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Dynamics of an Imperfect Microbeam Considering its Exact Shape

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ABSTRACT

We study the static and dynamic behavior of electrically actuated micromachined arches. First, we conduct experiments on micromachined polysilicon beams by driving them electrically and varying their amplitude and frequency of voltage loads. The results reveal several interesting nonlinear phenomena of jumps, hysteresis, and softening behaviors. Next, we conduct analytical and theoretical investigation to understand the experiments. First, we solve the Eigen value problem analytically. We study the effect of the initial rise on the natural frequency and mode shapes, and use a Galerkin-based procedure to derive a reduced order model, which is then used to solve both the static and dynamic responses.

We use two symmetric modes in the reduced order model to have accurate and converged results. We use long time integration to solve the nonlinear ordinary differential equations, and then modify our model using effective length to match experimental results.

To further improve the matching with the experimental data, we curve-fit the exact profile of the microbeam to match the experimentally measured profile and use it in the reduced-order model to generate frequency-response curves.

Finally, we use another numerical technique, the shooting technique, to solve the nonlinear ordinary differential equations. By using shooting and the curve fitted function, we found that we get good agreement with the experimental data.

1. INTRODUCTION

While arch beams are similar to doubly clamped beams, they have initial rise from the straight position. This initial rise is

considered to be an imperfection due to the fabrication. When we deposit more than one material on top of each other, each layer has a different thermal expansion coefficient. This will lead to thermal stresses (stress gradient) between layers that cause one layer to expand more than the other. This is one of the reasons why an initial curvature in the microbeam may occur.

Many groups studied the non-linear behavior for macro- and micro-arch beams. Schreyer et al. [1] studied the buckling of the arches. They analyzed buckling and post-buckling positions for circular clamped-clamped arch beams by solving nonlinear equilibrium equations.

Humphreys [2] studied the dynamic of macro-arch structures and defined the arch characteristics. Hsu et al. [3] studied the stability of clamped-clamped arches due to time-wise step load. Pi et al. [4] used the nonlinear finite element model to study buckling and post-buckling behaviors of arch beams. Tien et al. [5] used the method of averaging to study the nonlinear dynamics of arches due to periodic excitation. Matsunaga [6] examined the effect of axial loads on arch beams. They used a circular simply supported arch beam that was subjected to axial load to find the natural frequencies. They proved that the first natural frequency was affected by the curvature more than the second one. Lee et al. [7] studied the response of buckled clamped-clamped beams due to a sinusoidal excitation. Sulfridge et al. [8] presented a study that showed the different types of actuation that can be used in arch beams, such as electrostatic actuation, thermal actuation, piezoelectric actuation, and mechanical actuation. Xi et al. [9] studied the stability of clamped-clamped arches due to mechanical and

thermal actuation. Poon et al. [10] used the Runge-Kutta method to study the response of a curved clamped-clamped beam excited by a sinusoidal signal, and found softening and hardening behaviors. Zhang et al. [11] conducted some experimental and theoretical investigations of initially curved clamped-clamped beams when excited by a DC load, found a good agreement for the snap through motion, and examined how this was affected by the level of curvature of the microbeams and the electrostatic load. Buchaillet et al. [12] also did experimental and theoretical investigation of the dynamic of snap through motion for initially curved clamped-clamped beams when subjected to vibration. Krylov et al. [13] experimentally demonstrated the snap-through motion and pull-in instability. The same group [14] used the Galerkin procedure to solve their model after adding geometric and electric nonlinearities. In [15], they used the phase portrait method to understand the dynamics of an arch beam when actuated by a step voltage. They also studied the transient response and escape from one potential well to the other one. Das et al. [16] used finite and boundary element methods to study the transient analysis of arch beams, and showed that arch beams might face a softening effect before having a snap through motion. Younis et al. [17] used the Galerkin procedure to discretize the equation of motion for the shallow arch. They studied both the dynamic and static responses for the micro arch, and also compared their results to experimental ones to validate their model. Ouakad et al. [18] studied the non-linear behavior of the shallow arch when actuated by both DC and AC load, used Galerkin to simulate the static response and to solve the Eigen value problem under DC load, and used the multiple scale method in perturbation to get the forced vibration under both AC and DC loads. Mergen et al. [19] studied the nonlinear behavior of an electrically actuated MEMS resonator based on modified couple stress theory; they used the pseudo-arc length technique to examine the nonlinear static and dynamic behavior of the resonator. Harne and Wang [20] presented a method for mass sensing sensor based upon the coupling of a host linear structure to small bistable inclusion. Ruzziconi et al [21] developed an analysis to predict the experimental response for a clamped-clamped micobeam with a non-negligible imperfections, they observed the softening behavior by varying the frequency and the electrodynamic voltage. Gerson et al [22] did a finite element analysis on a shallow arch to study the stability limit for this structure.

2. THE MICRO STRUCTURE

We recall the same beam from our previous work [19], which considered as a rectangular microbeam acting as a parallel plate capacitor, so the lower electrode is located underneath the upper electrode with a gap filled of air between them, Fig. 1.

Figure 1. A top-view picture of the analyzed MEMS device.

From Fig. 1. We can see that there are the actuation pads at the edges of the microbeam, which are used to connect the beam with the electrostatic force to excite it with both V_{DC} and V_{AC} . The experimental set-up used for testing the MEMS device is shown in Fig. 2. It consists of a Micro System Analyzer (MSA), under which the microchip is mounted horizontally inside a special vacuum chamber. The chamber is designed to fit directly underneath it. The chamber is equipped with a viewport window located on the top that is made of glass, which enables the laser to penetrate without any distortion. It has some ports placed in the lateral sides, which serve to supply the pressure gauge and electrical connection. Specifically, the chamber is hooked up to a high vacuum pump. The AC and DC power sources are provided either by the MSA itself, which generates different type of signals that can be used to excite the microbeam, or by generating an electrical signal using the software LabVIEW. Then, this generated signal is passed through a data acquisition card, an amplifier, and a multimeter (to ensure the exact voltage) before reaching the micro-chip.

Figure 2. Experimental set-up used for testing the device: the MSA analyzed (up right), the vacuum chamber (down right) and the monitor (left).

To acquire the beam's dimension, we test the beam under an optical interferometer profilometer (WYKO). We use this device to know the topography of the beam. The WKYO profilometer does this by reflecting a light from a reference mirror that then combines with a light reflected from the sample to produce interference fringes. The contrast of the fringes affects the focus of the image. Fig. 3 (a) shows the

overall picture, whereby different colors schematically denote different elevations. From the longitudinal profile, Fig. 3 (b), we can see that the microbeam is not straight, but has a configuration that is curled up a few microns at $x = l/2$.

Figure 3. Actual external profile of the microbeam: (a) a top-view picture of the microbeam with the two section lines where the actual profile is acquired. Colors denote the elevation in the curled-up external profile; (b) the actual external profile in the

longitudinal direction (X Profile) showing, in microns, the length of the microbeam and the curled up configuration; (c) the actual external profile in the transversal direction (Y Profile) showing, in microns, the width of the microbeam and the total amount of thickness plus gap.

We measured the microbeam and detected its natural frequencies experimentally. By connecting the chip inside the vacuum chamber, through the electrical port, we connected one of the beams to the electrical source to excite it by a weak random signal to detect its natural frequencies, Fig. 4.

Figure 4. First four measured natural frequencies by white noise, with $V_{DC} = 1\text{V}$ and $V_{AC} = 1\text{V}$.

After we excited the beam with a white noise signal to detect its natural frequencies, we applied a signal from the LabVIEW program, and varied the voltages to get more controlled response. Next, we fixed the DC voltage at 0.7 V and varied the AC voltage. Fig. 5(a) shows that the 1st natural frequency of the beam is around 148 KHz. Then we fixed the DC load at 0.7 V and started increasing the AC load, such that we notice the softening behavior in our results, as shown in Fig. 5(b) through (e).

Figure 5. Experimental measured frequency response curves at (a) $V_{AC} = 0.75$ V; (b) $V_{AC} = 1$ V. (c) $V_{AC} = 2$ V. (d) $V_{AC} = 3$ V. (e) $V_{AC} = 4$ V. The forward and backward sweep are, respectively, in red stars and blue circles.

3. THEORETICAL STUDY OF ELECTRICALLY ACTUATED MICROMACHINED ARCHES

In this section, we investigate the response of the micromachined arches theoretically when actuated by both DC electrostatic load and AC harmonic load. We use reduced order model (R.O.M) to solve the force vibration response due to DC and AC load. We use two modes of a straight beam in the (R.O.M), and then we compare it by using two arch mode shapes. We modify our model by considering an effective length that accounts for the moving pads. We also use a curve fitted function to represent the initial curvature of the beam, and compare the response by using a classical function from the literature.

3.1 PROBLEM FORMULATION

The equation governing the micro- arch is derived. Consider a clamped-clamped micro arch as seen in Fig. 6(a) The equation

$$\hat{w}_o(\hat{x}) = \frac{b_o}{2} \left[1 - \cos\left(\frac{2\pi\hat{x}}{L}\right) \right]$$

governing the initial shape is

where b_o is the initial rise from the straight position. The micro-arch is actuated by an electrostatic force applied between the upper and the lower electrodes, and acts like a parallel plate capacitor. This electrostatic force is composed of a DC component superimposed with an AC harmonic load of

amplitude V_{AC} and a frequency $\hat{\Omega}$, the gap distance between the two electrodes d , and the length of the upper electrode L , with a width of b and thickness of h . We assume a shallow arch where $(d\hat{w}_o/d\hat{x})^2 \ll 1$ [24], so the parallel plate

assumption will be valid. Thus, the nonlinear equation of motion for the transverse deflection of the arch [25],[13] is:

$$EI \frac{\partial^4 \hat{w}}{\partial \hat{x}^4} + \rho A \frac{\partial^2 \hat{w}}{\partial \hat{t}^2} + \hat{c} \frac{\partial \hat{w}}{\partial \hat{t}} = \left[\frac{\partial^2 \hat{w}}{\partial \hat{x}^2} + \frac{d^2 \hat{w}_o}{d\hat{x}^2} \right] \left[\frac{EA}{2L} \int_0^L \left\{ \left(\frac{\partial \hat{w}}{\partial \hat{x}} \right)^2 + 2 \left(\frac{\partial \hat{w}}{\partial \hat{x}} \frac{d\hat{w}_o}{d\hat{x}} \right) \right\} d\hat{x} \right] - \frac{\epsilon b [V_{DC} + V_{AC} \cos(\hat{\Omega} \hat{t})]^2}{2(d + \hat{w}_o + \hat{w})^2} \quad (1)$$

Where E is the Young's modulus (modulus of elasticity); I is the moment of inertia for the beam, considering a rectangular

cross section, $I = \frac{bh^3}{12}$; ρ is the material density; A is the

cross sectional area, $A = bh$; $\hat{c}$ is the viscous damping coefficient; and $\mathcal{E}$ is the dielectric constant (here we use it for air).

Figure 6. (a) Electrically actuated clamped-clamped arch, (b) 3-D schematic for the arch.

From Fig. 6 the boundary conditions for the clamped-clamped arch are:

$$\begin{aligned} \hat{w}(0, \hat{t}) = 0, \frac{\partial \hat{w}}{\partial \hat{x}}(0, \hat{t}) = 0, \\ \hat{w}(L, \hat{t}) = 0, \frac{\partial \hat{w}}{\partial \hat{x}}(L, \hat{t}) = 0 \end{aligned} \quad (2)$$

Next, we introduce nondimensional variables:

$$w = \frac{\hat{w}}{d}, w_o = \frac{\hat{w}_o}{d}, x = \frac{\hat{x}}{L}, t = \frac{\hat{t}}{T} \quad (3)$$

$$\text{Where; } T = \sqrt{\frac{\rho A L^4}{EI}}$$

Next, we plug Eq. (3) into Eq. (1) and Eq. (2), which yields the following nondimensional equation of motion and nondimensional boundary conditions:

$$\begin{aligned} \frac{\partial^4 w}{\partial x^4} + \frac{\partial^2 w}{\partial t^2} + C \frac{\partial w}{\partial t} = \\ \alpha_1 \left[\frac{\partial^2 w}{\partial x^2} + \frac{d^2 w_o}{dx^2} \right] \left[\int_0^1 \left\{ \left(\frac{\partial w}{\partial x} \right)^2 + 2 \left(\frac{\partial w}{\partial x} \frac{dw_o}{dx} \right) \right\} dx \right] \\ - \frac{\alpha_2 [V_{DC} + V_{AC} \cos(\Omega t)]^2}{(1 + w_o + w)^2} \end{aligned} \quad (4)$$

$$\begin{aligned} w(0, t) = 0, \frac{\partial w}{\partial x}(0, t) = 0 \\ w(1, t) = 0, \frac{\partial w}{\partial x}(1, t) = 0 \end{aligned} \quad (5)$$

$$\text{Where } w_o(x) = \frac{b_o}{2d} [1 - \cos(2\pi x)] \quad (6)$$

And the nondimensional parameters:

$$\begin{aligned} C = \frac{\hat{c} L^4}{EIT}; \alpha_1 = 6 \left(\frac{d}{h} \right)^2; \alpha_2 = \frac{\epsilon b L^4}{2EI d^3}; \\ \Omega = \frac{\hat{\Omega}}{\omega_n}; \omega_n = 1/T \end{aligned} \quad (7)$$

3.2 EIGEN VALUE PROBLEM

We solve the Eigen value problem to get the natural frequencies and mode shapes at different initial rises. Thus, we need to solve the linearized undamped unforced equation from Eq. (1) as seen in [26]. The linearized undamped unforced equation:

$$\frac{\partial^4 w}{\partial x^4} + \frac{\partial^2 w}{\partial t^2} = \alpha_1 \frac{\partial^2 w_o}{\partial x^2} \int_0^1 2 \left(\frac{\partial w}{\partial x} \frac{dw_o}{dx} \right) dx \quad (8)$$

We plug Eq. (6) into Eq. (8) and use the separation of variables technique:

$$w(x, t) = \Phi(x) e^{i\omega t} \quad (9)$$

We end up with:

$$\Phi^{iv} - \omega^2 \Phi = 4\alpha_1 \frac{b_o^2}{d^2} \pi^3 \cos(2\pi x) \int_0^1 \sin(2\pi x) \Phi' dx \quad (10)$$

Eq. (10) has a solution consisting of two parts, the homogeneous part $\Phi_h(x)$ and the particular part $\Phi_p(x)$, following [26]:

$$\Phi(x) = \Phi_h(x) + \Phi_p(x) \quad (11)$$

Consider the homogeneous part the same as the mode shape of a straight beam [26]:

$$\begin{aligned} \Phi_h(x) = A \sin(\sqrt{\omega} x) + B \cos(\sqrt{\omega} x) + \\ C \sinh(\sqrt{\omega} x) + D \cosh(\sqrt{\omega} x) \end{aligned} \quad (12)$$

Then, we solve for the particular part:

$$\Phi_p(x) = c_5 \cos(2\pi x) \quad (13)$$

Substituting Eq. (13) into Eq. (10) gives the constant c_5

$$c_5 = \frac{4\alpha_1 \frac{b_o^2}{d^2} \pi^3 \int_0^1 \sin(2\pi x) \Phi_h'(x) dx}{16\pi^4 - \omega^2 + 4\alpha_1 \frac{b_o^2}{d^2} \pi^4} \quad (14)$$

Thus, c_5 is function of the first unknowns A, B, C, and D.

$$\begin{aligned}\Phi(x) &= A \sin(\sqrt{\omega}x) + B \cos(\sqrt{\omega}x) + \\ &C \sinh(\sqrt{\omega}x) + D \cosh(\sqrt{\omega}x) + \\ &\frac{4\alpha_1 \frac{b_o^2}{d^2} \pi^3 \int_0^1 \sin(2\pi x) \Phi_h'(x) dx}{16\pi^4 - \omega^2 + 4\alpha_1 \frac{b_o^2}{d^2} \pi^4} \cos(2\pi x)\end{aligned}\quad (15)$$

$$\begin{aligned}\Phi(0) &= 0, \Phi'(0) = 0, \\ \Phi(1) &= 0, \Phi'(1) = 0\end{aligned}\quad (16)$$

To solve the Eigen value problem, we apply the boundary conditions Eq. (16). We end up with 4 algebraic equations in the following form:

$$\begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\quad (17)$$

Each element in M-matrix is a function of the natural frequency ω . The determinant of this matrix will give the natural frequency (nondimensional). Then, we can get the values of three of the unknowns A, B, C and D, and calculate the unknown C5. Finally, we solve for the mode shape that is associated with each frequency.

3.3 REDUCED ORDER MODEL (ROM)

To solve Eq. (4) and get the response of the beam $w(x, t)$ we use the Galerkin procedure to discretize the equation of the beam [23]. So the deflection of the arch is approximated as:

$$w(x, t) = \sum_{i=1}^n \Phi_i(x) u_i(t) \quad (18)$$

Where $\Phi_i(x)$ is the trial function that satisfies the boundary conditions of the arch, and $u_i(t)$ is the modal coordinate.

For $\Phi_i(x)$, we can substitute any function that satisfies the boundary conditions, but most of the functions used in the literature are either the mode shapes of a straight beam ($b_o = 0$), or the exact mode shapes of the arch itself.

The mode shapes of a straight clamped-clamped beam from can be expressed as:

$$\begin{aligned}\Phi_i(x) &= \cosh(\sqrt{\omega_{i,non}} x) - \cos(\sqrt{\omega_{i,non}} x) \\ &- \sigma_i [\sinh(\sqrt{\omega_{i,non}} x) - \sin(\sqrt{\omega_{i,non}} x)]\end{aligned}\quad (19)$$

Where;

$$\sigma_1 = 0.982502, \sigma_2 = 1.00078, \sigma_3 = 0.999966, \sigma_4 = 1$$

And,

$$\begin{aligned}\omega_{1,non} &= 22.3733, \omega_{2,non} = 61.6728, \omega_{3,non} = 120.903, \\ \omega_{4,non} &= 199.859\end{aligned}$$

To get the exact mode shapes for the arch beam, we solve the Eigen value problem as we discussed before, and then plug them back in the Galerkin Eq. (18).

To solve Eq. (4) and derive the R.O.M., we follow the following procedure:

- Multiply Eq. (4) by the denominator $(1 + w_o + w)^2$, because from a computational point of view, it is too expensive to integrate a numerator over a denominator [28].

Eq. (4) becomes:

$$\begin{aligned}(1 + w_o + w)^2 \left(\frac{\partial^4 w}{\partial x^4} + \frac{\partial^2 w}{\partial t^2} + C \frac{\partial w}{\partial t} \right) = \\ \alpha_1 (1 + w_o + w)^2 \left[\frac{\partial^2 w}{\partial x^2} + \frac{d^2 w_o}{dx^2} \right] * \\ \left[\int_0^1 \left\{ \left(\frac{\partial w}{\partial x} \right)^2 + 2 \left(\frac{\partial w}{\partial x} \frac{dw_o}{dx} \right) \right\} dx \right] \\ - \alpha_2 [V_{DC} + V_{AC} \cos(\Omega t)]^2\end{aligned}\quad (20)$$

- Plug Eq. (18) into Eq. (20), and this will give us the following equation:

$$\begin{aligned}\left(1 + w_o + \sum_{i=1}^n \Phi_i(x) u_i(t) \right)^2 * \\ \left(\sum_{i=1}^n \Phi_i'''(x) u_i(t) + \sum_{i=1}^n \Phi_i(x) \ddot{u}_i(t) \right. \\ \left. + C \sum_{i=1}^n \Phi_i(x) \dot{u}_i(t) \right) = \\ \alpha_1 \left(1 + w_o + \sum_{i=1}^n \Phi_i(x) u_i(t) \right)^2 * \\ \left[\sum_{i=1}^n \Phi_i''(x) u_i(t) + \frac{d^2 w_o}{dx^2} \right] * \\ \left[\int_0^1 \left\{ \left(\sum_{i=1}^n \Phi_i'(x) u_i(t) \right)^2 \right. \right. \\ \left. \left. + 2 \left(\sum_{i=1}^n \Phi_i'(x) u_i(t) \frac{dw_o}{dx} \right) \right\} dx \right] \\ - \alpha_2 [V_{DC} + V_{AC} \cos(\Omega t)]^2\end{aligned}\quad (21)$$

- Multiply Eq. (21) by $\Phi_i(x)$ and integrate from 0 to 1, and we end up with a differential equation in terms of the modal coordinate $u_i(t)$.

- We solve for $u_i(t)$ and multiplied it with $\Phi_i(x)$ to get the whole response $w(x,t)$ from Eq. (18).

One method used to validate how many modes we should use in the force vibration is to solve the natural frequency using the R.O.M. We used one symmetric mode in the R.O.M. When we compared it to the exact solution (Eigen value problem) we found a discrepancy. We solved it again by adding another symmetric mode, which gives us better agreement to the exact solution. We also solved it using one and two modes from the arch beam mode shape. Fig. 7 shows this comparison. We can note that using one or two arch modes or two straight modes should lead to good results. We decided to use the arch modes (the first mode and the third mode of an arch beam).

Figure 7. Comparison between the exact solution and the R.O.M. in solving for the frequency vs. initial rise, using straight beam mode shapes and the arch mode shapes.

4. COMPARISON BETWEEN THEORY AND EXPERIMENT

In this section, we try to match experimental data with results generated from our model. Assuming the same beam length as that obtained from the WYKO, we could not get a good agreement with the measurement. However, we noticed that part of the pad is moving as we excite the arch with the electrostatic force. We modified our model by considering the moving part as a part of the arch. This means that the length of the beam that we measured from the WYKO is not the effective length. We tried more than one value until we had acceptable results.

First, we solved the Eigen value problem for our arch in order to know its theoretical natural frequency and compare it to the experimental one. Fig. 8 shows the first mode of our arch with its non-dimensional natural frequency.

Figure 8. First mode for our arch, and its corresponding natural frequency $\omega_n=67.77$.

We converted this nondimensional frequency to the dimensional one $\omega_{Hz} = \frac{\omega_n}{2\pi} \sqrt{\frac{EI}{\rho AL^4}}$. Substituting our values

will give $\omega_{Hz} = 288,716$ Hz, which is far from the experimental one (148250 Hz). We tried more than one value until we reached very close number to the experimental one. The effective length we used was $L_{eff}=1.395 \times \text{original length}$. To simulate the forced vibration using the R.O.M., we have to calculate the yet-unknown damping term, the quality factor (Q) is used to calculate the damping based on Fig. 4. Based on this we found $Q = 974$.

We used these values and the effective length by applying different values of V_{DC} and V_{AC} to simulate the forced response and compare to experimental data. Fig. 9 shows both the simulation and the experimental results. The first two symmetric arch beam mode shapes were used in the following simulation. Note that we used the abbreviation L-T I (Long-Time integration) in the legend of the following figures.

Figure 9. The measured frequency response curve and the simulated one of an arch (a) $V_{AC} = 1V$. (b) $V_{AC} = 2V$. (c) $V_{AC} = 3V$. (d) $V_{AC} = 4V$. (e) $V_{AC} = 5V$.

Fig. 9 shows qualitative matching with the experiment, but not quantitative matching. Therefore, we need to enhance the results more in order to aim toward better matching. One way to do this is to use a curve-fitted function that represents the exact initial profile for the arch instead of the assumed one of Eq. (6). We discretized the initial profile using the WYKO images as shown in Fig. 10, and then used to curve fit the data and have a profile function to use it as new $w_o(x)$ Eq. (22).

Figure 10. Plot of the discretized points based on the measured arch profile.

The function that we obtained is expressed as:

$$W_o(x) = -0.0940438 + 3.94325x + 40.1152x^2 + 15.9357x^3 - 254.592x^4 + 280.601x^5 - 85.9134x^6 \quad (22)$$

The following figures show both the simulation and the experimental results, and compare them using the curve-fitted function Eq. (22) and using Eq. (6).

Figure 11. The measured frequency response curve and the simulated one of an arch (a) $V_{AC} = 1V$. (b) $V_{AC} = 2V$. (c) $V_{AC} = 3V$. (d) $V_{AC} = 4V$. (e) $V_{AC} = 5V$, using the curve fitted

function. The abbreviation L-T I C-F stands for (Long-Time integration curve fitted function).

Fig. 11 shows qualitative matching with the experiment, but not quantitative matching. It shows that using the curve-fitted equation enhanced the results a little bit; the upper curves become closer to the experimental one, but still the discrepancy is considerable.

5. THE SHOOTING TECHNIQUE

In the previous simulations, we used the long-time integration technique to solve the differential equations; this technique depends on the domain of the attraction. To have a stable solution, the domain should be big enough; otherwise, this technique will lead to an unstable solution, which is not necessary to be true (it might not be physically unstable). In order to predict the entirety of the solutions, we need to use another numerical technique to solve the differential equations. One of the most robust techniques is the shooting technique, which can capture the periodic solution more accurately than long-time integration, and can predict stable and unstable solutions better. We followed the procedure of [29], which clarified this method in detail.

Next, we show results using the classical shape of the initial curvature, Eq. (6), and then compare to the results obtained using the exact shape, Eq. (22). the results are depicted in Fig. 12. The figure compares frequency-response curves obtained using the shooting technique for several values of V_{DC} and V_{AC} , to those obtained experimentally.

Figure 12. The measured frequency response curve and the simulated one of an arch (a) $V_{AC} = 1V$. (b) $V_{AC} = 2V$. (c) $V_{AC} = 3V$. (d) $V_{AC} = 4V$. (e) $V_{AC} = 5V$, using shooting technique.

Fig. 12 shows much better match with the experiments using the shooting technique. They also show the difference between using the curve-fitted function to the classical one, and how the results were improved significantly by using the fitted one. This indicates the importance of accounting for the exact shape of the arch in the model.

It may also be noted that there is still a difference between the simulated results and the experimental ones, and the upper simulated curve extended more than the experimental ones.

This can be explained by analyzing the basin of attraction of these upper curves [30].

6. SUMMARY AND CONCLUSIONS

In this study we investigated the dynamic behavior of an imperfect (arch) microbeam theoretically and experimentally. We used several analytical techniques to match the theoretical results with the experimental measurements of the frequency-response curves of the arch. We found that using an effective length of the arch, to account for the flexible pads, the shooting technique, to capture most of the resonant curves especially near bifurcation points, and the exact shape of the arch, as captured through measurements, lead to improved matching with the experiments. Future research needs to be conducted to perform basin of attraction analysis to further improve the matching.

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