Efficient and stable extrapolation of prestack wavefields

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SUMMARY

The double-square-root (DSR) relation offers a platform to perform prestack imaging using an extended single wavefield that honors the geometrical configuration between sources, receivers and the image point, or in other words, prestack wavefields. Extrapolating such wavefields in time, nevertheless, is a big challenge because the radicand can be negative, thus reduce to a complex phase velocity, which will make the rank of the mixed domain matrix very high. Using the vertical offset between the sources and receivers, we introduce a method for deriving the DSR formulation, which gives us the opportunity to derive approximations for the mixed domain operator. The method extrapolates prestack wavefields by combining all data into one wave extrapolation procedure, allowing both upgoing and downgoing wavefields since the extrapolation is done in time, and doesn't have the $v(z)$ assumption in the offset axis of the media. Thus, the imaging condition is imposed by taking the zero-time and zero-offset slice from the multi-dimensional prestack wavefield. Unlike reverse time migration (RTM), no crosscorrelation is needed and we also have access to the subsurface offset information, which is important for migration velocity analysis. Numerical examples show the capability of this approach in dealing with complex velocity models and can provide a better quality image compared to RTM more efficiently.

INTRODUCTION

Wave equation depth migration methods are commonly divided into two types: one way for wave extrapolation in depth and two way for wave extrapolation in time or reverse time migration (Etgen et al. (2009)). Conventionally, both methods are applied on individual shot gathers. With the one way approach, it's possible to combine all data (multiple shot gathers) into one wave extrapolation procedure with the survey-sinking or DSR formulation of the wave equation (Claerbout (1985); Popovici (1997); De Hoop et al. (2003)). The limitation of the DSR formulation is the one-way nature of wave extrapolation, which limits the imaging accuracy at large structural dips. An alternative is to extend the survey-sinking approach to extrapolation in time rather than depth in the full source and receiver wavefield (Alkhalifah and Fomel (2010)). Application of two way extrapolators to modeling and migration follows the exploding reflector concept (Loewenthal et al. (1976); Claerbout (1985)) and allows for upgoing as well as downgoing wavefields. However, a major hinder in time extrapolating the DSR equation is an inherent singularity for horizontally traveling waves (Biondi (2002); De Hoop et al. (2003); Alkhalifah and Fomel (2010)). So most of the discussions above are based on the assumption that the velocity $v(x, z)$ doesn't depend on x , which will reduce the accuracy when the velocity varies a lot in the horizontal layer.

The key element missing in utilizing the DSR formulation for time extrapolation is an efficient and stable spectral implementation. In this abstract, we start by defining a five dimensional wavefield (for 2D media), which will satisfy two wave equations (one for the source and the other for the receiver location). Under the assumption that our depth coordinate of the source and receivers must be at same level, which means we solve the equation in a four dimension subspace, we eliminate one variable from the original equation and then derive out the same DSR dispersion relation as Alkhalifah and Fomel (2010) for time-space domain extrapolation of DSR equation. We can also eliminate the square root in the original formulation by utilizing the vertical offset wavenumber. Despite, its approximation nature, its accuracy is demonstrated on the synthetic data including the Marmousi model.

THEORY

Consider a seismic survey $P(t, s, r, z_s, z_r)$ as a function of time t , source locations s , receiver location r at the surface, source depth z_s and receiver depth z_r . Our ultimate goal is to extrapolate the five dimensional wavefield in time with the boundary condition given by the data:

$$P(t, s, r, z_s = 0, z_r = 0) = D(s, r, t), \quad (1)$$

where $D(s, r, t)$ is the seismic signal recorded at location $(r, 0)$ and excited at $(s, 0)$. The extended prestack wavefield $P(t, s, r, z_s, z_r)$, describes the data at receiver location (r, z_r) and time t initiated at a source located at (s, z_s) at time zero scattered from potential image points. As we move the source and receiver to an image point the time and offset of the receiver coincides with the source and equal to zero, the imaging condition. None co-incident source and receivers at time zero represent subsurface offsets used from MVA.

Figure 1: The physical meaning of the prestack wavefield $P(t, s, r, z_s, z_r)$. The received wavefield at (r, z_r) with time t initiated at time $t = 0$ at the source (s, z_s) and travels to and back from the reflector.

In the general VTI or TTI case, the prestack exploding reflector equation can be effectively extended with the help of the acoustic approximation (Alkhalifah, 1998). For simplicity, we limit the following discussion to the isotropic case. Based on the above definition, the prestack wavefield satisfies the fol-

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lowing equations:

$$\frac{\partial^2 P}{\partial t^2} = v_s^2 \left(\frac{\partial^2 P}{\partial s^2} + \frac{\partial^2 P}{\partial z_s^2} \right), \quad (2)$$

$$\frac{\partial^2 P}{\partial t^2} = v_r^2 \left(\frac{\partial^2 P}{\partial r^2} + \frac{\partial^2 P}{\partial z_r^2} \right). \quad (3)$$

In which, $v_s = v(s, z_s)$ and $v_r = v(r, z_r)$ are the velocity at source and receiver location, respectively. The imaging condition is imposed by extracting the zero-time and zero-offset slice of the prestack wavefield,

$$I(x, z) = P(0, s = x, r = x, z_s = z, z_r = z). \quad (4)$$

Denote $\vec{x} = (s, r, z_s, z_r)$ be the generalized coordinate and $\vec{k} = (k_s, k_r, k_{z_s}, k_{z_r})$ be the relative wavenumber, then

$$\hat{P}(t, \vec{k}) = \int P(t, \vec{x}) e^{-i\vec{k} \cdot \vec{x}} \quad (5)$$

be the Fourier transform of $P(t, \vec{x})$. The wave extrapolation operator satisfies that (Zhang and Zhang (2009), Wards et al. (2008))

$$P(t + \Delta t, \vec{x}) + P(t - \Delta t, \vec{x}) \approx 2 \int \hat{P}(t, \vec{k}) \cos(\omega(\vec{v}, \vec{k})\Delta t) e^{i\vec{k} \cdot \vec{x}}, \quad (6)$$

where $\vec{v} = (v_s, v_r)$ are the velocity at source and receiver location and ω is the angular frequency.

In the frequency domain, the extrapolation is defined by the dispersion relations

$$\begin{aligned} \omega^2 &= v_s^2 (k_s^2 + k_{z_s}^2), \\ \omega^2 &= v_r^2 (k_r^2 + k_{z_r}^2). \end{aligned} \quad (7)$$

Take a coordinate transform

$$z = \frac{z_s + z_r}{2}, \beta = \frac{z_s - z_r}{2}, \quad (8)$$

with the corresponding wavenumber transform

$$k_{z_s} = \frac{k_z + k_\beta}{2}, k_{z_r} = \frac{k_z - k_\beta}{2}. \quad (9)$$

Equations (7) become

$$\omega^2 = v_s^2 \left(k_s^2 + \left(\frac{k_z + k_\beta}{2} \right)^2 \right), \quad (10)$$

$$\omega^2 = v_r^2 \left(k_r^2 + \left(\frac{k_z - k_\beta}{2} \right)^2 \right). \quad (11)$$

We reformulate the above equation as

$$\begin{aligned} \frac{\omega^2}{v_s^2} - \frac{\omega^2}{v_r^2} - (k_s^2 - k_r^2) &= k_z k_\beta \\ \frac{\omega^2}{v_s^2} + \frac{\omega^2}{v_r^2} - (k_s^2 + k_r^2) &= \frac{k_z^2 + k_\beta^2}{2} \end{aligned} \quad (12)$$

These dispersion relations do not suffer from the singularity that accompanies the conventional DSR form, as we have not yet imposed the zero vertical offset assumption. However, it is hard to solve the above equations for the 5-dimensional wavefield with the current computational capabilities, and thus, we reduce the dimensionality of the wavefield to four. Since at the

boundary typically $z_s = z_r = 0$, especially for most marine acquisitions and the imaging condition also requires $z_s = z_r = z$, we can solve the wavefield in the subspace $z_s - z_r = 0$, which means $\beta = 0$. In the case that the depth of source and receiver z_s, z_r are not the same, the equation (12) can be used to extrapolate the data to the same datum and then do the migration. Eliminating the wavenumber k_β relative β in equation (12), we get the dispersion equation:

$$\begin{aligned} v_-^4 \omega^4 + 2v_s^2 v_r^2 \left[v_-^2 (k_s^2 - k_r^2) - v_+^2 k_z^2 \right] \omega^2 \\ + v_s^4 v_r^4 \left[(k_s^2 + k_r^2 + k_z^2)^2 - 4k_s^2 k_r^2 \right] = 0. \end{aligned} \quad (13)$$

This is the same formulation derived by Alkhalifah and Fomel (2010) starting from DSR equation. It's a second order polynomial with respect to ω^2 , and thus,

$$\omega^2 = \frac{v_s^2 v_r^2 \left[(k_s^2 + k_r^2 + k_z^2)^2 - 4k_s^2 k_r^2 \right]}{v_+^2 k_z^2 - v_-^2 (k_s^2 - k_r^2) + 2D}, \quad (14)$$

where

$$D = \sqrt{k_z^2 \left[v_s^2 v_r^2 (k_s^2 + k_r^2 + k_z^2) - v_r^4 k_r^2 - v_s^4 k_s^2 \right]}, \quad (15)$$

with the shortened notation $v_+^2 = v_s^2 + v_r^2, v_-^2 = v_s^2 - v_r^2$.

We can use the low rank approximation introduced in Fomel et al. (2010) or optimized expansion method Wu and Alkhalifah (2013) to solve the above equation. The difficulty of equation (15) comes from that the function D is not defined for all wavenumbers. The square root in D may produce complex D values, which will make the formulation obsolete and unstable. Luckily, we can revert to equation (7) to redefine D as follows:

$$\begin{aligned} D &= \sqrt{k_z^2 \left[v_r^2 v_s^2 k_z^2 + (v_s^2 - v_r^2)(-v_r^2 k_{z_r}^2 + v_s^2 k_{z_s}^2) \right]} \\ &= \sqrt{k_z^2 \left[v_r^2 v_s^2 k_z^2 + v_-^2 \left(\left(\frac{v_s(k_z + k_\beta)}{2} \right)^2 - \left(\frac{v_r(k_z - k_\beta)}{2} \right)^2 \right) \right]} \\ &= \sqrt{k_z^2 \left[v_r^2 v_s^2 k_z^2 + v_-^2 \left(\frac{v_-^2 k_z^2}{4} + \frac{v_+^2 k_z k_\beta}{2} + \frac{v_-^2 k_\beta^2}{4} \right) \right]} \\ &= \sqrt{k_z^2 \left[\frac{v_+^4 k_z^2}{4} + \frac{v_+^2 v_-^2 k_z k_\beta}{2} + \frac{v_-^4 k_\beta^2}{4} \right]} \\ &= \frac{|k_z| |v_+^2 k_z + v_-^2 k_\beta|}{2}. \end{aligned}$$

Now we have D well posed for any wavenumber, which was not the case in equation (15). However, we have an additional wavenumber corresponding to the vertical offset. Assuming, a subspace of the wavefield given (like in DSR) by $\beta = 0$, the energy flux along the boundary of this subspace is expected to be small for $|v_-^2| \leq v_+^2$. So we can set $k_\beta = 0$, and thus

$$D \approx k_z^2 \frac{v_+^2}{2}. \quad (16)$$

Plugging this approximate D into equation (14), we obtain a new dispersion relation free of the weaknesses described above, and given by:

$$\omega^2 = \frac{v_s^2 v_r^2 \left[(k_s^2 + k_r^2 + k_z^2)^2 - 4k_s^2 k_r^2 \right]}{2v_+^2 k_z^2 - v_-^2 (k_s^2 - k_r^2)}. \quad (17)$$

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For horizontal wave propagation, the $k_z = 0$ resulted in an essential singularity in the original formulation of the prestack wavefield extrapolation. Setting $k_z = 0$ in equation 17 yields

$$\omega^2 = -\frac{v_s^2 v_r^2 (k_s^2 - k_r^2)}{v_-^2}, \quad (18)$$

which is clearly not singular and corresponds to the accurate dispersion relation for horizontal waves traveling in the same plane containing the sources and receivers.

In the zero-offset case, $v_s = v_r = v$, $k_s = k_r = \frac{k_x}{2}$ and equation 17 admits

$$\omega^2 = \frac{v^2 (k_x^2 + k_z^2)}{4}, \quad (19)$$

which is appropriate for post-stack exploding-reflector reverse time migration Baysal et al. (1983).

The corresponding dynamic form for this new dispersion relation 17 is given by

$$2v_+^2 \frac{\partial^2 \nabla_z^2 P}{\partial t^2} - v_-^2 \frac{\partial^2 (\nabla_s^2 P - \nabla_r^2 P)}{\partial t^2} = v_s^2 v_r^2 \left[\nabla^4 P - 4 \nabla_r^2 \nabla_s^2 P \right],$$

which can be used for a variety of finite difference or finite element numerical schemes.

EXAMPLE

Our first example is a simple two-layer model. The velocity above and below are, respectively, 1000m/s and 2000m/s. The number of grid points are $n_x = 400, n_z = 256, n_h = 401$. We show the migration result of RTM and the new DSR extrapolator in Figures 2(b) and 2(a), respectively. Figure 2(a) shows higher resolution than Figure 2(b) as a result of the smoothness imposed by the correlation of two wavefield in RTM. Figure 3 shows the strong scalability test result of our code. It's almost linearly scalable from 1024 to 2048 processors. The RTM can only be scalable up to 800 processors at most, as it is limited by the number of shots. Further discussions in the matter is below.

Our next example is the famous benchmark Marmousi dataset (Versteeg (1994)). We will compare the migration result of our method with RTM. We generate the dataset corresponding to the Marmousi model using a conventional 8th-order in space and 2nd order in time finite difference approximation to the wave equation. The modeling parameters are $\Delta t = 0.4ms$, $\Delta x = \Delta z = 4m$ and a Ricker wavelet source with peak frequency of 30Hz. The shot interval is 8m, and the receiver range satisfy

$$-2512m \leq r - s \leq 2512m \quad (20)$$

for each shot. The receiver spacing is 16m. Figure 5 shows the Marmousi velocity model. We extrapolate the prestack wavefield using our DSR extrapolation in time with a discrete mesh given by $\Delta x = \Delta z = \Delta h = 8m$ and $\Delta t = 0.4ms$. The prestack wavefield at time $t = 0$ in the depth-midpoint-offset domain is shown in Figure 4, which demonstrates how the energy is focussed at zero offset. Figure 6(a) shows the wavefield at time

$t = 0$ and offset $h = 0$ (migration image). For comparison, we migrate the same dataset with RTM with the same mesh size $\Delta x = \Delta z = 8m$ and $\Delta t = 0.4ms$. Figure 6(b) shows the migration image generated by RTM. The flat and steep interfaces are well-imaged with high resolution in Figure 6(a), while the interfaces are slightly blurred and even indistinguishable in Figure 6(b).

COMPARISON WITH RTM

Our first intuition that the method is expensive considering the four dimensional wavefield. However, solutions for all shots and receivers are obtained simultaneously in one extrapolation. For a domain given by n_x samples in the horizontal direction, n_z in the depth direction, n_h in the offset direction and n_t in the time axis, the cost of this algorithm is of $O(n_x n_z n_h n_t)$. The cost of a standard RTM is $O(2n_x n_z n_t n_s)$, where n_s is the number of shots. This does not include the cost of the crosscorrelation. RTM also lacks the natural subsurface offset information admitted by the prestack wavefield naturally.

On the other hand RTM can be easily executed in parallel using multi CPUs, with different processors dealing with different shots. However, this parallelization is usually limited by the number of shots. In extrapolating prestack data we solve for one wavefield. The parallelization need to be done along the lines we tend to do for the Helmholtz solver, in which we develop communication boundaries that allow the segments to exchange information. A smart implementation of that can result in higher parallelization than the common shot one for RTM. In our first numerical experiment, the number of shots is 800, so the standard RTM can only be scalable to at most 800 processors. However, as shown in Figure 3, our code for the new method can still be scalable up to 4096 processors. In addition, prestack wavefields does not require us to store wavefields, like RTM, as the imaging condition does not require a crosscorrelation step. Even though, some RTM methods (Clapp (2009)) have been proposed to solve this issue, but at an additional cost.

CONCLUSIONS

Starting from the five dimensional prestack wavefield for 2D media, we derive the space-time DSR dispersion relation. In addition, the extended domain allows us to derive a new formulation free of $k_z = 0$ singularity hampering the conventional DSR. Numerical results show the new method has some advantages over RTM.

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Figure 2: Migration image for two-layer model produced by (a) PERM and (b) RTM.

Figure 3: Strong scalability test results of out code for PERM on two-layer model.

Figure 4: Prestack Image Gather for the marmousi model produced by PERM.

Figure 5: Marmousi velocity model.

Figure 6: Migration result comparison between PERM and RTM on marmousi model. (a) PERM. (b)RTM.

EDITED REFERENCES

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