR with respect to treating topography. The resulting seven-dimensional wavefield in 3D is difficult to characterize or extrapolate. It, however, provides a platform to introduce assumptions that allow us to reduce the dimensionality of this extended prestack domain in favor of certain applications. One of these applications is the conventional DSR formulation for imaging. In this paper, we develop a generalized formulation that extends the prestack wavefield to include any source and receiver configurations. The resulting high dimensionality of the problem can be reduced by imposing assumptions similar to those resulting in the DSR formulation. We also define an efficient and stable extrapolation in the DSR formulation. Other assumptions can be used to treat other imaging issues, such as datuming and constructing efficient 3D wavefield solutions in 2.5D media.

EXTENDED PRESTACK WAVEFIELD

Each of the source and receiver wavefields separately, described in a three dimensional space coordinate system (x, y, z) , satisfy the following dispersion relations:

$$\begin{aligned}\omega^2 &= v_s^2 |\mathbf{k}_s|^2, \\ \omega^2 &= v_r^2 |\mathbf{k}_r|^2,\end{aligned}\tag{1}$$

where ω is the angular temporal frequency, v_s and v_r are the velocities corresponding to the source and the receiver positions, respectively, and $\mathbf{k}_s$ and $\mathbf{k}_r$ are the source and receiver 3D wavenumbers, respectively. These wavenumbers correspond to the source, $\mathbf{s}$, and the receiver, $\mathbf{r}$, locations in an extended formulation, which allows us to move the sources and receivers in 3D and describe wavefields as they are related to a potential scatterer (or image point). The symbol $|\cdot|$ stands for the l_2 norm. We define the 3D midpoint and offset vectors as

$$\mathbf{m} = \frac{\mathbf{s} + \mathbf{r}}{2}, \quad \mathbf{h} = \frac{\mathbf{s} - \mathbf{r}}{2},\tag{2}$$

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which implies

$$\mathbf{k}_s = \frac{\mathbf{k}_m + \mathbf{k}_h}{2}, \quad \mathbf{k}_r = \frac{\mathbf{k}_m - \mathbf{k}_h}{2} \quad (3)$$

and

$$\omega^2 = v_s^2 \left(\frac{|\mathbf{k}_m + \mathbf{k}_h|}{2} \right)^2, \quad \omega^2 = v_r^2 \left(\frac{|\mathbf{k}_m - \mathbf{k}_h|}{2} \right)^2. \quad (4)$$

We then reformulate equations 4 to obtain

$$\begin{aligned} \frac{\omega^2}{v_s^2} - \frac{\omega^2}{v_r^2} &= \mathbf{k}_m \cdot \mathbf{k}_h, \\ \frac{\omega^2}{v_s^2} + \frac{\omega^2}{v_r^2} &= \frac{|\mathbf{k}_m|^2 + |\mathbf{k}_h|^2}{2}. \end{aligned} \quad (5)$$

The above equations could be used to extrapolate two separate wavefields. If used together, they can extrapolate a seven-dimensional wavefield, $P(\mathbf{m}, \mathbf{h}, t)$, for all sources and receivers in 3D. Considering the typical size of seismic data and the size of the corresponding image domain, such an implementation is currently completely impractical. Recognizing that the sources and receivers tend to share the same horizontal plane during surface data acquisition as well as imaging, we can set the vertical component of offset, h_z to zero and eliminate k_{hz} from equations 5 to arrive at the following single equation:

$$\begin{aligned} &\left(\frac{\omega^2}{v_s^2} - \frac{\omega^2}{v_r^2} - k_{mx}k_{hx} - k_{my}k_{hy} \right)^2 + k_{mz}^4 \\ &= 2k_{mz}^2 \left(\frac{\omega^2}{v_s^2} + \frac{\omega^2}{v_r^2} - \frac{k_{mx}^2 + k_{my}^2 + k_{hx}^2 + k_{hy}^2}{2} \right). \end{aligned} \quad (6)$$

Solving equation 6 for k_{mz} yields the conventional DSR formulation in the offset-midpoint domain in 3D, which is used for conventional downward continuation (Biondi, 2002). Solving equation 6 with respect to ω provides the phase shift operator required to extrapolate the prestack wavefield in time in a process that Alkhalifah and Fomel (2010) referred to as prestack exploding reflector modeling and imaging.

EXTRAPOLATION IN TIME

We can approximately extrapolate the prestack wavefield in time with a mixed-domain phase shift operator. Let $\mathbf{g}$ represent the space coordinates $\mathbf{g} = \{\mathbf{m}, h_x, h_y\}$, the wavefield described at an advanced time is given by

$$P(t + \Delta t, \mathbf{g}) = \int P(t, \mathbf{k}_g) e^{i\phi(\mathbf{g}, \mathbf{k}_g)\Delta t} e^{i\mathbf{k}_g \cdot \mathbf{g}} d\mathbf{k}_g, \quad (7)$$

where $P(t, \mathbf{k}_g)$ is the n -dimensional Fourier transform of $P(t, \mathbf{g})$:

$$P(t, \mathbf{k}_g) = \frac{1}{(2\pi)^n} \int P(t, \mathbf{x}) e^{-i\mathbf{k}_g \cdot \mathbf{g}} d\mathbf{g}. \quad (8)$$

In the geometrical (high-frequency) approximation, the function $\phi(\mathbf{g}, \mathbf{k}_g)$ appearing in equation (7) is given by

solving for ω in equation 6. Alternatively, we can use the second order extrapolation in time that admits the following form:

$$P(t + \Delta t, \mathbf{g}) + P(t - \Delta t, \mathbf{g}) \approx 2 \int D(t, \mathbf{k}_g) \cos[\phi(\mathbf{g}, \mathbf{k}_g), \Delta t] e^{i\mathbf{k}_g \cdot \mathbf{g}} d\mathbf{k}_g. \quad (9)$$

Equation (9) is accurate in homogeneous media and a good approximation for inhomogeneous media (Zhang and Zhang, 2009). The cost of extrapolating the wavefield using equation (9) is $O(N^2)$, where N is the number of grid points in our extended domain. Luckily, the mixed space-wavenumber domain operator is inherently low rank (Fomel et al., 2010), and thus, can be approximated by separating the variables, which allows us to use Fast Fourier transforms (FFT). Let $W(\mathbf{v}, \mathbf{k}_g) = 2 \cos(\Delta t \phi(\mathbf{v}, \mathbf{k}_g))$. The optimized expansion approach allows us to approximate this operator by solving

$$\begin{aligned} &\min_{\mathbf{v}_i, \mathbf{k}_{g_j}} \min_{a_{ij}} \max_{(\mathbf{v}, \mathbf{k}_g) \in \Omega} |W(\mathbf{v}, \mathbf{k}_g) \\ &- \sum_{i=1, j=1}^{i=M, j=N} a_{i,j} W(\mathbf{v}_i, \mathbf{k}_{g_j}) W(\mathbf{v}, \mathbf{k}_g)|, \end{aligned} \quad (10)$$

to get the best fit $\mathbf{v}_i, \mathbf{k}_{g_j}, a_{ij}$. Here, Ω defines an approximation domain restricted by the physical nature of the wavefield. The integers M, N corresponds to the desired rank, and thus, the desired efficiency. Specifically, M corresponds to the number of inverse FFTs needed at each time step, the major cost in the extrapolation. Considering the smooth nature of the extrapolation operator, and thus, the objective function, the inversion for the optimized representation can be conducted on a sparse grid.

Let $\mathbf{v}_{min}, \mathbf{v}_{max}$ and $\mathbf{k}_{g_{min}}, \mathbf{k}_{g_{max}}$ be the relative minimum and maximum values of the velocity in our model and desired wavenumbers, we set

$$\Omega = \{(\mathbf{v}, \mathbf{k}_g) | \mathbf{v}_{min} \leq \mathbf{v} \leq \mathbf{v}_{max}, \mathbf{k}_{g_{min}} \leq \mathbf{k}_g \leq \mathbf{k}_{g_{max}}, \omega(\mathbf{v}, \mathbf{k}_g) \leq 2\pi f_{max}\} \quad (11)$$

where f_{max} is the maximum frequency of the source wavelet. Since $-2 \leq W(\mathbf{v}, \mathbf{k}_g) \leq 2$, the choice of coefficients should satisfy this condition. This can be enforced by using a scale factor, a_{ij} , in equation (10) to guarantee that the approximate representation falls within the range. However, this step alone will not guarantee the full stability of this algorithm. For example, in the conventional isotropic case, the optimization is done with respect to one material parameter velocity, v , and one wavenumber $k_g = |\mathbf{k}_g|$. As shown in Figure 1, our mixed domain extrapolation operator (and our wavefield) need to satisfy $\omega(v, k_g) = vk_g \leq 2\pi f_{max}$, and thus,

$$\hat{P}(t, \mathbf{k}_g) = 0, \quad \forall |\mathbf{k}_g| > \frac{2\pi f_{max}}{v_{min}}. \quad (12)$$

In a numerical implementation, the extrapolation is unstable if

$$\hat{P}(t, \mathbf{k}_g) \neq 0, \quad \text{for some } \omega(v, k_g) > \frac{\pi}{\Delta t}. \quad (13)$$

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Figure 1: The wavenumber and velocity range considered in the extrapolation of the wavefield for the conventional isotropic medium case. The area in green is the source of instability and corresponds to unphysical situations.

As a result, we redefine our extrapolation operator so that

$$W(v, \mathbf{k}_g) = 0 \quad \text{for} \quad |\mathbf{k}_g| > \frac{\pi}{\Delta t v_{max}}, \quad (14)$$

which is used as a constraint in the optimized expansion (10). The changes to the dispersion relation does not effect the numerical accuracy if the time step is taken to satisfy

$$\Delta t \leq \frac{v_{min}}{v_{max}} \frac{1}{2f_{max}} \quad (15)$$

In the homogeneous medium case, this constraint degenerates to $\Delta t \leq \frac{1}{2f_{max}}$, which corresponds to the Nyquist frequency with a generous time sampling.

APPLICATION

Using prestack wavefields enables imaging prestack data directly. We illustrate this property using the benchmark 2D Marmousi model (Versteeg, 1994) using time extrapolation migration. The approximate dispersion relation is given by

$$\omega^2 = \frac{v_r^2 v_s^2 (k_{hx}^2 + k_{mz}^2) (k_{mx}^2 + k_{mz}^2)}{2v_+^2 k_{mz}^2 - v_-^2 k_{hx} k_{mx}}, \quad (16)$$

where $v_s = v(s, z)$, $v_r = v(r, z)$ is the velocity at source and receiver locations, $v_+^2 = v_s^2 + v_r^2$ and $v_-^2 = v_s^2 - v_r^2$. This formulation is a good approximation of the full operator (Alkhalifah, 2012).

Using the Marmousi model (Figure 2(a)), we generate synthetic data by solving the wave equation using a finite-difference method with $\Delta t = 0.4$ ms, $\Delta x = \Delta z = 4$ m and a Ricker wavelet with a peak frequency of 30 Hz. The shot interval is 8 m. The receiver coordinate satisfies $-2512 \text{ m} \leq r - s \leq 2512 \text{ m}$ for each shot, and the receiver interval is 16 m. Figure 2(a) shows the Marmousi velocity model. We extrapolate the prestack wavefield using equation 9 with a discrete mesh of $\Delta x = \Delta z = \Delta h = 8$ m and $\Delta t = 0.4$ ms. The

starting time is the maximum time of the recorded data at $t = 3$ s. The prestack wavefield at time $t = 0$ in the depth-midpoint-offset domain is shown in Figure 3, which shows that the energy is mainly focused at zero offset. Figure 2(b) shows the wavefield at time $t = 0$ and offset $h = 0$ (migration image), which is clearly of high quality.

Figure 2: (a) The Marmousi velocity model. (b) The resulting image from time extrapolating back to zero time using equations 9 and 16.

ADDITIONAL POSSIBILITIES

Equation 5 provides considerable flexibility and possibilities allowing for many similar dimensionality reduction options taking the imaging condition and data acquisition realities into account. The corresponding seven dimensional prestack wavefield (three for each of the sources and receivers space representation and one for time) poses considerable challenges in its definition and treatment. As such, we typically reduce the dimensionality by recognizing that some dimensions can be set to zero. One option is to set the offset to zero, which allows for a dispersion relation corresponding to zero offset migration. In the prestack case, the recorded data lies at

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Figure 3: The subsurface offset-midpoint gathers. The front frame is the wavefield at $h_x = 0$. The right frame is the wavefield at the midpoint, $m_x = 4904m$, for different offsets. The above frame is the wavefield at the depth $z = 800m$ for different offsets and midpoints

zero depth ($s_z = r_z = 0$), while the image is defined at zero time and offset ($\mathbf{h} = 0$). Thus, if we contend to setting only the vertical offset between the source and receiver to zero, we obtain, as shown above, the conventional DSR formula with data at $z = 0$ and the image at $t = 0$. However, the general extended formulation allows for other possibilities.

One choice is furnished by assuming that the lateral offset between the sources and receivers equals zero for the image for even data collected on irregular topography with $s_z \neq r_z$. This assumption allows us to eliminate k_z from equation 5, which yields a dispersion relation given in terms of k_{hz} , similar to the DSR formula but with opposite signs for the two square roots. By placing our data at the proper midpoint m_z location, we can extrapolate the wavefield using the conventional DSR or its time extrapolation implementation all the way to zero time. At that point, we extract the zero offset section and place our energy at the proper h_z location and use the new DSR formulation to extrapolate the wavefield to zero h_z . Considering that h_z for the recorded data is much smaller than z corresponding to the target depth and smaller than h_x and h_y , this approximation might be acceptable.

If the data is acquired on a flat surface defined at zero depth in a medium dominated by horizontal reflectors in the y direction (2.5D model), we can set $m_y = 0$ and eliminate k_m from equation 5 by setting it to zero. As a result, we obtain the following dispersion relation:

$$\omega^2 = \frac{v_r^2 v_s^2 (k_{hx}^2 + k_{hy}^2 + k_{mz}^2) (k_{mx}^2 + k_{mz}^2)}{2v_+^2 k_{mz}^2 - v_-^2 k_{hx} k_{mx}}. \quad (17)$$

Along with equation 9, it would allow us to extrapolate a 2D image along the x direction to a full azimuth coverage data set efficiently. We can also set $h_y = 0$, and thus, do a 2D time modeling extrapolation to a prestack

2D data set at $z = 0$. We can then set $k_z = 0$, and use equation 5 to eliminate k_{my} , and solve for k_{hy} , which yields

$$k_{hy}^2 = \frac{1}{2} \left(2\omega^2 \left(\frac{1}{v_g^2} + \frac{1}{v_s^2} \right) - k_{hx}^2 - k_{mx}^2 \right) \pm \frac{\sqrt{(v_g^2 (k_{hx} - k_{mx})^2 - 4\omega^2) (v_s^2 (k_{hx} + k_{mx})^2 - 4\omega^2)}}{v_g v_s} \quad (18)$$

The resulting dispersion relation can be used to extend the 2D prestack data to full azimuth for a 2.5D model, by extrapolating in h_y . Both steps involve extrapolating a 3D wavefield. In the first step, such a wavefield is given as a function of offset, midpoint (in the x -direction), and depth and enables extrapolating the wavefield (using, for example, equation 16) forward in time starting from the zero-time image. In the second step, the wavefield is given as a function of offset, midpoint (in the x -direction), and time and we extrapolate wavefield using equation 18 to positive h_y offset direction starting from the 2D data set at zero y offset. The result is a full azimuth 3D data set for a 2.5D model obtained with optimal efficiency.

In summary, the offset components of the extended domain define the data (the experiment) and can be used to utilize constraints on the acquisition. Meanwhile, the midpoint components of this extended domain define the intended image and can be used to impose constraints on the image (or model). The DSR formulation resulted from utilizing the fact that the sources and receivers often share the same horizontal plane in acquisition. The 2.5D speedy extrapolation, described above, resulted from imposing a homogeneous model in the y -direction.

CONCLUSIONS

We extend the prestack wavefield to include all possible source and receiver configuration, and treat the sources and receivers 3D spatial dimensions as independent variables in a 7D wavefield for 3D space. Such a formulation provides flexibility to derive more practical dispersion relations treating certain problems in wavefield imaging. Among the most utilized reduction of dimensionality is the assumption that the sources and receivers share the same horizontal plane yielding the familiar DSR dispersion relation. We implement an efficient and stable time extrapolation of the DSR dispersion relation. We also suggest other dimensionality reduction possibilities that can be used for wavefield extrapolation in a 2.5D model.

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EDITED REFERENCES

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