Prestack traveltimes for dip-constrained TI media

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SUMMARY

The double-square-root (DSR) formula is an integral part of many wavefield based imaging tools. A transversely isotropic medium with a tilted symmetry axis (TI) version of the DSR formula is nearly impossible to obtain analytically. As a result, we develop an approximate version of the DSR formula valid for media with the symmetry axis normal to the dip of the reflector (DTI). The accuracy of this approximate solution is enhanced using Shanks transform to a point where the errors are extremely small for practical anisotropic values. Under this assumption, we also do not need to compute the symmetry axis field as it is inherently included in the formulation.

INTRODUCTION

Conventionally, wave-extrapolation techniques reverse time migration (RTM) are applied on individual shot gathers. However, the survey-sinking approach (Claerbout, 1985) allows for one-way wavefield extrapolation in depth for multiple shot gathers using the double-square-root (DSR) formulation of the wave equation. Within this approach we combine both up- and down-going wavefields in one equation. The DSR equation was first described in Russian literature by Belonosova and Alekseev (1967). Later it was applied in the form of a pseudo-differential dispersion relation (Claerbout, 1985) to prestack migration methods (Yilmaz, 1979; Popovici, 1996; De Hoop et al., 2003).

For a long time, the DSR equation was applied to isotropic and low-dip media. That was good enough for isotropic or low-anisotropic horizontally layered formations. Today, the main challenge for imaging are complex models including highly-anisotropic media with large dips around salt bodies. Recent studies show that using transversely isotropic model with tilted axis of symmetry (TTI) gives very good results for imaging such complex media (Huang et al., 2008; Swanston et al., 2011; Reta-Tang et al., 2011). However, the complexity of the DSR equation rises significantly with taking anisotropy into account even for one of the simplest anisotropic model - transversely isotropic model with vertical axis of symmetry (VTI).

One reasonable simplification for the general TTI model is to incorporate a transversely isotropic medium with a tilted symmetry axis normal to the reflector (DTI) in the imaging formulation as described by Alkhalifah and Sava (2010). In this case, we assume the symmetry axis normal to the reflector. This assumption simplifies our equations considering the reduced number of the model parameters and because the reflection and incidence angles are equal in DTI similar to that in the isotropic case or the VTI case for horizontal reflections. The constrain of symmetry axis being normal to the reflector is of-

ten used as a first approximation for complex model analysis. The schematic plot of DTI media is shown in Figure 1.

In this abstract, we derive the DSR equation for 2D DTI media in weak-anelliptic (small η). This approximation results in relatively simple equations which, however, provide good accuracy for a wide range of offsets. We show also traveltime parameters for DTI media and our first attempt for model parameter estimation. We will incorporate the new DSR equation into existing prestack phase-shift migration methods.

Figure 1: Schematic view of the DTI model. Symmetry axis of TI, θ , is equal to the interface dip ϕ .

THEORY

The TTI slowness surface in the acoustic approximation in the coordinate system linked to the symmetry axis direction is similar to the VTI slowness surface (Alkhalifah, 1998):

$$F_{TTI}(q', p') = q'^2 - \frac{1}{v_0^2} \frac{1 - (1 + 2\delta)(1 + 2\eta)p'^2 v_0^2}{1 - 2\eta(1 + 2\delta)p'^2 v_0^2} = 0, \quad (1)$$

where q' and p' are vertical and horizontal slownesses in the coordinate system linked to the symmetry axis direction, respectively; v_0 is the P-wave velocity in the direction of symmetry axis; $\eta = (\epsilon - \delta)/(1 + 2\delta)$; ϵ and δ are Thomsen (1986) anisotropic parameters. We assume that the symmetry axis is measured from the vertical axis given by the angle θ . To define the TTI slowness surface in the acquisition coordinate system, we apply the rotation operator

$$\begin{pmatrix} p' \\ q' \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix}. \quad (2)$$

The resulting TTI slowness surface is given by a fourth-order polynomial equation with an algebraically complicated solu-

tion:

$$F_{TTI}(q, p) = v_0^2(q \cos \theta + p \sin \theta)^2(1 - 2\eta(1 + 2\delta)(p \cos \theta - q \sin \theta)^2 v_0^2) + (1 + 2\delta)(1 + 2\eta)(p \cos \theta - q \sin \theta)^2 v_0^2 - 1 = 0. \quad (3)$$

Since both up- and down-going waves (or, in other terms, source and receiver waves) must be solutions of equation (3), we can rewrite equation (3) as

$$\begin{cases} F_{TTI}(q_s, p_s) = 0 \\ F_{TTI}(q_r, p_r) = 0 \end{cases} \quad (4)$$

Next, we perform the following variable changes to incorporate the DTI model:

$$\sin \theta = \frac{p_x}{\sqrt{p_x^2 + q^2}} = \frac{p_s + p_r}{\sqrt{(p_s + p_r)^2 + (q_s + q_r)^2}}, \quad (5)$$

$$\cos \theta = \frac{q}{\sqrt{p_x^2 + q^2}} = \frac{q_s + q_r}{\sqrt{(p_s + p_r)^2 + (q_s + q_r)^2}}, \quad (6)$$

Substituting (5) and (6) into (4), we end up with a system of two non-linear equations with two unknowns q_s and q_r :

$$\begin{cases} F_{DTI1}(q_s, q_r, p_s, p_r) = 0 \\ F_{DTI2}(q_s, q_r, p_s, p_r) = 0 \end{cases} \quad (7)$$

where F_{DTI1} and F_{DTI2} are 8th-order polynomials with respect to q_s and q_r , respectively. One can solve these equations either numerically considering it as a minimization problem or analytically using perturbation theory. The exact numerical solution of equations (7) is plotted in Figure 2. We found that the DTI form reduces to the VTI one when $p_x = 0$ and the isotropic one when $p_h = 0$. The isotropic solution was considered as an initial model for our numerical minimization (7).

Figure 2: The exact DSR surface based on the numerical solution of equations (7) for the DTI model with $v_0 = 2 \text{ km/s}$, $\eta = 0.2$, $\delta = 0.1$. The color bar corresponds to the $q(p_x, p_h)$ values in s/km.

ANALYTICAL SOLUTION USING PERTURBATION THEORY

Numerical solutions of equations (7) are quite expensive and often not stable, especially near the critical values for horizontal slowness. Therefore, we search for approximate solutions which can give acceptable accuracy. Assuming η small, we use a Taylor series expansion in η as a trial solution for both source and receiver vertical slownesses

$$\begin{cases} q_s = q_{s0} + q_{s1}\eta + q_{s2}\eta^2 \\ q_r = q_{r0} + q_{r1}\eta + q_{r2}\eta^2 \end{cases} \quad (8)$$

The final approximate solution of the equations (7) is given by

$$q = q_s + q_r = q_{s0} + q_{r0} + (q_{s1} + q_{r1})\eta + (q_{s2} + q_{r2})\eta^2, \quad (9)$$

where the elliptical coefficients for the source and receiver parts are given by

$$q_{s0} = \frac{1}{v_0} \sqrt{\frac{1 - (1 + 2\delta)(p_h - p_x)^2 v_0^2 + 2\delta A^2}{(1 + 2\delta)}}, \quad (10)$$

$$q_{r0} = \frac{1}{v_0} \sqrt{\frac{1 - (1 + 2\delta)(p_h + p_x)^2 v_0^2 + 2\delta A^2}{(1 + 2\delta)}}, \quad (11)$$

where

$$A = \frac{1}{2} \left(\sqrt{(1 - p_x v_0)^2 - (1 + 2\delta)p_h^2 v_0^2} + \sqrt{(1 + p_x v_0)^2 - (1 + 2\delta)p_h^2 v_0^2} \right). \quad (12)$$

The other coefficients q_{s1} , q_{r1} , q_{s2} and q_{r2} are not shown here due to their size and complexity. We also set A to be real.

$$|p_h \sqrt{1 + 2\delta}| + |p_x| \leq \frac{1}{v_0}. \quad (13)$$

Special cases

In order to test our approximation, we write $q = q_s + q_r$ for two special cases: $p_h = 0$ and $p_x = 0$.

$$q(p_h = 0) = \frac{2\sqrt{1 - p_x^2 v_0^2}}{v_0}, \quad (14)$$

$$q(p_x = 0) = \frac{2\sqrt{1 - (1 + 2\delta)p_h^2 v_0^2}}{v_0} - 2\eta \frac{(1 + 2\delta)^2 p_h^4 v_0^3}{\sqrt{1 - (1 + 2\delta)p_h^2 v_0^2}} - \frac{(1 + 2\delta)^3 p_h^6 v_0^5 (4 - 3(1 + 2\delta)p_h^2 v_0^2)}{(1 - (1 + 2\delta)p_h^2 v_0^2)^{3/2}} \eta^2. \quad (15)$$

As expected from the numerical solution (Figure 2), we have an isotropic slowness surface when $p_h = 0$ and an approximate VTI slowness surface as a series in η when $p_x = 0$.

Accuracy study

In order to test the accuracy of our approximation, we compare our series with different approximations (only elliptical part, elliptical plus linear and all coefficients given by equation 9) with the exact solution. We also test Shanks transform

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(Bender and Orszag, 1978), used to improve our approximation. All approximations give zero error when $p_h = 0$ and we obtain accuracy variation only along the p_x axis. We illustrate the behavior of the relative error with changing p_h and compare the order of error for the different approximations for $p_x = 0$ in Figure 3. We can see that the elliptical approximation is good enough only for small-offsets, while Shanks transformation gives good accuracy almost up to critical values of p_h .

Figure 3: Relative errors in the DTI slowness surface approximation for $p_x = 0$. The critical slowness for this model is $p_{hc} \approx 0.385 \text{ s/km}$.

DTI TRAVELTIME PARAMETERS

The traveltime function at zero offset typically provides the Taylor series coefficients for a traveltime (or traveltime squared) expansion as a function of offset. Thus, the series coefficients can be obtained from the derivatives of the slowness surface at a given horizontal component of the slowness vector. Let us expand our solution into a Taylor series expansion in p_h

$$q = q_0 + q_1 p_h + q_2 p_h^2 + q_3 p_h^3 + q_4 p_h^4, \quad (16)$$

where

$$q_0 = \frac{2\sqrt{1 - p_x^2 v_0^2}}{v_0}, \quad (17)$$

$$q_2 = -\frac{v_0(1 + 2\delta)}{(1 - p_x^2 v_0^2)^{3/2}}, \quad (18)$$

$$q_4 = -\frac{v_0^3(1 + 2\delta)^2(1 + 4p_x^2 v_0^2 + 8\eta(1 - p_x^2 v_0^2))}{4(1 - p_x^2 v_0^2)^{7/2}}, \quad (19)$$

and $q_1 = q_3 = 0$ due to the symmetry. Using series coefficients (17)-(19) and assuming that we know the distance to the reflector z , we can compute the traveltime parameters: two-way zero-offset traveltime t_0 , normal moveout velocity squared v_n^2 and heterogeneity factor S_2 as

$$t_0 = zq_0 = \frac{2z}{v_0} \sqrt{1 - p_x^2 v_0^2}, \quad (20)$$

$$v_n^2 = -2 \frac{q_2}{q_0} = \frac{v_0^2(1 + 2\delta)}{(1 - p_x^2 v_0^2)^2}, \quad (21)$$

$$S_2 = -2 \frac{q_0 q_4}{q_2^2} = 1 + 4p_x^2 v_0^2 + 8\eta(1 - p_x^2 v_0^2). \quad (22)$$

The obtained traveltime parameters have simple forms in comparison to the traveltime parameters for tilted elliptical TI media (Golikov and Stovas, 2011a) or general TTI media (Golikov and Stovas, 2011b). In the special case when $p_x = 0$, the traveltime parameters given by equations (20)-(22) reduce to ones for VTI media with horizontal interface. The traveltime parameters as functions of p_x for isotropic, VTI and DTI media are plotted in Figure 4. One can see that t_0^2 for the DTI model is the same as the isotropic one, while v_n^2 and S_2 for DTI model are equal to the VTI ones only when $p_x = 0$ and are between isotropic and VTI values for non-zero p_x .

Inversion of the traveltime parameters

Having traveltime parameters for the DTI model given by equations (20)-(22) in a closed and simple form helps us perform inversion for the model parameters. In general, we have an ill-posed problem since we have only three measured traveltime parameters t_0 , v_n^2 , S_2 and five independent model parameters: zero-offset reflection depth z , elastic parameters linked to the symmetry axis v_0 , δ and η and the horizontal component of the midpoint slowness, p_x , which control the interface dip. In order to perform a unique inversion, one must have additional information about the reflection interface, including interface position and orientation or equivalently assumptions on the media parameters. For example, let us assume that the reflection interface position is known a priori. Specifically, we know the zero-offset reflection depth z and interface dip, which is linked to p_x . Thus, we can easily obtain model parameters v_0 , δ and η from equations (20)-(22) as

$$v_0 = \frac{2z}{\sqrt{t_0^2 + 4z^2 p_x^2}}, \quad (23)$$

$$\delta = \frac{v_n^2(1 - p_x^2 v_0^2)^2}{2v_0^2} - 1, \quad (24)$$

$$\eta = \frac{S_2 - 1 - 4p_x^2 v_0^2}{8(1 - p_x^2 v_0^2)}. \quad (25)$$

These equations can be used for an iterative update of the image and velocity model, simultaneously. In the general case, when more than three model parameters are unknown, inversion can be performed using a least-squares minimization process.

THE IMPULSE RESPONSE

To demonstrate features of the phase-shift migration operator for the new DSR equation for DTI media, we show in Figure 5 the prestack imaging impulse response. The input data is given by five pulses at zero-offset and vertical time 0.6, 1.2, 1.8, 2.4 and 3 seconds. We use a vertically heterogeneous velocity model, $v(z) = 1.5 + 0.6z$, with anisotropic parameters $\delta = 0$ and $\eta = 0.2$.

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Figure 4: Traveltime parameters plotted versus p_x for isotropic (dashed line), VTI (dashed-dotted line) and DTI (solid line) models with parameters $v_0 = 2$ km/s, $\eta = 0.2$, $\delta = 0.1$ and $z = 1$ km. Note that t_0^2 for isotropic and DTI cases are the same.

Figure 5: The impulse response for the prestack DTI phase-shift migration of 5 pulses at zero offset in a vertically inhomogeneous DTI model with $v(z) = 1.5 + 0.6z$ and anisotropic parameters $\delta = 0$ and $\eta = 0.2$. The three sections represent slices in the prestack cube with the black lines representing the location of these slices relative to each other.

One can see that, unlike the TTI case, the response is symmetric because in our DTI model tilt is always normal to the reflector. The CMP sections (side plot) looks similar to the ones for the isotropic case with higher velocity for far offsets.

CONCLUSIONS

We develop a DSR formulation for a 2D DTI model based on a weak-anelliptic approximation. Assuming that the symmetry axis is normal to the reflector allows us to obtain relatively simple solutions, which is quite accurate for even strong anisotropy. In fact, the Shanks transform based approximation has relative errors of less than one percent up to almost critical values for offset component of the horizontal slowness vector p_h . For the small offsets one can use the elliptical approximation, which is simple and easy to understand. By expanding our solution into a series in p_h , we obtain simple equations for the traveltime parameters for the DTI model. A comparison between traveltime parameters for isotropic, VTI and DTI cases shows that for non-horizontal interfaces the DTI traveltime parameters differ from the isotropic and VTI ones.

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EDITED REFERENCES

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2012 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

REFERENCES

- Alkhalifah, T., 1998, Acoustic approximations for processing in transversely isotropic media: *Geophysics*, **63**, 623–631.
- Alkhalifah, T., and P. Sava, 2010, A transversely isotropic medium with a tilted symmetry axis normal to the reflector: *Geophysics*, **75**, no. 3, A19–A24.
- Belonosova, A. V., and A. S. Alekseev, 1967, About one formulation of the inverse kinematic problem of seismics for a two-dimensional continuously heterogeneous medium, *in* Some methods and algorithms for interpretation of geophysical data (in Russian): Nauka, 137–154.
- Bender, C. M., and S. A. Orszag, 1978, Advanced mathematical methods for scientists and engineers: McGraw-Hill.
- Claerbout, J. F., 1985, Imaging the earth's interior: Blackwell Scientific Publications.
- De Hoop, M. V., J. H. Le Rousseau, and B. L. Biondi, 2003, Symplectic structure of wave-equation imaging: a path-integral approach based on the double-square-root equation: *Geophysical Journal International*, **153**, 52–74.
- Golikov, P., and A. Stovas, 2011a, Traveltime parameters in a tilted elliptical anisotropic medium: *Geophysical Prospecting*, **60**, 433–443, doi: 10.1111/j.1365-2478.2011.01003.x.
- Golikov, P., and A. Stovas, 2011b, Traveltime parameters in tilted TI media: 81st Annual International Meeting, SEG, Expanded Abstracts, 315–319.
- Huang, T., S. Xu, J. Wang, G. Ionescu, and M. Richardson, 2008, The benefit of TTI tomography for dual azimuth data in Gulf of Mexico: 78th Annual International Meeting, SEG, Expanded Abstracts, 222–226.
- Popovici, A. M., 1996, Prestack migration by split-step DSR: *Geophysics*, **61**, 1412–1416.
- Reta-Tang, C., J. Simmons, W. Whiteside, J. Cai, R. Camp, and Y. He, 2011, A case study: Improved subsalt imaging through TTI model building and imaging of a WAZ survey in the Gulf of Mexico: 81st Annual International Meeting, SEG, Expanded Abstracts, 3943–3947.
- Swanston, A. M., M. D. Mathias, and C. A. Barker, 2011, Wide-azimuth TTI imaging at Tahiti: Reducing structural uncertainty of a major deepwater subsalt field: *Geophysics*, **76**, no. 5, WB67–WB78.
- Thomsen, L., 1986, Weak elastic anisotropy: *Geophysics*, **51**, 1954–1966.
- Yilmaz, O., 1979, Pre-stack partial migration: Ph.D. thesis, Stanford University.