3D Plane-wave Least-squares Kirchhoff Migration

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SUMMARY

A three dimensional least-squares Kirchhoff migration (LSM) is developed in the prestack plane-wave domain to increase the quality of migration images and the computational efficiency. Due to the limitation of current 3D marine acquisition geometries, a cylindrical-wave encoding is adopted for the narrow azimuth streamer data. To account for the mispositioning of reflectors due to errors in the velocity model, a regularized LSM is devised so that each plane-wave or cylindrical-wave gather gives rise to an individual migration image, and a regularization term is included to encourage the similarities between the migration images of similar encoding schemes. Both synthetic and field results show that: 1) plane-wave or cylindrical-wave encoding LSM can achieve both computational and IO saving, compared to shot-domain LSM, however, plane-wave LSM is still about 5 times more expensive than plane-wave migration; 2) the regularized LSM is more robust compared to LSM with one reflectivity model common for all the plane-wave or cylindrical-wave gathers.

INTRODUCTION

It has been shown that least-squares migration (Nemeth et al., 1999; Duquet et al., 2000) can provide better image quality than Kirchhoff Migration (KM) if an accurate migration velocity is used. However, one of the drawbacks of LSM is its high computational cost, especially for 3D prestack migration. Romero et al. (2000) proposed a blended source method for conventional migration by encoding and stacking different shot gathers into a supergather. This algorithm can be applied to LSM with Kirchhoff migration (Dai et al., 2011; Wang and Schuster, 2012), wave-equation migration (Huang and Schuster, 2012; Tang, 2009), and reverse time migration (Dai et al., 2010, 2012). Wave-equation LSM can achieve high computational efficiency (Dai et al., 2011; Huang and Schuster, 2012) by modeling and migrating the encoded supergather with one finite-difference solution to the wave equation for a large distribution of encoded point sources. For Kirchhoff LSM, however, the computational cost is determined by the total number of traces, which cannot be decreased by the blended encoding of shot gathers. As a less expensive alternative, a linear time-shift phase encoding which is identical to the tau-p transform, can transfer the shot-domain data to the plane-wave domain. By replacing the large number of shots with a smaller number of ray parameters, a significant computational saving is achieved for Kirchhoff LSM. For the narrow azimuth streamer data, a cylindrical-wave encoding (Duquet and Lailly, 2006; Vigh and Starr, 2008) is adopted. Our empirical tests show that 3D plane-wave LSM is about 4 times more expensive than shot domain KM, and 5 times less expensive than standard

LSM for the models tested.

Due to velocity model errors, reflectors from different plane-wave gathers can be positioned differently, so stacking the prestack migration images will blur the image of the reflectors. To overcome this problem, individual migration images are computed for different plane-wave or cylindrical-wave gathers. Since the images from plane-wave gathers with slightly different incidence angles should be similar, a regularization term is applied to encourage such similarities.

This rest of this paper is organized into the following three sections. The first part presents the theory of 3D regularized plane-wave and cylindrical-wave least-squares migration (RPWLSM and RCWLSM). The next section presents synthetic and field data results that demonstrate the efficiency and effectiveness of RPWLSM and RCWLSM. Finally, a summary is provided.

THEORY

3D plane-wave LSM

The 3D plane-wave encoding (Figure1b) can be expressed as:

$$d(x_g, y_g, t; p_x, p_y) = \sum_{x_s} \sum_{y_s} d(x_g, y_g, t; x_s, y_s) * \delta(t - p_x \cdot (x_s - x_0) - p_y \cdot (y_s - y_0)), \quad (1)$$

where the shot-domain data $d(x_g, y_g, t; x_s, y_s)$ are encoded with a time-shift function $\delta(t - p_x \cdot (x_s - x_0) - p_y \cdot (y_s - y_0))$ and stacked together. The time shift $p_x \cdot (x_s - x_0) + p_y \cdot (y_s - y_0)$ is a linear function of the projected distance between source position (x_s, y_s) and the reference position (x_0, y_0) , and p_x and p_y are the ray parameters in x and y directions.

Assuming the reflectivity model $\mathbf{m}$ is independent of the ray parameter, for a dataset with $N_{px} \times N_{py}$ plane-waves the modeling operation can be expressed as

$$[\mathbf{d}] = \begin{bmatrix} \mathbf{d}_{1,1} \\ \vdots \\ \mathbf{d}_{1,N_{py}} \\ \mathbf{d}_{2,1} \\ \vdots \\ \mathbf{d}_{N_{px},N_{py}} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_{1,1} \\ \vdots \\ \mathbf{L}_{1,N_{py}} \\ \mathbf{L}_{2,1} \\ \vdots \\ \mathbf{L}_{N_{px},N_{py}} \end{bmatrix} [\mathbf{m}], \quad (2)$$

and the migration operation can be expressed as

$$\mathbf{m}_{mig} = \begin{bmatrix} \mathbf{L}_{1,1}^T \cdots \mathbf{L}_{1,N_{py}}^T \mathbf{L}_{2,1}^T \cdots \mathbf{L}_{N_{px},N_{py}}^T \end{bmatrix} \begin{bmatrix} \mathbf{d}_{1,1} \\ \vdots \\ \mathbf{d}_{1,N_{py}} \\ \mathbf{d}_{2,1} \\ \vdots \\ \mathbf{d}_{N_{px},N_{py}} \end{bmatrix}, \quad (3)$$

where the final image is the stack of migration images from all of the individual plane-waves gathers. Here $\mathbf{d}_{i,j}$ ($\mathbf{L}_{i,j}$) represents the response of the system to (modeling operator for) the plane-wave source with the i th ray parameter in the x direction and j th ray parameter in the y direction. PWLSM is formulated to find the $\mathbf{m}$ that minimizes the misfit functional

$$f(\mathbf{m}) = \frac{1}{2} \sum_{i=1}^{N_{px}} \sum_{j=1}^{N_{py}} \|\mathbf{L}_{i,j} \mathbf{m} - \mathbf{d}_{i,j}\|^2 + \mathbf{R}, \quad (4)$$

where $\mathbf{R}$ is the regularization term, and $\mathbf{m}$ is defined as the stacked migration image.

If the migration velocity is not accurate, the prestack images from different plane-wave gathers are dissimilar and simple stacking will blur the image and slow the convergence. In order to improve the robustness of plane-wave LSM in the presence of migration velocity errors, we assume that each plane-wave gather $\mathbf{d}_{i,j}$ is associated with its own reflectivity model $\mathbf{m}_{i,j}$, so an ensemble of prestack images $\hat{\mathbf{m}}$ can be defined as

$$\hat{\mathbf{m}} = \begin{bmatrix} \mathbf{m}_{1,1} \\ \vdots \\ \mathbf{m}_{1,N_{py}} \\ \mathbf{m}_{2,1} \\ \vdots \\ \mathbf{m}_{N_{px},N_{py}} \end{bmatrix}, \quad (5)$$

and the modeling and migration equations can be expressed as

$$\begin{bmatrix} \mathbf{d}_{1,1} \\ \vdots \\ \mathbf{d}_{1,N_{py}} \\ \mathbf{d}_{2,1} \\ \vdots \\ \mathbf{d}_{N_{px},N_{py}} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_{1,1} & & & \\ & \vdots & & \\ & & \mathbf{L}_{1,N_{py}} & \\ & & & \mathbf{L}_{2,1} \\ & & & \vdots \\ & & & & \mathbf{L}_{N_{px},N_{py}} \end{bmatrix} [\hat{\mathbf{m}}], \quad (6)$$

and

$$\begin{bmatrix} \mathbf{m}_{mig,1,1} \\ \vdots \\ \mathbf{m}_{mig,1,N_{py}} \\ \mathbf{m}_{mig,2,1} \\ \vdots \\ \mathbf{m}_{mig,N_{px},N_{py}} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_{1,1}^T & & & \\ & \vdots & & \\ & & \mathbf{L}_{1,N_{py}}^T & \\ & & & \mathbf{L}_{2,1}^T \\ & & & \vdots \\ & & & & \mathbf{L}_{N_{px},N_{py}}^T \end{bmatrix} [\mathbf{d}] \quad (7)$$

Figure 1: Different encoding strategies: a) 2D plane-wave encoding, b) 3D plane-wave encoding and c) 3D cylindrical-wave encoding.

The regularized misfit functional with the ensemble of prestack images is defined as

$$f(\hat{\mathbf{m}}) = \frac{1}{2} \sum_{i=1}^{N_{px}} \sum_{j=1}^{N_{py}} \|\mathbf{L}_{i,j} \mathbf{m}_{i,j} - \mathbf{d}_{i,j}\|^2 + \mathbf{R}. \quad (8)$$

The regularization term $\mathbf{R}$ is defined as a function to penalize the difference between migration images computed with slightly different incidence angles (Dai and Schuster, 2013), and it is defined as

$$\mathbf{R} = \frac{1}{2} \gamma \sum_{i=2}^{N_{px}-1} \sum_{j=2}^{N_{py}-1} \|\mathbf{m}_{i+1,j} + \mathbf{m}_{i-1,j} + \mathbf{m}_{i,j+1} + \mathbf{m}_{i,j-1} - 4 * \mathbf{m}_{i,j}\|^2, \quad (9)$$

where γ is the damping coefficient that is chosen by trial-and-error testing. We denote this regularized plane-wave LSM as regularized plane-wave least-squares migration (RPWLSM).

3D cylindrical-wave LSM

For narrow azimuth streamer data acquired in the current 3D marine geometry, the source sampling in the crossline direction is usually too sparse to support a full plane-wave encoding, so an inline-only plane-wave encoding, which is equivalent to a cylindrical-wave encoding (Figure 1c), is adopted and can be expressed as:

$$d(x_g, y_g, t; p_x, y_s) = \sum_{x_s} d(x_g, y_g, t; x_s, y_s) * \delta(t - p_x \cdot (x_s - x_0)), \quad (10)$$

Equations 6 and 7 for the cylindrical-wave encoding can be expressed as:

$$\begin{bmatrix} \mathbf{d}_1 \\ \mathbf{d}_2 \\ \vdots \\ \mathbf{d}_{N_{px}} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_1 & & & \\ & \mathbf{L}_2 & & \\ & & \vdots & \\ & & & \mathbf{L}_{N_{px}} \end{bmatrix} \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \vdots \\ \mathbf{m}_{N_{px}} \end{bmatrix}, \quad (11)$$

and

$$\begin{bmatrix} \mathbf{m}_{mig,1} \\ \mathbf{m}_{mig,2} \\ \vdots \\ \mathbf{m}_{mig,N_{px}} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_1^T & & & \\ & \mathbf{L}_2^T & & \\ & & \ddots & \\ & & & \mathbf{L}_{N_{px}}^T \end{bmatrix} \begin{bmatrix} \mathbf{d}_1 \\ \mathbf{d}_2 \\ \vdots \\ \mathbf{d}_{N_{px}} \end{bmatrix}. \quad (12)$$

Equation 4 can be rewritten as:

$$f(\mathbf{m}) = \frac{1}{2} \sum_{i=1}^{N_{px}} \|\mathbf{L}_{i,j} \mathbf{m} - \mathbf{d}_i\|^2 + \mathbf{R}, \quad (13)$$

and we denote this as the cylindrical-wave LSM (CWLSM).

Similarly, equations 8 and 9 for a cylindrical-wave encoding can be written as:

$$f(\hat{\mathbf{m}}) = \frac{1}{2} \sum_{i=1}^{N_{px}} \|\mathbf{L}_i \mathbf{m}_i - \mathbf{d}_i\|^2 + \mathbf{R}, \quad (14)$$

and

$$\mathbf{R} = \frac{1}{2} \gamma \sum_{i=1}^{N_{px}-1} \|\mathbf{m}_i - \mathbf{m}_{i-1}\|^2. \quad (15)$$

We denote this as the regularized cylindrical-wave least-squares migration (RCWLSM).

NUMERICAL RESULTS

3D synthetic test of plane-wave LSM

The PWLSM and RPWLSM algorithms are tested on the 3D SEG overthrust model with a OBS acquisition geometry. Synthetic data are generated by a finite-difference solver for the 3D wave-equation with a 15-Hz peak-frequency Ricker wavelet. The model is 2.5 km in depth, and 5 km $\times$ 5 km on the surface. Four hundred OBS stations are evenly planted on the water bottom (0.375km), with both inline and crossline spacings of 250 m. Fifty sail lines (with a crossline spacing of 100 m) are designed to shoot on the surface, and each sail line consists of 99 shots with an inline spacing of 50 m, so the total shot number is 4950. Figure 2 shows the conventional CSG-domain KM images for three different slices. The total of 4950 CSGs are transformed into 961 plane-wave gathers, with a range of ray parameters between -0.23 ms/m and 0.23 ms/m in both the inline and crossline directions. Figures 3 - 5 show, respectively, the images of PWKM, PWLSM and RPWLSM at the same slice position. Both PWLSM and RPWLSM are stopped after 10 iterations. The PWKM image provides most of the subsurface information as the CSG KM does, but with noise caused by an insufficient plane-waves. Zhang et al. (2005) also showed that more plane-waves are needed when the source sampling is sparse. Both PWLSM and RPWLSM provide high-resolution images as delimited in the green circles. PWLSM can provide even higher resolution images than RPWLSM when the migration velocity is accurate, because the prestack images are consistent in this case. This result is consistent with our 2D plane-wave LSM test (Wang et al., 2013). A computational and IO speed up of $\frac{4950}{961} = 5.15$

Figure 2: CSG Kirchhoff migration images.

Figure 3: PW Kirchhoff migration images.

Figure 4: PWLSM images.

Figure 5: RPWLSM images.

is achieved, and the CPU costs of both PWLSM and RPWLSM after 10 iterations are 4 times that of the CSG-domain KM.

3D field data test of cylindrical-wave LSM

CWLSM and RCWLSM are tested on 3D narrow azimuth streamer data recorded over a deep body of water. The imaging model size is 5 km (Z) $\times$ 8.3 km (Y) $\times$ 1 km (X). Fifty-one cylindrical-waves with a range of ray parameters between -0.23 ms/m and 0.23 ms/m in the inline direction are constructed from 257 shots, and each shot has 2744 receivers distributed on 8 streamers. Figures 6 a)-d) show, respectively, the images of CSG-domain KM, CW KM, CWLSM and RCWLSM. All the images are at the crossline position of 0.46 km. Both CWLSM and RCWLSM are stopped after 10 iterations. CW-domain KM provides the same quality of images as CSG-domain KM, but with only $\frac{51}{257} = 20\%$ the CPU cost. CWLSM and RCWLSM can produce images of high resolution (delimited in green circles) at a 5 times the CPU cost compared to the CSG-domain KM. However, CSG KM can provide higher quality image than LSM does at some deeper depth (blue circles).

DISCUSSION AND CONCLUSION

A 3D plane-wave least-squares migration algorithm is proposed to efficiently produce high quality migration images. By transforming CSG data into the plane-wave domain, the computation and IO costs are significantly decreased. For the narrow azimuth marine acquisition, a cylindrical-wave encoding is adopted. Synthetic and field results show that PWLSM (or CWLSM) can provide high quality images, when the migration velocity is accurate, and this is consistent with our previous results for 2D plane wave migration. To improve the robustness of this algorithm, a regularized plane-wave (cylindrical-wave) least-squares migration method is proposed and shown to give the most focused images and a better convergence rate.

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Figure 6: 3D streamer data results. a) CSG KM, b) CW KM, c) CWLSM and d) RCWLSM. All the images are at the cross line position of 0.46 km

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